# MA2160 Sections 6 and 8 Exam 3

April 21, 2004

Do all problems

Show all work

1. Evaluate the following series exactly.

   (a) $\Sigma_{n=5}^{\infty}\left(\frac{1}{3}\right)^n$

   (b) $1+3+\frac{3^2}{2!}+\frac{3^3}{3!}+\frac{3^4}{4!}+\cdots$

2. Find the first four non-zero terms in the Taylor expansion of $f(x) = cosx$ about $x = \pi/4$.

3. (a) Find the first four non-zero terms in the Taylor expansion of $f(x) = \frac{1}{1-x}$ about $x = 0$.

   (b) Integrate the formula you found in part 3(a) to find the first four non-zero terms in the Taylor expansion of $f(x) = ln(1-x)$ about $x = 0$.

   (c) Use the formula you found in part 3(b) to find an approximation of $ln(.9)$.

4. Using the slope field below for the differential equation $y' = 1 + xy$, plot the solution that passes through the point $(1, 1)$.

5. Solve the initial value problem $y' = -x^2/y^2, y(0) = 1$ exactly.

6. (a) Write down the formula that is used in the Euler approximation to the solution of the initial value problem $y' = f(x, y), y(x_0) = y_0$.

(b) Consider the initial value problem $y' = x + 1, y(0) = 1$. Use Euler's method with two steps to find an approximation to the solution of the problem at $x = 0.2$

(c)Find the exact solution of the problem in part (b). What is $y(.2)$? What is the error in the approximation of part (b) (that is, $|y(x_2) - y_2|$)?

7. At time $t = 0$, a bottle of juice at $80°F$ is placed in a mountain stream whose temperature is $50°F$. After 4 minutes, its temperature is $72°F$. Let $H(t)$ denote the temperature of the juice at time $t$, in minutes.

(a) Write a differential equation for $H(t)$, using Newton's Law of Cooling.

(b) Solve the differential equation, showing the steps.

(c) When will the temperature of the juice reach $60°F$?