1. The population of a certain town doubles in 14 years. How long will it take for the population to triple? Assume that the rate of increase of the population is proportional to the population.

   Select the correct answer.

   (a) 18.2 years
   (b) 20.2 years
   (c) 22.2 years
   (d) 23.2 years
   (e) 24.2 years

2. The half-life of radium is 1700 years. Assume that the decay rate is proportional to the amount. An initial amount of 5 grams of radium deca to 3 grams in

   Select the correct answer.

   (a) 850 years
   (b) 1050 years
   (c) 1150 years
   (d) 1250 years
   (e) 1350 years

3. An object is taken out of a 65°F room and placed outside where the temperature is 35°F room. Five minutes later the temperature is 63°F. It cools according to Newton's Law. The temperature of the object after one hour is

   Select the correct answer.

   (a) 50.5°F
   (b) 49.9°F
   (c) 49.3°F
   (d) 48.7°F
   (e) 48.1°F

4. A chicken is taken out of the freezer (0°C) and placed on a table in a 20°C room. Ten minutes later the temperature is 2°C. It warms according to Newton's Law. How long does it take before the temperature reaches 15°C?

   Select the correct answer.

   (a) 122 minutes
   (b) 127 minutes
   (c) 132 minutes
   (d) 137 minutes
   (e) 142 minutes

5. In Newton's Law of cooling, $\frac{dT}{dt} = k(T - T_m)$ the constant k is

   Select the correct answer.

   (a) a constant of integration evaluated from an initial condition
   (b) a constant of integration evaluated from another condition
   (c) a proportionality constant evaluated from an initial condition
   (d) a proportionality constant evaluated from another condition

6. A tank contains 50 gallons of water in which 2 pounds of salt is dissolved. A brine solution containing 1.5 pounds of salt per gallon of water is pumped into the tank at the rate of 4 gallons per minute, and the well-stirred mixture is pumped out at the same rate. Let $A(t)$ represent the amount of salt in the tank at time t. The correct initial value problem for $A(t)$ is

   Select the correct answer.

   (a) $\frac{dA}{dt} = 6 - 2A/25,\ A(0) = 2$
   (b) $\frac{dA}{dt} = 6 + 2A/25,\ A(0) = 2$
   (c) $\frac{dA}{dt} = 4 + 2A/25,\ A(0) = 2$
   (d) $\frac{dA}{dt} = 1.5 - 2A/25,\ A(0) = 0$
   (e) $\frac{dA}{dt} = 4 - 2A/25,\ A(0) = 0$

7. In the previous problem, how much salt will there be in the tank after a long period of time?

   Select the correct answer.

   (a) 2 pounds
   (b) 50 pounds
   (c) 75 pounds
   (d) 200 pounds
   (e) none of the above

8. In the previous two problems, the amount of salt in the tank at time t is

   Select the correct answer.

   (a) $A(t) = -75 + 77e^{2t/25}$
   (b) $A(t) = 75 - 73e^{-2t/25}$
   (c) $A(t) = 50 - 48e^{-2t/25}$
   (d) $A(t) = -50 + 52e^{2t/25}$
   (e) $A(t) = 75 - 73e^{2t/25}$

9. A ball is thrown upward from the top of a 200 foot tall building with a velocity of 40 feet per second. Take the positive direction upward and the origin of the coordinate system at ground level. What is the initial value problem for the position, $x(t)$, of the ball at time t?

Select the correct answer.

(a) $\frac{d^2x}{dt^2} = 40,\ x(0) = 200,\ \frac{dx}{dt}(0) = 40$

(b) $\frac{d^2x}{dt^2} = -40,\ x(0) = 200,\ \frac{dx}{dt}(0) = 40$

(c) $\frac{d^2x}{dt^2} = 32,\ x(0) = 200,\ \frac{dx}{dt}(0) = 40$

(d) $\frac{d^2x}{dt^2} = -32,\ x(0) = 200,\ \frac{xA}{dt}(0) = 40$

(e) $\frac{d^2x}{dt^2} = 200,\ x(0) = 32,\ \frac{dx}{dt}(0) = 40$

10. In the previous problem, the solution of the initial value problem is

Select the correct answer.

(a) $x = 16t^2 + 40t + 200$

(b) $x = -16t^2 + 200t + 40$

(c) $x = -16t^2 + 40t + 200$

(d) $x = 32t^2 + 40t + 200$

(e) $x = -32t^2 + 40t + 200$

11. In the logistic model for population growth, $\frac{dP}{dt} = P(8 - 2P)$, the carrying capacity of the population $P(t)$ is

Select the correct answer.

(a) 8

(b) 2

(c) 4

(d) 1/4

(e) 16

12. The solution of the logistic equation $\frac{dP}{dt} = P(8 - 2P)$ with initial condition $P(0) = 2$ is

Select the correct answer.

(a) $P = 4/(2 - e^{-8t})$
(b) $P = 2/(8 + e^{-8t})$
(c) $P = 8/(2 + e^{-8t})$
(d) $P = 8/(2 - e^{-8t})$
(e) $P = 4/(1 + e^{-8t})$

13. Two chemicals, A and B, are combined, forming chemical C. The rate of the reaction is jointly proportional to the amounts of A and B not yet converted to C. Initially, there are 200 grams of A and 300 grams of B, and, during the reaction, for each gram of A used up in the conversion, there are three grams of B used up. An experiments shows that 75 grams of C are produced in the first ten minutes. After a long period of time, how much of A and of B remains, and how much of C has been produced?

Select the correct answer.

(a) 200 grams of A, 0 grams of B, 300 grams of C
(b) 0 grams of A, 0 grams of B, 500 grams of C
(c) 100 grams of A, 0 grams of B, 400 grams of C
(d) 0 grams of A, 100 grams of B, 400 grams of C
(e) 0 grams of A, 200 grams of B, 300 grams of C

14. In the previous problem, the amount of chemical C, $X(t)$, produced by time t is

Select the correct answer.

(a) $X = 800(1 - e^{-400kt})/(2 - e^{-400kt})$, where $k = \ln(29/26)/4000$
(b) $X = 800(1 - e^{-800kt})/(4 - e^{-800kt})$, where $k = \ln(29/20)/8000$
(c) $X = 400(1 - e^{-800kt})/(4 - e^{-800kt})$, where $k = \ln(29/20)/8000$
(d) $X = 400(1 - e^{-400kt})/(4 - e^{-400kt})$, where $k = \ln(29/26)/4000$
(e) $X = 600(1 - e^{-800kt})/(4 - e^{-800kt})$, where $k = \ln(29/20)/8000$

15. Radioactive element X decays to element Y with decay constant -0.3. Y decays to stable element Z with decay constant -0.2. What is the system of differential equations for the amounts, $x(t)$, $y(t)$, $z(t)$ of the elements X, Y, Z, respectively, at time t.

Select the correct answer.

(a) $\frac{dx}{dt} = -0.2x,\ \frac{dy}{dt} = 0.2x - 0.3y,\ \frac{dz}{dt} = 0.2y$

(b) $\frac{dx}{dt} = -0.2x,\ \frac{dy}{dt} = 0.2x - 0.3y,\ \frac{dz}{dt} = 0.3y$

(c) $\frac{dx}{dt} = -0.3x,\ \frac{dy}{dt} = 0.3x - 0.2y,\ \frac{dz}{dt} = 0.2y$

(d) $\frac{dx}{dt} = -0.3x,\ \frac{dy}{dt} = 0.3x - 0.2y,\ \frac{dz}{dt} = 0.3y$

(e) $\frac{dx}{dt} = -0.3y,\ \frac{dy}{dt} = 0.3x - 0.2z,\ \frac{dz}{dt} = 0.2y$

16. The solution of the system of differential equations in the previous problem is

Select the correct answer.

(a) $x = c_1 e^{-0.3t},\ y = -3c_1 e^{-0.3t} + c_2 e^{-0.2t},\ z = c_3 - 3c_1 e^{-0.3t} - c_2 e^{-0.2t}$

(b) $x = c_1 e^{-0.2t},\ y = -2c_1 e^{-0.2t} + c_2 e^{-0.3t},\ z = c_3 + 2c_1 e^{-0.3t} - c_2 e^{-0.2t}$

(c) $x = c_1 e^{-0.3t},\ y = -3c_1 e^{-0.3t} + c_2 e^{-0.2t},\ z = c_3 + 2c_1 e^{-0.3t} + c_2 e^{-0.2t}$

(d) $x = c_1 e^{-0.3t},\ y = -3c_1 e^{-0.3t} + c_2 e^{-0.2t},\ z = c_3 + 2c_1 e^{-0.3t} - c_2 e^{-0.2t}$

(e) $x = c_1 e^{-0.2t},\ y = -2c_1 e^{-0.2t} + c_2 e^{-0.3t},\ z = c_3 + 2c_1 e^{-0.3t} - c_2 e^{-0.2t}$

17. Tank A contains 80 gallons of water in which 20 pounds of salt has been dissolved. Tank B contains 30 gallons of water in which 5 pounds of salt has been dissolved. A brine mixture with a concentration of 0.5 pounds of salt per gallon of water is pumped into tank A at the rate of 4 gallons per minute. The well-mixed solution is then pumped from tank A to tank B at the rate of 6 gallons per minute. The solution from tank B is also pumped through another pipe into tank A at the rate of 2 gallons per minute, and the solution from tank B is also pumped out of the system at the rate of 4 gallons per minute. The correct differential equations with initial conditions for the amounts, $x(t)$ and $y(t)$, of salt in tanks A and B, respectively, at time t are

Select the correct answer.

(a) $\frac{dx}{dt} = 2 - x/40 + y/5,\ \frac{dy}{dt} = x/40 - y/3,\ x(0) = 20,\ y(0) = 5$

(b) $\frac{dx}{dt} = 2 - 3x/40 + y/15,\ \frac{dy}{dt} = 3x/40 - y/5,\ x(0) = 20,\ y(0) = 5$

(c) $\frac{dx}{dt} = 4 - 3x/40 + y/15,\ \frac{dy}{dt} = 3x/40 - y/5,\ x(0) = 20,\ y(0) = 5$

(d) $\frac{dx}{dt} = 4 - x/40 + y/5,\ \frac{dy}{dt} = x/40 - y/3,\ x(0) = 20,\ y(0) = 5$

(e) $\frac{dx}{dt} = 2 - 3x/40 + y/5,\ \frac{dy}{dt} = x/40 - y/5,\ x(0) = 20,\ y(0) = 5$

18. In the previous problem, how much salt will there be in tanks A and B after a long period of time?

Select the correct answer.

(a) 5 pounds in A, 20 pounds in B
(b) 20 pounds in A, 5 pounds in B
(c) 5 pounds in A, 40 pounds in B
(d) 40 pounds in A, 15 pounds in B
(e) none of the above

19. In the Lotka-Volterra predator-prey model $\frac{dx}{dt} = -ax + bxy$, $\frac{dy}{dt} = ey - cxy$, where $x(t)$ is the predator population and $y(t)$ is the prey population, the coefficient c represents which of the following:

Select the correct answer.

(a) the predator die-off rate
(b) the prey growth rate
(c) the increase in the predator population due to interactions with the prey
(d) the decrease in the prey population due to interactions with the predator
(e) none of the above

20. In the competition model $\frac{dx}{dt} = ax - bxy$, $\frac{dy}{dt} = cy - dxy$, where $x(t)$ and $y(t)$ are the populations of the competing species, moose and deer, respectively, the coefficient c represents which of the following:

Select the correct answer.

(a) the moose growth rate
(b) the deer growth rate
(c) the decrease in the moose population due to interactions with the deer
(d) the decrease in the deer population due to interactions with the moose
(e) none of the above