MA5630 Numerical Optimization Course

Instructor: Prof. Allan A. Struthers

A Term Project report on

State Estimation of Power Systems Using Weighted Least Squares Optimization Technique

Submitted by

Alorf, Abdulaziz A.

Banerjee, Bibaswan

Spring 2011

Michigan Technological University

Table of Contents

1. Background and Introduction..3
    1.1 Background 3
    1.2 Introduction..3

2. Literature Review 5
    2.1 Past Research 5
    2.2 State Estimation Methods...5

3. Approach 7
    3.1 Objective..7
    3.2 Method used for implementation 7

4. Results..9
    4.1 System model for the state estimation problem using WLS method..9
    4.2 Obtained Results for the case when no data is missing 12
    4.3 Obtained results for the missing data cases.....................................12

5. Conclusion 15

6. References...16

Chapter 1 – Background and Introduction

1.1 Background:

State Estimation is the process of assigning a value to an unknown system state variable based on measurements from that system according to some criteria. A commonly used and familiar criterion is that of minimizing the sum of the squares of the differences between the estimated and measured values of a function. The idea of this least squares estimation have been known and used since the early part of the nineteenth century. The major developments in this area have taken place in the twentieth century in a in the aerospace field. In these developments the basic problems have involved the location of an aerospace vehicle and the estimation of its trajectory given redundant and imperfect measurements of its position and velocity vector. In many applications these measurements are based on optical observations and/or radar signals that may be contaminated with noise and may contain system measurement errors. State estimators may be both static and dynamic. Both types of estimators have been developed for power systems.

As the size and complexity of transmission systems have increased over the past few years, electric utilities have experienced a need to increase the number of real time electrical measurements to be used for monitoring and control. One natural approach to this problem would be to measure every variable of interest. However, this is not only expensive but unnecessary since many variables can be calculated from others using a digital computer on an on-line basis. While there are many variables that provide useful information about a transmission system, two of particular interest are the voltage magnitude and phase angle at each bus. In addition to being useful quantities in themselves, these variables can be used directly to calculate such information as power injected at the buses and line flow power levels. A number of papers on this subject have been proposed and published recently and several iterative techniques have been proposed for performing these calculations [1-11].

1.2 Introduction:

The state vector of a system is a non-redundant set of variables that completely describes the state of the system. In a power system this state vector is composed of the voltage magnitude and the phase angle at each of the buses (nodes) of the system. A static state estimator is a digital data processing algorithm that converts telemetered measurements taken at the nodes of the transmission network, into the “best" estimate of the state vector. The computation time required for solving complex equations is more than that needed for solving similar real equations. This project presents an efficient method in which the least square estimation technique is used to find the voltage magnitudes and angles. The elements of the state vector can be estimated from various sets of measurements. The direct measurement of the phase angle at each bus is considered economically unfeasible. It is more practical to measure the active and reactive power flow in each line. If these power flows are measured at the two ends of each line there is sufficient redundancy to detect bad data and for filtering measurement error. Advantages of using redundant power flow measurements to estimate the state vector include:

1. High accuracy of the measurements.
2. Ease of deriving the state vector from measurements.
3. Ease of detecting and identifying bad data.
4. Relative insensitivity of the state estimation to measurement errors.

Chapter 2 – Literature Review

2.1 Past Research:

The power system state estimation (PSEE) problem has been the subject of many works since the late sixties. Prof. Schweppe, the leading researcher of the Power Systems Engineering Group at MIT, was the first to propose and develop the idea of state estimation for power systems monitoring. Since then, the subject has called the attention of many researchers from universities, research centers and industry. The first paper directly related to this topic was published as early as January 1970 by F. C. Schweppe, J. Wildes and D. Rom, and at the same time also by R. E. Larson, W. F. Tinney, J. Peschon, L. P. Hajdu, and D. S. Piercy. The fundamental approach to state estimation is to use Weighted Least Squares (WLS) method. Since the publication of Schweppe's paper several different alternatives to the WLS approach have been investigated. Among these are the usage of maximum likelihood criterion, minimum variance criterion, sequential estimators, transformation methods and fast decoupled estimators. A number of papers have also been devoted to the study of bad data processing starting from 1975 to the recent years. In the recent years Genetic Algorithm (GA) application to state estimation is also gaining much popularity.

2.2 State Estimation Methods:

The past and recent works related to state estimation methods have witnessed several different techniques to bring about the process of calculation and giving an estimate of the state of the power system. Many iterative techniques have been proposed for the purpose. The fundamental approach to state estimation is the basic weighted least squares (WLS) method. Other methods of state estimation like the maximum likelihood criterion, minimum variance criterion, sequential estimators, transformation methods, fast decoupled estimators and Genetic algorithm application to state estimation are prominently used.

Regardless of which technique is chosen, it will be quite important to determine how errors in the model of the system will affect the resulting state estimates. A sensitivity analysis of this nature will indicate the accuracy that can be obtained and will show which parameters will have' a critical effect. Indeed, without performing such a study it is quite difficult to determine how much confidence should be placed in the state estimates or how they will affect the subsequent calculation of other variables, such as unmeasured power levels.

The sensitivity analysis described here examines the statistical weighted least-squares method of state estimation. Some of the advantages of this method are stated below:

1. The properties of the optimum estimate are well defined, i.e., assuming the measurement errors are Gaussian, the average error will be zero and the dispersion of the individual errors about this average will be a minimum.
2. There is no need to model the time behavior of either the system or the measurement noises (which, incidentally, eliminates an extensive amount of conjecture).
3. It is possible to achieve reasonable computation time and memory requirements by using sparse matrix methods similar to those found in load flow programs.

4. The method is not restricted to any one type of measurement; i.e., bus injections, line flows, and voltage magnitudes can all be used in various combinations.

Chapter 3 – Approach

3.1 Objective:

The objective for this project is to provide an estimate of the system state using the Weighted Least Squares (WLS) method.

3.2 Method used for implementation:

This section describes the Weighted Least Squares (WLS) method for state estimation in detail.

The objective of WLS method for state estimation is to minimize the sum of the squares of the weighted deviations of the estimated measurements from the actual measurements. WLS involves solving the solution for normal equations by iterative procedure. The objective function to be minimized is

$$J(\hat{x}) = \sum_{i=1}^{m} \frac{(z_i - h_i(\hat{x}))^2}{\sigma_i^2}$$

Where,

$\hat{x}$ is the estimated state vector of dimension n.

$h_i(\hat{x})$ is the estimated measurement i.

z_i is the measure value of the measurement i.

$\sigma_i^2 = R_{ii}$ is the variance in the error in the measurement i.

It is necessary to generate an initial state vector x^0. When using power flow, this will correspond to flat voltage profile.

1. Start iterations, set the iteration index k=0.

2. Initialize the state vector x^k, typically, as a flat start.
3. Calculate the gain matrix, $G(x^k)$.
4. Calculate the right hand side $t^k = H(x^k)^T R^{-1}\left(z - h(x^k)\right)$.
5. Decompose $G(x^k)$ and solve for Δx^k.
6. Test for convergence, max $\left|\Delta x^k\right| < \varepsilon$?.
7. If no, update $x^{k+1} = x^k + \Delta x^k$, $k = k + 1$, and go to step 3. Else, Stop!.

The above algorithm involves the following computations in each iteration, k:

1. Calculation of the expression $H(x^k)^T R^{-1}(z - h(x^k))$.
   a. Calculating the measurement function $h(x^k)$.
   b. Building the measurement Jacobian $H(x^k)$.
2. Calculation of $G(x^k)$ and solution of the equation $t^k = H(x^k)^T R^{-1}(z -$ hxk.
   a. Building the gain matrix, $G(x^k)$.
   b. Decomposing $G(x^k)$ into its Cholesky factors.
   c. Performing the forward/back substitution to solve for Δx^{k+1}.

Chapter 4 – Results

4.1 System model for the State Estimation Problem using WLS method IEEE 14 bus system from [11]:

The IEEE 14 bus system is shown in the following figure:

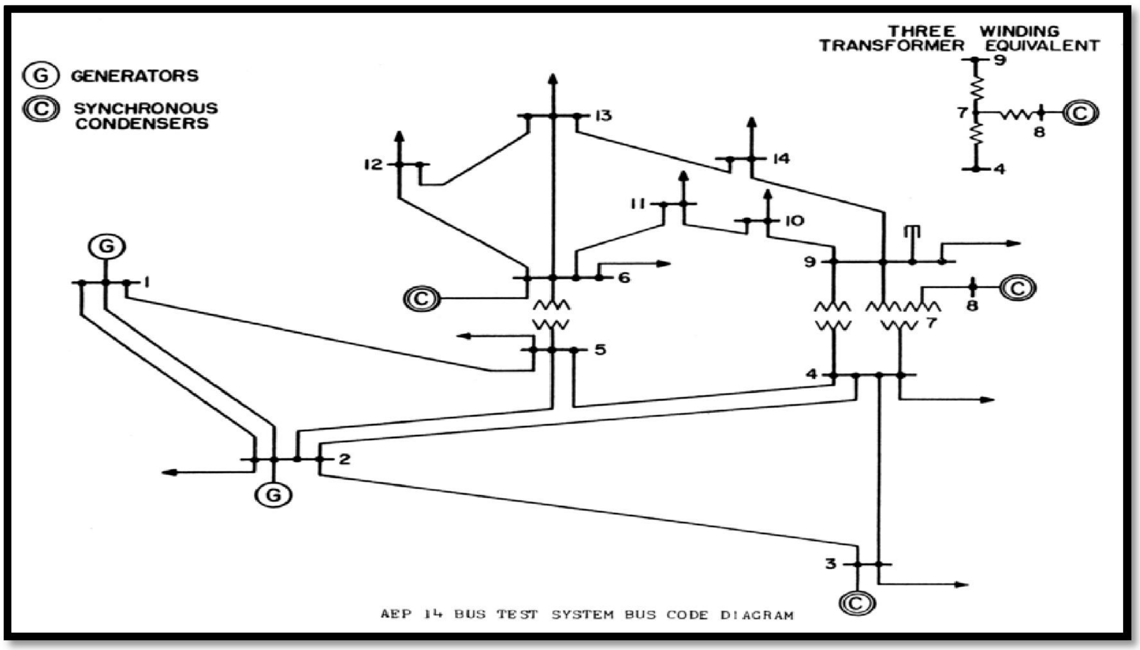

Figure 4.1: IEEE 14 bus system [11].

The system considered for the state estimation is the IEEE 14 bus system in [11] shown in Fig 4.1. The line data and the bus data for the system are available in [11] as shown in the Tables 4.1 and 4.2. The measured data (Zdata) for the problem is obtained from a power flow solution based on the system data of Tables 4.1 and 4.2. To the power flow solution a normally distributed random noise is added. The noise is assumed to have a zero mean and a standard deviation of the form of σ. The measurement data with this error component σ is given in the Table 4.3.

The line data for IEEE 14 bus system is shown in the following table:

Table 4.1: IEEE 14 bus system line data.

Sr. No.	From Bus	To Bus	R (p.u)	X (p.u)	B/2 (p.u)	Transformer tap (a)
1	1	2	0.01938	0.05917	0.0264	1
2	1	5	0.05403	0.22304	0.0246	1
3	2	3	0.04699	0.19797	0.0219	1
4	2	4	0.05811	0.17632	0.0170	1
5	2	5	0.05695	0.17388	0.0173	1
6	3	4	0.06701	0.17103	0.0064	1
7	4	5	0.01335	0.04211	0.0000	1
8	4	7	0.0000	0.20912	0.0000	0.978
9	4	9	0.0000	0.55618	0.0000	0.969
10	5	6	0.0000	0.25202	0.0000	0.922
11	6	11	0.09498	0.19890	0.0000	1
12	6	12	0.12291	0.25581	0.0000	1
13	6	13	0.06615	0.13027	0.0000	1
14	7	8	0.0000	0.17615	0.0000	1
15	7	9	0.0000	0.11001	0.0000	1
16	9	10	0.03181	0.08450	0.0000	1
17	9	14	0.12711	0.27038	0.0000	1
18	10	11	0.08205	0.19207	0.0000	1
19	12	13	0.22092	0.19988	0.0000	1
20	13	14	0.17093	0.34802	0.0000	1

The bus data for IEEE 14 bus system is shown in the following table:

Table 4.2: IEEE 14 Bus data.

Bus	Type	Vsp	θ	PGi	QGi	PLi	QLi	Qmin	Qmax
1	1	1.060	0	0	0	0	0	0	0
2	2	1.045	0	40	42.4	21.7	12.7	-40	50
3	2	1.010	0	0	23.4	94.2	19.0	0	40
4	3	1.0	0	0	0	47.8	-3.9	0	0
5	3	1.0	0	0	0	7.6	1.6	0	0
6	2	1.070	0	0	12.2	11.2	7.5	-6	24
7	3	1.0	0	0	0	0.0	0.0	0	0
8	2	1.090	0	0	17.4	0.0	0.0	-6	24
9	3	1.0	0	0	0	29.5	16.6	0	0
10	3	1.0	0	0	0	9.0	5.8	0	0
11	3	1.0	0	0	0	3.5	1.8	0	0
12	3	1.0	0	0	0	6.1	1.6	0	0
13	3	1.0	0	0	0	13.5	5.8	0	0
14	3	1.0	0	0	0	14.9	5.0	0	0

Where,

Slack/Swing bus – Type 1

Load bus/PQ bus – Type 2

Generator bus/PV bus – Type 3

The measured data for IEEE 14 bus system is shown in the following table:

Table 4.3: IEEE 14 bus system measurement data.

Sr. No	Measurement type	Value (p.u)	From bus	To bus	$\sigma_i = \sqrt{R_{ii}}$
1	1	1.06	1	0	9e-4
2	2	0.1830	2	0	1e-4
3	2	-0.9420	3	0	1e-4
4	2	0.00	7	0	1e-4
5	2	0.00	8	0	1e-4
6	2	-0.0900	10	0	1e-4
7	2	-0.0350	11	0	1e-4
8	2	-0.0610	12	0	1e-4
9	2	-0.1490	14	0	1e-4
10	3	0.3523	2	0	1e-4
11	3	0.0876	3	0	1e-4
12	3	0.0000	7	0	1e-4
13	3	0.2103	8	0	1e-4
14	3	-0.0580	10	0	1e-4
15	3	-0.0180	11	0	1e-4
16	3	-0.0160	12	0	1e-4
17	3	-0.0500	14	0	1e-4
18	4	1.5708	1	2	64e-6
19	4	0.7340	2	3	64e-6
20	4	-0.5427	4	2	64e-6
21	4	0.2707	4	7	64e-6
22	4	0.1546	4	9	64e-6

23	4	-0.4081	5	2	64e-6
24	4	0.6006	5	4	64e-6
25	4	0.4589	5	6	64e-6
26	4	0.1834	6	13	64e-6
27	4	0.2707	7	9	64e-6
28	4	-0.0816	11	6	64e-6
29	4	0.0188	12	13	64e-6
30	4	0.05634	13	14	64e-6
31	5	-0.1748	1	2	64e-6
32	5	0.0594	2	3	64e-6
33	5	0.0213	4	2	64e-6
34	5	-0.1540	4	7	64e-6
35	5	-0.0264	4	9	64e-6
36	5	-0.0193	5	2	64e-6
37	5	-0.1006	5	4	64e-6
38	5	-0.2084	5	6	64e-6
39	5	0.0998	6	13	64e-6
40	5	0.1480	7	9	64e-6
41	5	-0.0864	11	6	64e-6
42	5	0.0141	12	13	64e-6
43	5	0.01694	13	14	64e-6

Note: 1 = Voltage, 2 = Real power injection, 3 = Reactive power injection, 4 = Real power flow, 5 = Reactive power flow

4.2 Obtained results for the case when no data is missing:

Result obtained for IEEE 14 bus system for the reference case is shown in the following table.

Table 4.4: WLS result for IEEE 14 bus system.

Bus No.	Voltage (p.u)	Bus Angles θ (deg)
1	1.0084	0.0000
2	0.9916	-5.5085
3	0.9535	-14.1561
4	0.9596	-11.3767
5	0.9631	-9.7254
6	1.0184	-16.0092
7	0.9945	-14.7122
8	1.0313	-14.7130
9	0.9793	-16.4687
10	0.9780	-16.6975
11	0.9939	-16.4770
12	1.0004	-16.9482
13	0.9932	-16.9807
14	0.9689	-17.8608

Note: The results (MATLAB output) are shown in Appendix 2.

4.3 Obtained results for the missing data cases:

I. All Data for Bus No. 14 missing:

The results are shown in the following table:

Table 4.5: WLS result for IEEE 14 bus system for missing data I.

Bus No.	Voltage (p.u)	Bus Angles θ (deg)
1	1.0068	0.0000
2	0.9899	-5.5265
3	0.9518	-14.2039
4	0.9579	-11.4146
5	0.9615	-9.7583
6	1.0185	-16.0798
7	0.9919	-14.7510
8	1.0287	-14.7500
9	0.9763	-16.5125
10	0.9758	-16.7476
11	0.9932	-16.5397
12	1.0009	-17.0203
13	0.9940	-17.0583

Note 1: The code has provision for any missing data. Here since all the data pertaining to bus no 14 is missing, the system matrix is reconstructed ignoring those measurements and give rise to the estimate for the remaining 13 buses. For this case, the estimate for the bus no 14 will be of no use and will be incorrect. It may be observed that the estimates for this missing data case in table 4.5 when compared to those in table 4.4 are almost similar for the remaining buses.

Note 2: The results (MATLAB output) are shown in Appendix 2.

II. Some Random Line Measurement Data is missing:

The results are shown in the following table:

Table 4.6: WLS result for IEEE 14 bus system for missing data II.

Bus No.	Voltage (p.u)	Bus Angles θ (deg)
1	1.0308	0.0000
2	1.0141	-5.2644
3	0.9736	-10.8803
4	0.9829	-10.8803
5	0.9865	-9.3051
6	1.0408	-11.1377
7	1.0138	-14.0619
8	1.0492	-14.0608
9	0.9975	-15.7489
10	0.9978	-14.7092
11	1.0220	-11.6952
12	1.0238	-12.0232
13	1.0172	-12.0531
14	1.0068	-12.7111

Note 1: Some random measurements are considered to be missing. Care was taken such that the system matrix does not become singular because of these missing measurements. The code has provision for reconstructing the system Jacobian and then finding out the estimates. However, it may be noted that the estimates in Table 4.4 for the reference case and those in the Table 4.6 are somewhat different from each other. This points out to the

fact that the estimates when some random measurement is not considered may not be very accurate because we are relying on lesser number of measured data for the estimate.

Note 2: The results (MATLAB output) are shown in Appendix 2.

A plot showing the variation of the objective function with iteration is shown in the following figure:

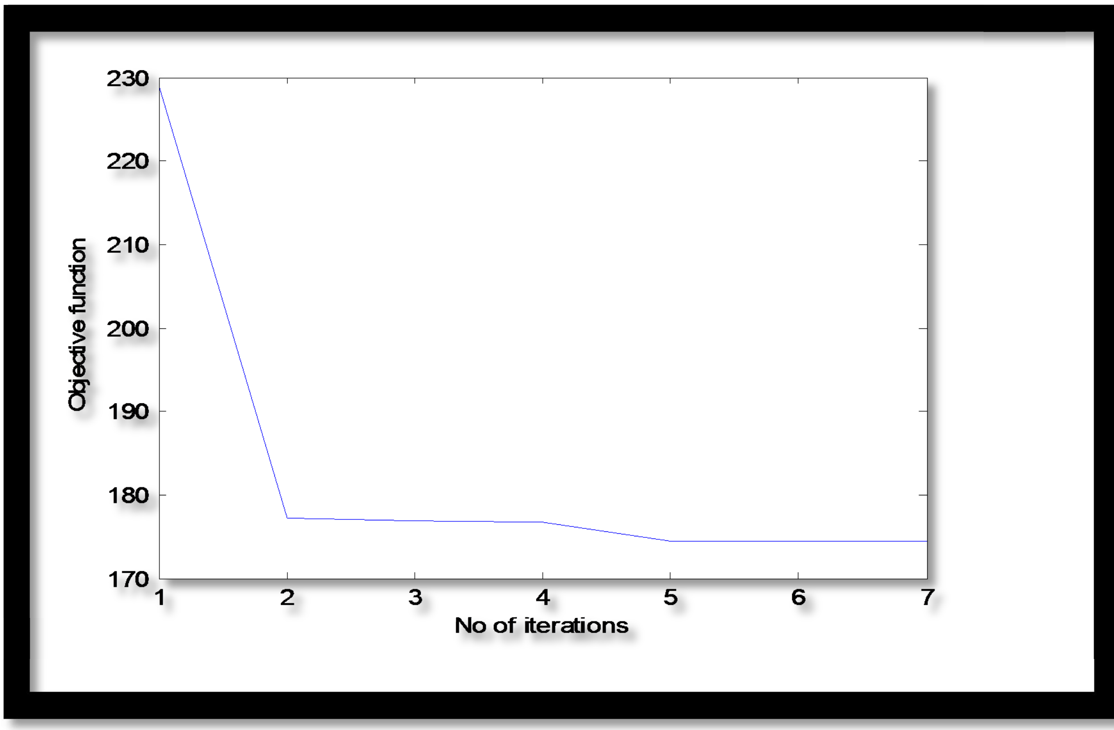

Figure 4.2: Minimization of the Objective function.

It can be seen from the above plot that the objective function reaches its minimum value after 6 iterations which is an acceptable solution for any Newton-Raphson based iterative method for power flow and estimation methods. Initially the value of the function is high but after the first iteration, the code brings down the value relatively close to the solution.

The check for the code is that the bus voltage magnitudes should lie between 0.95 pu to 1.05 pu. The following Fig 4.3 illustrates the fact that for the set of measurements considered and the code used, the voltage magnitudes for all the 14 buses lie between this threshold pointing to the fact that the system is running under normal and stable operating conditions. A deviation from this threshold would suggest a problem in the system.

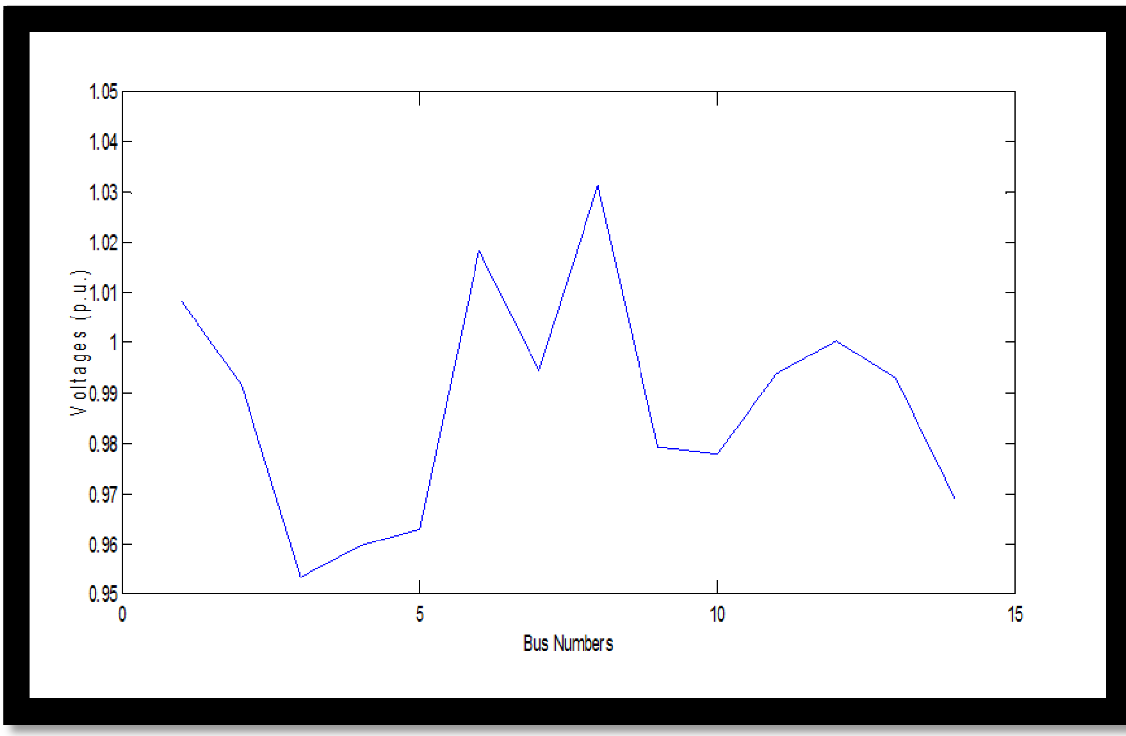

Figure 4.3: Bus Voltage magnitudes for complete measured data

Chapter 5 –Conclusion

5.1 Conclusions:

The following conclusions can be drawn after observing and analyzing the results obtained:

1. The S.E. problem can be solved by WLS method with a good amount of accuracy.
2. However, the presence of any bad measurement cannot be identified in this algorithm.
3. When the measurements pertaining to one particular bus is missing the WLS algorithm reconstructs the system and give the estimates for the remaining buses.
4. When some random measurements are missing, they are not considered, keeping in mind that the system matrix does not become singular, and the WLS algorithm reconstructs the system and forms the estimates. These estimates when compared to the reference case (when all data are present), do not however give very good estimates as we are relying on lesser number of measured data for the estimates and this is bound to have some amount of error incorporated in it.

References:

[1] Power Generation Operation and Control, Allen J. Wood, Bruce F. Wollenberg, Wiley and Sons, 1984.

[2] Power System State Estimation, Theory and Implementation, Ali Abur, Antonio Go`mez Expo`sito,Marcel Dekker, Inc., 2004.

[3] Magdy A. Fahim, C. N. Weygandt, Senior Member, IEEE, K. A. Fegley, Fellow, IEEE, 'Design of Power System State Estimator', *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-100, No. 6, June 1981, pp 3077 –3086.

[4] M. E. El-Harway, Editor, 'Bad Data Detection of Unequal Magnitudes in State Estimation of Power systems', *Power Engineering Letters, IEEE Power Engineering Review*, April 2002.

[5] F. Broussolle, 'State Estimation in Power systems: Detecting Bad Data through the Sparse Inverse Matrix Method', *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-97, No. 3, May/June 1978, pp. 678 – 682.

[6] Robert E. Larson, Member IEEE, William F. Tinney, Senior Member, IEEE, Laszlo P. Hajdu, Senior Member, IEEE, and Dean S. Piercy, 'State Estimation in Power Systems Part II: Implementation and Application', *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-89, No. 3, March 1970, pp. 353 – 363.

[7] Computational Methods for Electric Power Systems, Mariesa Crow, CRC press, 2003.

[8] Computer Aided Power Systems Analysis, George L. Kusic, Prentice Hall, Englewood Cliffs, New Jersey, 1986.

[9] Calculations and Programs for Power System Networks, Y. Wallach, Prentice Hall, Englewood Cliffs, New Jersey, 1986.

[10] Fred C. Schweppe, *Member*, *IEEE*, and Edmund J. Handschin, *Member*, *IEEE*, ‘Static State Estimation in Electric Power Systems’, *Proceedings of the IEEE*, vol. 62, No. 7, July 1984, pp. 972 – 982.

[11] http://www.ee.washington.edu/research/pstca/pf14/pg_tca14bus.htm