Spring Semester 2007

MA2160 Final Exam - PART 1

Calculators are NOT allowed on Part 1.

Turn in Part 1 BEFORE you take out your calculator.

Name:(print) Solutions ID#: ______

Score - Part 1: ___ / 50

Score - Part 2: ___ / 50

Score - Total: ___ / 100

Circle your section number:

Section	Time	Instructor	Section	Time	Instructor
01	08:05	S. Tao	07	11:05	A. Niu
02	12:05	H. Wang	08	12:05	S. Butler
03	09:05	R. Targove	09	14:05	K. Feigl
04	08:05	D. Yorgov	10	15:05	L. Erlebach
05	10:05	A. Roy	11	16:05	L. Erlebach
06	11:05	H. Wang	12	14:05	S. Butler

- Justify all answers and show all work! No work, no credit!
- Frame your answers.

1. (5 pts) Evaluate $\int \frac{x+2}{2x^2+3x+1}\,dx$. Partial Fractions

$$\frac{x+2}{(2x+1)(x+1)} = \frac{A}{2x+1} + \frac{B}{x+1} = \frac{A(x+1)+B(2x+1)}{(2x+1)(x+1)}$$

Equate numerators: $x+2 = A(x+1) + B(2x+1)$

$x=-1$: $-1+2 = B(-2+1) \Rightarrow 1 = -B \Rightarrow B=-1$

$x=-1/2$: $-\frac{1}{2}+2 = A(-\frac{1}{2}+1) \Rightarrow \frac{3}{2} = \frac{1}{2}A \Rightarrow A=3$

$$\int \frac{3}{2x+1}\,dx - \int \frac{1}{x+1}\,dx = \boxed{\frac{3}{2}\ln|2x+1| - \ln|x+1| + C}$$

2. (6 pts) Evaluate $\int_0^1 x^3\sqrt{x^4+2}\,dx$. substitution

$w = x^4+2$

$dw = 4x^3\,dx \Rightarrow \frac{1}{4}\,dw = x^3\,dx$

$$\int x^3\sqrt{x^4+2}\,dx = \frac{1}{4}\int w^{1/2}\,dw = \frac{1}{4}\cdot\frac{2}{3}w^{3/2}+C = \frac{1}{6}w^{3/2}+C$$

$$= \frac{1}{6}(x^4+2)^{3/2}+C$$

$$\Rightarrow \int_0^1 x^3\sqrt{x^4+2}\,dx = \frac{1}{6}(x^4+2)^{3/2}\Big|_0^1 = \boxed{\frac{1}{6}(3)^{3/2} - \frac{1}{6}(2)^{3/2}}$$

3. (5 pts) Evaluate $\int \theta \cos(3\theta)\, d\theta$.

Integration by Parts

$u = \theta \qquad v' = \cos(3\theta)$

$u' = 1 \qquad v = \frac{1}{3}\sin(3\theta)$

$\frac{1}{3}\theta\sin(3\theta) - \int \frac{1}{3}\sin(3\theta)\, d\theta$

$$\boxed{\frac{1}{3}\theta\sin(3\theta) + \frac{1}{9}\cos(3\theta) + C}$$

4. (6 pts) Find the function $y(t)$ that solves the initial value problem: $\frac{dy}{dt} = y^2(1+t)$, $y(1) = 2$.

Separation of variables

$\frac{1}{y^2}\,dy = (1+t)\,dt \quad , y \neq 0$

$\int \frac{1}{y^2}\,dy = \int (1+t)\,dt$

$-\frac{1}{y} = t + \frac{t^2}{2} + C \Rightarrow y = \frac{-1}{t + \frac{t^2}{2} + C}$

Note: By substitution into DE, $y = 0$ is also a solution.

IC: $2 = y(1) = \frac{-1}{1 + \frac{1}{2} + C} \Rightarrow C = -2$

$$\boxed{y(t) = \frac{-1}{\frac{t^2}{2} + t - 2} = \frac{-2}{t^2 + 2t - 4}}$$

5. Write each improper integral in terms of well-defined operations using limits of proper integrals. Then find the value of the integral, if it converges, or state that it does not converge.

(a) (5 pts) $\int_{-\infty}^{-1} \frac{1}{(2x+1)^2}\,dx$ $= \boxed{\lim_{b\to-\infty} \int_b^{-1} \frac{1}{(2x+1)^2}\,dx}$

$$= \lim_{b\to-\infty} \left[-\frac{1}{2}\left(\frac{1}{2x+1}\right)\Bigg|_b^{-1} \right]$$

$$= \lim_{b\to-\infty} \left[-\frac{1}{2}(-1) + \frac{1}{2}\underbrace{\left(\frac{1}{2b+1}\right)}_{\to 0} \right] = \boxed{\frac{1}{2}}$$

(b) (5 pts) $\int_{-1}^{1} \frac{1}{x}\,dx$ $= \boxed{\lim_{b\to 0^-} \left[\int_{-1}^{b} \frac{1}{x}\,dx\right] + \lim_{b\to 0^+} \left[\int_{b}^{1} \frac{1}{x}\,dx\right]}$

Consider:

$$\lim_{b\to 0^-} \left[\int_{-1}^{b} \frac{1}{x}\,dx\right] = \lim_{b\to 0^-} \left[\ln|x|\Big|_{-1}^{b}\right] = \lim_{b\to 0^-} \left[\underbrace{\ln|b|}_{\to -\infty} - 0\right]$$

$\Rightarrow$ Diverges

Since one integral diverges, the original improper integral diverges.

6. Consider the vectors $\vec{v} = 2\vec{j} + \vec{k}$, $\vec{w} = -\vec{i} + \vec{j} - 2\vec{k}$ and $\vec{u} = 3\vec{i} - \vec{j} + \vec{k}$.

(a) (3 pts) Are $\vec{v}$ and $\vec{w}$ perpendicular? Why or why not?

$$\vec{v} \cdot \vec{w} = (0)(-1) + (2)(1) + (1)(-2) = 0 + 2 - 2 = 0$$

$$\boxed{\text{Yes, since } \vec{v} \cdot \vec{w} = 0}$$

(b) (3 pts) Find a vector of length 2 that points in the same direction as $\vec{u}$.

$$\|\vec{u}\| = \sqrt{3^2 + (-1)^2 + 1^2} = \sqrt{11}$$

$$\Rightarrow \frac{2}{\|\vec{u}\|}\vec{u} = \frac{2}{\sqrt{11}}(3\vec{i} - \vec{j} + \vec{k})$$

$$\boxed{\begin{aligned} &= \frac{6}{\sqrt{11}}\vec{i} - \frac{2}{\sqrt{11}}\vec{j} + \frac{2}{\sqrt{11}}\vec{k} \\ &= \frac{6\sqrt{11}}{11}\vec{i} - \frac{2\sqrt{11}}{11}\vec{j} + \frac{2\sqrt{11}}{11}\vec{k} \end{aligned}}$$

(c) (3 pts) Find the angle between $\vec{w}$ and $\vec{u}$.

$$\vec{w} \cdot \vec{u} = -3 - 1 - 2 = -6$$

$$\|\vec{w}\| = \sqrt{1 + 1 + 4} = \sqrt{6}, \quad \|\vec{u}\| = \sqrt{9 + 1 + 1} = \sqrt{11}$$

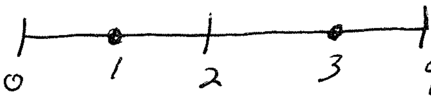
$$\vec{w} \cdot \vec{u} = \|\vec{w}\| \, \|\vec{u}\| \cos\theta$$

$$\Rightarrow -6 = \sqrt{6}\sqrt{11}\cos\theta \Rightarrow \cos\theta = \frac{-6}{\sqrt{6}\sqrt{11}} = \frac{-\sqrt{6}}{\sqrt{11}}$$

$$\boxed{\theta = \arccos\left(\frac{-\sqrt{6}}{\sqrt{11}}\right) = \arccos \frac{-\sqrt{66}}{11}}$$

7. (3 pts) Use the Midpoint Rule to estimate the value of $\int_0^4 (x^2+2)\,dx$ using $n=2$ subintervals.

$\Delta x = \frac{4-0}{2} = 2$



$F(x) = x^2+2$

$MID(2) = \Delta x\left[F(1) + F(3)\right] = 2(3+11) = \boxed{28}$

8. Find the sum of each geometric series, if it converges, or state why it does not converge.

(a) (3 pts) $\sum_{k=1}^{\infty}\left(\frac{2}{3}\right)^k = \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \cdots$

First term : $a = \frac{2}{3}$

Common multiple : $x = \frac{2}{3}$

Since $|x| = \frac{2}{3} < 1 \Rightarrow$ series converges to

$S = \frac{a}{1-x} = \frac{\frac{2}{3}}{1-\frac{2}{3}} = \boxed{2}$

(b) (3 pts) $-2+3-\frac{9}{2}+\frac{27}{4}-\frac{81}{8}+\cdots$

First term : $a = -2$

Common multiple : $x = -\frac{3}{2}$

$\boxed{\text{Since } |x| = \frac{3}{2} > 1 \Rightarrow \text{ Series does not converge}}$