1. State the order of the differential equation and tell whether it is linear or nonlinear.

$$y''' + 3yy'' + 2y' = 3x + 1.$$

2. State the order of the differential equation and tell whether it is linear or nonlinear.

$$y'' + 2y' - \sin x\ y = e^x.$$

3. Verify that the indicated function is an explicit solution of the given differential equation.

$$y'' - 3y' + 2y = 0; y = e^{2x}.$$

4. Verify that the indicated family of functions is a solution of the given differential equation.

$$y'' - 6y' + 9y = 0; y = c_1e^{3x} + c_2xe^{3x}.$$

5. Verify that the pair of functions is a solution of the system of differential equations.

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -2x + 3y$$

$$x = e^t + 3e^{2t}$$

$$y = e^t + 6e^{2t}$$

6. $y = 1/(x^2 + c)$ is a one parameter family of solutions of the differential equation $y' + 2xy^2 = 0$. Find a solution satisfying the initial condition $y(1) = 2$.

7. $y = c_1\cos x + c_2\sin x$ is a two parameter family of solutions of the differential equation $y'' + y = 0$. Find a solution that also satisfies the initial conditions $y(\pi/4) = 2, y'(\pi/4) = 1$.

8. $y = c_1e^x + c_2e^{-x}$ is a two parameter family of solutions of the differential equation $y''-y = 0$. Find a solution that also satisfies the initial conditions $y(0) = 3, y'(0) = -1$.

9. Determine by inspection at least two solutions of the initial value problem $xy' = 2y, y(0) = 0$.

10. Determine whether Theorem 1.1 guarantees that the differential equation $y' = \sqrt{y^2 - 4}$ has a unique solution through the point $(1, 3)$.

11. Determine whether Theorem 1.1 guarantees that the differential equation $y' = \sqrt{y^2 - 4}$ has a unique solution through the point $(-1, 2)$.

12. Determine a region of the $xy-$plane for which the given differential equation has a unique solution passing through a point (x_0, y_0) in the region.

$$y' = \sqrt{xy}.$$

13. Determine a region of the $xy-$plane for which the given differential equation has a unique solution passing through a point (x_0, y_0) in the region.

$$x(y-3)y' = 1.$$

14. The population of a certain country grows at a rate proportional to its population. If immigration into the country at a rate $r > 0$ is allowed, write down the differential equation for the population as a function of time.

15. A chicken is taken out of the freezer (at 32°F) and placed on the kitchen table (at 70°F). Write down the differential equation and initial condition for the temperature of the chicken at time t. Assume that Newton's Law of warming applies.

16. A large tank initially contains 100 gallons of pure water. A brine solution with a concentration of 2 pounds of salt per gallon is poured into the tank at the rate of 3 gallons per minute. The well-stirred mixture is then pumped out at the same rate. Write down the differential equation and initial condition for the amount of salt in the tank at time t.

17. Write down the differential equation for the charge on the capacitor in an R-C circuit to which a 12 volt battery is attached.

18. Write down the differential equation for the charge on the capacitor in an L-R circuit to which a voltage of $100\sin t$ is applied.

19. A rock is thrown upward from the top of a 200 foot tall building with an upward speed of 20 feet per second. Write down the initial value problem for the position of the rock as a function of time.

20. A body falls toward earth under the force of gravity. Assume that there is a damping force due to air resistance that is proportional to the velocity. Write down the differential equation for the velocity as a function of time.

ANSWER KEY
Zill Differential Equations 9e Chapter 1 Form A

1. Third order, nonlinear

2. Second order, linear

3. $y = e^{2x}, \qquad y' = 2e^{2x}, \qquad y'' = 4e^{2x},$
$y'' - 3y' + 2y = 4e^{2x} - 3(2e^{2x}) + 2e^{2x} = (4 - 6 + 2)e^{2x} = 0$

4. $y = c_1e^{3x} + c_2xe^{3x},$
$y' = 3c_1e^{3x} + 3c_2xe^{3x} + c_2e^{3x},$
$y'' = 9c_1e^{3x} + 9c_2xe^{3x} + 6c_2e^{3x},$
$y''-6y'+9y = 9c_1e^{3x}+9c_2xe^{3x}+6c_2e^{3x}-6(3c_1e^{3x}+3c_2xe^{3x}+c_2e^{3x})+9(c_1e^{3x}+c_2xe^{3x}) =$
$(9 - 18 + 9)c_1e^{3x} + (9 - 18 + 9)c_2xe^{3x} + (6 - 6)c_2e^{3x} = 0$

5. $\frac{dx}{dt} = e^t + 6e^{2t} = y,$
$-2x + 3y = -2e^t - 6e^{2t} + 3e^t + 18e^{2t} = e^t + 12e^{2t} = \frac{dy}{dt}$

6. $y = \frac{1}{x^2-1/2}$

7. $y = \sqrt{2}\cos x/2 + 3\sqrt{2}\sin x/2$

8. $y = e^x + 2e^{-x}$

9. $y = 0, y = x^2$

10. Yes

11. No, because $\sqrt{y^2 - 4}$ is neither continuous nor differentiable there.

12. $\{(x, y) : x > 0, y > 0\}$ or $\{(x, y) : x < 0, y < 0\}$

13. $\{(x, y) : x > 0, y > 3\}$ or three other regions

14. $\frac{dP}{dt} = kP + r$, where $P(t)$ is the population at time t and k is the constant of proportionality.

15. $\frac{dT}{dt} = k(T - 70), T(0) = 32$, where $T(t)$ is the temperature of the chicken at time t, and k is the constant of proportionality.

16. $\frac{dA}{dt} = 6 - \frac{3A}{100}, A(0) = 0$, where $A(t)$ is the amount of salt in the tank at time t.

17. $R\frac{dq}{dt} + \frac{1}{C}q = 12$, where q is the charge on the capacitor, and R and C are the resistance and capacitance, respectively.

18. $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} = 100\sin t$, where q is the charge on the capacitor, and L and R are the inductance and resistance, respectively.

19. $\frac{d^2s}{dt^2} = -32, s(0) = 200, \frac{ds}{dt}(0) = 20$, where $s(t)$ is the position of the rock at time t relative to the ground.

20. $m\frac{dv}{dt} = mg - kv$, where v is velocity at time t, m is mass, g is the gravitational acceleration, and k is the constant of proportionality. The positive direction is downward.

1. State the order of the differential equation and tell whether it is linear or nonlinear.
   $6y'' + 3y' - 8y = \cos x$.

2. State the order of the differential equation and tell whether it is linear or nonlinear.
   $y'''' + 2y'y'' - e^x y = \tan x$.

3. Verify that the indicated function is an explicit solution of the given differential equation.

$$y'' - 5y' + 4y = 0; y = e^{4x}.$$

4. Verify that the indicated family of functions is a solution of the given differential equation.

$$y'' - 6y' + 5y = 0; y = c_1 e^{5x} + c_2 e^x.$$

5. Verify that the pair of functions is a solution of the system of differential equations.

$$\frac{dx}{dt} = 2x + y$$
$$\frac{dy}{dt} = 2x + 3y$$
$$x = -2e^t + 2e^{4t}$$
$$y = 2e^t + 4e^{4t}$$

6. $y = 1/(1 + ce^{-x})$ is a one parameter family of solutions of the differential equation $y' = y - y^2$. Find a solution satisfying the initial condition $y(2) = 3$.

7. $y = c_1 \cos x + c_2 \sin x$ is a two parameter family of solutions of the differential equation $y'' + y = 0$. Find a solution that also satisfies the initial conditions $y(\pi/2) = -1, y'(\pi/2) = 4$.

8. $y = c_1 e^x + c_2 e^{-x}$ is a two parameter family of solutions of the differential equation $y'' - y = 0$. Find a solution that also satisfies the initial conditions $y(1) = -2, y'(1) = -3$.

9. Determine by inspection at least two solutions of the initial value problem $xy' = 3y, y(0) = 0$.

10. Determine whether Theorem 1.1 guarantees that the differential equation $y' = \sqrt{y - 3}$ has a unique solution through the point $(1, 3)$.

11. Determine whether Theorem 1.1 guarantees that the differential equation $y' = \sqrt{y - 3}$ has a unique solution through the point $(-1, 4)$.

12. Determine a region of the $xy-$plane for which the given differential equation has a unique solution passing through a point (x_0, y_0) in the region.

$$y' = \sqrt{x(y - 2)}.$$

13. Determine a region of the $xy-$plane for which the given differential equation has a unique solution passing through a point (x_0, y_0) in the region.

$$(x-1)^2(y+2)y' = 1.$$

14. The population of a certain country grows at a rate proportional to its population. If emigration out of the country at a rate $r > 0$ is allowed, write down the differential equation for the population as a function of time.

15. A baked chicken is taken out of the oven (at 350°F) and placed in the refrigerator (at 40°F). Write down the differential equation and initial condition for the temperature of the chicken at time t. Assume that Newton's Law of cooling applies.

16. A large tank initially contains 50 gallons of a brine solution with a concentration of 2 pounds of salt per gallon of water. Pure water is pumped into the tank at the rate of 4 gallons per minute. The well-stirred mixture is then pumped out at the same rate. Write down the differential equation and initial condition for the amount of salt in the tank at time t.

17. Write down the differential equation for the charge on the capacitor in an R-C circuit to which a voltage of $10\cos t$ is applied.

18. Write down the differential equation for the charge on the capacitor in an L-R circuit to which a 12 volt battery is attached.

19. A rock is thrown from the top of a 500 foot tall building with a downward speed of 30 feet per second. Write down the initial value problem for the position of the rock as a function of time.

20. A body falls toward earth under the force of gravity. Assume that there is a damping force due to air resistance that is proportional to the square of the velocity. Write down the differential equation for the velocity as a function of time.

ANSWER KEY
Zill Differential Equations 9e Chapter 1 Form B

1. Second order, linear

2. Fourth order, nonlinear

3. $y = e^{4x}, \qquad y' = 4e^{4x}, \qquad y'' = 16e^{4x},$
$y'' - 5y' + 4y = 16e^{4x} - 5(4e^{4x}) + 4e^{4x} = (16 - 20 + 4)e^{4x} = 0$

4. $y = c_1e^{5x} + c_2e^x,$
$y' = 5c_1e^{5x} + c_2e^x,$
$y'' = 25c_1e^{5x} + c_2e^x,$
$y'' - 6y' + 5y = 25c_1e^{5x} + c_2e^x - 6(5c_1e^{5x} + c_2e^x) + 5(c_1e^{5x} + c_2e^x) = (25 - 30 + 5)c_1e^{5x} +$
$(1 - 6 + 5)c_2e^x = 0$

5. $2x + y = -4e^t + 4e^{4t} + 2e^t + 4e^{4t} = -2e^t + 8e^{4t} = \frac{dx}{dt},$
$2x + 3y = -4e^t + 4e^{4t} + 6e^t + 12e^{4t} = 2e^t + 16e^{4t} = \frac{dy}{dt}$

6. $y = \frac{1}{1-2e^{2-x}/3}$

7. $y = -4\cos x - \sin x$

8. $y = -5e^{x-1}/2 + e^{1-x}/2$

9. $y = 0, y = x^3$

10. No, because $\sqrt{y-3}$ is neither continuous nor differentiable there.

11. Yes

12. $\{(x, y) : x > 0, y > 2\}$ or $\{(x, y) : x < 0, y < 2\}$

13. $\{(x, y) : x > 1, y > -2\}$ or three other regions

14. $\frac{dP}{dt} = kP - r$, where $P(t)$ is the population at time t and k is the constant of proportionality.

15. $\frac{dT}{dt} = k(T - 40), T(0) = 350$, where $T(t)$ is the temperature of the chicken at time t, and k is the constant of proportionality.

16. $\frac{dA}{dt} = -\frac{2A}{25}, A(0) = 100$, where $A(t)$ is the amount of salt in the tank at time t.

17. $R\frac{dq}{dt} + \frac{1}{C}q = 10\cos t$, where q is the charge on the capacitor, and R and C are the resistance and capacitance, respectively.

18. $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} = 12$, where q is the charge on the capacitor, and L and R are the inductance and resistance, respectively.

19. $\frac{d^2s}{dt^2} = -32, s(0) = 500, \frac{ds}{dt}(0) = -30$, where $s(t)$ is the position of the rock at time t relative to the ground.

20. $m\frac{dv}{dt} = mg - kv^2$, where v is velocity at time t, m is mass, g is the gravitational acceleration, and k is the constant of proportionality. The positive direction is downward.

1. The differential equation $y'' + 2y' + 3y = 0$ is
   Select the correct answer.

   (a) first order linear
   (b) second order linear
   (c) third order linear
   (d) first order nonlinear
   (e) second order nonlinear

2. The differential equation $y'' + 2yy' + 3y = 0$ is
   Select the correct answer.

   (a) first order linear
   (b) second order linear
   (c) third order linear
   (d) first order nonlinear
   (e) second order nonlinear

3. The differential equation $y' + 3y = \sin x$ is
   Select the correct answer.

   (a) first order linear
   (b) second order linear
   (c) third order linear
   (d) first order nonlinear
   (e) second order nonlinear

4. The differential equation $y'' + 2y' + 3y = \sin y$ is
   Select the correct answer.

   (a) first order linear
   (b) second order linear
   (c) third order linear
   (d) first order nonlinear
   (e) second order nonlinear

5. The differential equation $y''' + 2y'' + 3xy' - 4e^x y = \sin x$ is
   Select the correct answer.

   (a) first order linear
   (b) second order linear
   (c) third order linear
   (d) first order nonlinear
   (e) second order nonlinear

6. The values of m for which $y = e^{mx}$ is a solution of $y'' - 5y' + 6y = 0$ are
   Select the correct answer.

   (a) 2 and 4
   (b) -2 and -3
   (c) 3 and 4
   (d) 2 and 3
   (e) 1 and 5

7. The values of m for which $y = x^m$ is a solution of $x^2y'' - 5xy' + 8y = 0$ are
   Select the correct answer.

   (a) 2 and 4
   (b) -2 and -4
   (c) 3 and 5
   (d) 2 and 3
   (e) 1 and 5

8. The values of c for which $y = c$ is a constant solution of $y' = y^2 + 3y - 4$ are
   Select the correct answer.

   (a) 1 and 4
   (b) -1 and -3
   (c) 1 and -4
   (d) -1 and 3
   (e) 1 and 3

9. The values of m for which $y = e^{mx}$ is a solution of $y'' - 4y' - 5y = 0$ are

   Select the correct answer.

   (a) 1 and 4
   (b) -1 and 4
   (c) 2 and 3
   (d) -2 and -3
   (e) -1 and 5

10. The population of a town increases at a rate proportional to its population. its initial population is 1000. The correct initial value problem for the population, $P(t)$, as a function of time, t, is

    Select the correct answer.

    (a) $\frac{dP}{dt} = kP, P(0) = 1000$
    (b) $\frac{dP}{dt} = kP^2, P(0) = 100$
    (c) $\frac{dP}{dt} = kP, P(0) = 100$
    (d) $\frac{dP}{dt} = kP(1 - P), P(0) = 100$
    (e) $\frac{dP}{dt} = kP^2, P(0) = 1000$

11. The solution of the initial value problem $y' = 3y, y(0) = 2$ is $y = ce^{3x}$, where $c =$

    Select the correct answer.

    (a) 2
    (b) -2
    (c) 3
    (d) -3
    (e) 1

12. The solution of the initial value problem $y' = 2y + x, y(1) = 1/4$ is $y = -x/2 - 1/4 + ce^{2x}$, where $c =$

    Select the correct answer.

    (a) 2
    (b) e^{-2}
    (c) e^{-1}
    (d) $e^{-2}/2$
    (e) 1

13. The initial value problem $y' = \sqrt{y^2 - 9}, y(x_0) = y_0$ has a unique solution guaranteed by Theorem 1.1 if

    Select the correct answer.

    (a) $y_0 = 3$
    (b) $y_0 = -3$
    (c) $y_0 = 5$
    (d) $y_0 = 0$
    (e) $y_0 = 1$

14. The temperature of a cup of coffee obeys Newton's law of cooling. The initial temperature of the coffee is 150°F and one minute later, it is 135°F. The ambient temperature of the room is 70°F. If $T(t)$ represents the temperature of the coffee at time t, the correct differential equation for the temperature with side conditions is

    Select the correct answer.

    (a) $\frac{dT}{dt} = k(T - 135)$
    (b) $\frac{dT}{dt} = k(T - 150)$
    (c) $\frac{dT}{dt} = k(T - 70)$
    (d) $\frac{dT}{dt} = T(T - 150)$
    (e) $\frac{dT}{dt} = T(T - 70)$

15. In the previous problem, after a long period of time, the temperature of the coffee approaches

    Select the correct answer.

    (a) 120°F
    (b) 100°F
    (c) 70°F
    (d) 65°F
    (e) 0°F

16. In the LRC circuit problem in the text, C stands for

    Select the correct answer.

    (a) capacitance
    (b) resistance
    (c) current
    (d) inductance
    (e) charge on the capacitor

17. A large mixing tank initially contains 100 gallons of water in which 30 pounds of salt have been dissolved. Another brine solution is pumped into the tank at the rate of 4 gallons per minute, and the resulting mixture is pumped out at the same rate. The concentration of the incoming brine solution is 2 pounds of salt per gallon. If $A(t)$ represents the amount of salt in the tank at time t, the correct differential equation for A is

    Select the correct answer.

    (a) $\frac{dA}{dt} = 8 - .02A$
    (b) $\frac{dA}{dt} = 8 - .04A$
    (c) $\frac{dA}{dt} = 4 - .04A$
    (d) $\frac{dA}{dt} = 2 - .04A$
    (e) $\frac{dA}{dt} = 4 - .08A$

18. In the previous problem, over a long period of time, the total amount of salt in the tank will approach

    Select the correct answer.

    (a) 30 pounds
    (b) 50 pounds
    (c) 100 pounds
    (d) 200 pounds
    (e) 300 pounds

19. In the LRC circuit problem in the text, the units of inductance, L, are

    Select the correct answer.

    (a) ohms
    (b) farads
    (c) amperes
    (d) henrys
    (e) coulombs

20. In the falling body problem, the units of acceleration might be

    Select the correct answer.

    (a) meters per second
    (b) feet per second
    (c) meters per second per second
    (d) kilograms per meter
    (e) kilograms per meter per second

ANSWER KEY
Zill Differential Equations 9e Chapter 1 Form C

1. b
2. e
3. a
4. e
5. c
6. d
7. a
8. c
9. e
10. a
11. a
12. b
13. c
14. c
15. c
16. a
17. b
18. d
19. d
20. c

1. The differential equation $y''' + 2y'' + 3y' + 7y = 0$ is
   Select the correct answer.

   (a) first order linear
   (b) second order linear
   (c) third order linear
   (d) first order nonlinear
   (e) second order nonlinear

2. The differential equation $y'' + 2yy' + 3y = 0$ is
   Select the correct answer.

   (a) first order linear
   (b) second order linear
   (c) third order linear
   (d) first order nonlinear
   (e) second order nonlinear

3. The differential equation $y' + 3y = \sin x$ is
   Select the correct answer.

   (a) first order linear
   (b) second order linear
   (c) third order linear
   (d) first order nonlinear
   (e) second order nonlinear

4. The differential equation $y'' + 2y' + 3y = \sin y$ is
   Select the correct answer.

   (a) first order linear
   (b) second order linear
   (c) third order linear
   (d) first order nonlinear
   (e) second order nonlinear

5. The differential equation $y''' + 2y'' + 3xy' - 4e^x y = \sin x$ is
   Select the correct answer.

   (a) first order linear
   (b) second order linear
   (c) third order linear
   (d) first order nonlinear
   (e) second order nonlinear

6. The values of m for which $y = e^{mx}$ is a solution of $y'' - 9y' + 20y = 0$ are
   Select the correct answer.

   (a) 4 and -5
   (b) -4 and -5
   (c) 3 and 6
   (d) 4 and 5
   (e) 3 and 5

7. The values of m for which $y = x^m$ is a solution of $x^2y'' - 7xy' + 12y = 0$ are
   Select the correct answer.

   (a) -3 and 4
   (b) -2 and -6
   (c) 3 and 4
   (d) 2 and 6
   (e) 3 and -4

8. The values of c for which $y = c$ is a constant solution of $y' = y^2 + 5y - 6$ are
   Select the correct answer.

   (a) 1 and 6
   (b) -1 and 6
   (c) 1 and -6
   (d) -2 and 3
   (e) 2 and 3

9. The values of m for which $y = e^{mx}$ is a solution of $y'' - 6y' - 7y = 0$ are

   Select the correct answer.

   (a) 1 and 7
   (b) -1 and 6
   (c) 1 and 6
   (d) 1 and -6
   (e) -1 and 7

10. The population of a town increases at a rate proportional to its population. its initial population is 5000. The correct initial value problem for the population, $P(t)$, as a function of time, t, is

    Select the correct answer.

    (a) $\frac{dP}{dt} = kP, P(0) = 5000$
    (b) $\frac{dP}{dt} = kP^2, P(0) = 500$
    (c) $\frac{dP}{dt} = kP, P(0) = 500$
    (d) $\frac{dP}{dt} = kP(1 - P), P(0) = 5000$
    (e) $\frac{dP}{dt} = kP^2, P(0) = 5000$

11. The solution of the initial value problem $y' = 5y, y(1) = 3$ is $y = ce^{5x}$, where $c =$

    Select the correct answer.

    (a) $3e^{-5}$
    (b) 3
    (c) $3e^5$
    (d) $-3e^5$
    (e) -3

12. The solution of the initial value problem $y' = 2y + x, y(-1) = 1/2$ is $y = -x/2 - 1/4 + ce^{2x}$, where $c =$

    Select the correct answer.

    (a) 2
    (b) $e^2/4$
    (c) e^2
    (d) $e^2/2$
    (e) 1

13. The initial value problem $y' = \sqrt{y^2 - 16}, y(x_0) = y_0$ has a unique solution guaranteed by Theorem 1.1 if

    Select the correct answer.

    (a) $y_0 = 4$
    (b) $y_0 = -4$
    (c) $y_0 = 0$
    (d) $y_0 = 8$
    (e) $y_0 = 1$

14. The temperature of a cup of coffee obeys Newton's law of cooling. The initial temperature of the coffee is 140°F and one minute later, it is 125°F. The ambient temperature of the room is 65°F. If $T(t)$ represents the temperature of the coffee at time t, the correct differential equation for the temperature is

    Select the correct answer.

    (a) $\frac{dT}{dt} = k(T - 125)$
    (b) $\frac{dT}{dt} = k(T - 140)$
    (c) $\frac{dT}{dt} = k(T - 65)$
    (d) $\frac{dT}{dt} = T(T - 140)$
    (e) $\frac{dT}{dt} = T(T - 65)$

15. In the previous problem, after a long period of time, the temperature of the coffee approaches

    Select the correct answer.

    (a) 125°F
    (b) 100°F
    (c) 65°F
    (d) 50°F
    (e) 0°F

16. In the LRC circuit problem in the text, R stands for

    Select the correct answer.

    (a) capacitance
    (b) resistance
    (c) current
    (d) inductance
    (e) charge on the capacitor

17. A large mixing tank initially contains 1000 gallons of water in which 40 pounds of salt have been dissolved. Another brine solution is pumped into the tank at the rate of 5 gallons per minute, and the resulting mixture is pumped out at the same rate. The concentration of the incoming brine solution is 3 pound of salt per gallon. If $A(t)$ represents the amount of salt in the tank at time t, the correct differential equation for A is

    Select the correct answer.

    (a) $\frac{dA}{dt} = 3 - .005A$
    (b) $\frac{dA}{dt} = 5 - .05A$
    (c) $\frac{dA}{dt} = 15 - .005A$
    (d) $\frac{dA}{dt} = 3 - .05A$
    (e) $\frac{dA}{dt} = 15 + .05A$

18. In the previous problem, over a long period of time, the total amount of salt in the tank will approach

    Select the correct answer.

    (a) 300 pounds
    (b) 500 pounds
    (c) 1000 pounds
    (d) 3000 pounds
    (e) 5000 pounds

19. In the LRC circuit problem in the text, the units for C are

    Select the correct answer.

    (a) ohms
    (b) farads
    (c) amperes
    (d) henrys
    (e) coulombs

20. In the falling body problem, the units of acceleration might be

    Select the correct answer.

    (a) centimeters per second
    (b) feet per second
    (c) feet per second per second
    (d) kilograms per centimeter
    (e) kilograms per centimeter per second

ANSWER KEY
Zill Differential Equations 9e Chapter 1 Form D

1. c
2. e
3. a
4. e
5. c
6. d
7. d
8. c
9. e
10. a
11. a
12. b
13. d
14. c
15. c
16. b
17. c
18. d
19. b
20. c

1. The differential equation $y' + 3y = e^x$ is

   Select the correct answer.

   (a) first order linear
   (b) second order linear
   (c) third order linear
   (d) second order nonlinear
   (e) third order nonlinear

2. The differential equation $y''' + 2\sin x\ y'' + 3e^x\ y' - 5\ln x\ y = 0$ is

   Select the correct answer.

   (a) first order linear
   (b) second order linear
   (c) third order linear
   (d) first order nonlinear
   (e) second order nonlinear

3. Verify that $y = e^{2x}$ is a solution of $y' = 2y$.

4. Verify that $y = x^2 - 2 + \cos x - \sin x$ is a solution of $y'' + y = x^2$.

5. Verify that $y = \tan(2x)$ is a solution of $y' = 2y^2 + 2$.

6. The values of m for which $y = e^{mx}$ is a solution of $y'' - 3y' + 2y = 0$ are

   Select the correct answer.

   (a) 2 and -3
   (b) -2 and -3
   (c) 1 and 2
   (d) 2 and 3
   (e) -1 and -2

7. By inspection, find a one parameter family of solutions of the differential equation $xy' = y$. Verify that each member of this family also satisfies the initial condition $y(0) = 0$.

8. $y = \frac{1}{x^2+c}$ is a one parameter family of solutions of $y' + 2xy^2 = 0$. Find a solution of this differential equation that also satisfies the initial condition $y(2) = 3$.

9. Consider the differential equation $y' = 1/((y^2 - 4)(x - 1))$. Determine a region in the $xy-$plane for which it has a unique solution whose graph passes through the point (x_0, y_0) in the region.

10. The values of m for which $y = x^m$ is a solution of $x^2y'' + 2xy' - 2y = 0$ are

    Select the correct answer.

    (a) 1 and 2
    (b) -1 and -3
    (c) 1 and -2
    (d) -1 and 2
    (e) 1 and 3

11. The solution of the initial value problem $y' = 2y + x, y(1) = 1/2$ is $y = -x/2 - 1/4 + ce^{2x}$, where $c =$

    Select the correct answer.

    (a) 2
    (b) $5e^{-2}/4$
    (c) $3e^{-2}/4$
    (d) $e^{-2}/2$
    (e) 1

12. The initial value problem $y' = \sqrt{y^2 - 16}, y(x_0) = y_0$ has a unique solution guaranteed by Theorem 1.1 if

    Select the correct answer.

    (a) $y_0 = 4$
    (b) $y_0 = -4$
    (c) $y_0 = 0$
    (d) $y_0 = 6$
    (e) $y_0 = 1$

13. Verify that $4x^2 - 3y^2 = c$ is a one parameter family of solutions of the differential equation $ydy/dx = 4x/3$.

14. Determine whether Theorem 1.1 guarantees a unique solution of the initial value problem $y' = \sqrt{1 - y^2}, y(1) = 1$.

15. A new technology is introduced into a community of 10000 individuals. If the rate at which the technology spreads through the community is jointly proportional to the number of people who have adopted the technology and the number of individuals who have not adopted it, write down the differential equation for the number of people, $x(t)$, who have adopted the technology by time t.

16. A large mixing tank initially contains 100 gallons of water in which 20 pounds of salt have been dissolved. Another brine solution is pumped into the tank at the rate of 6 gallons per minute, and the resulting mixture is pumped out at the same rate. The concentration of the incoming brine solution is 3 pounds of salt per gallon. If $A(t)$ represents the amount of salt in the tank at time t, the correct differential equation for A is

    Select the correct answer.

    (a) $\frac{dA}{dt} = 6 - .03A$
    (b) $\frac{dA}{dt} = 18 - .06A$
    (c) $\frac{dA}{dt} = 3 - .06A$
    (d) $\frac{dA}{dt} = 6 + .03A$
    (e) $\frac{dA}{dt} = 18 + .06A$

17. In the previous problem, over a long period of time, the total amount of salt in the tank will approach

    Select the correct answer.

    (a) 30 pounds
    (b) 50 pounds
    (c) 100 pounds
    (d) 200 pounds
    (e) 300 pounds

18. In the LRC circuit problem in the text, C stands for

    Select the correct answer.

    (a) capacitance
    (b) resistance
    (c) current
    (d) inductance
    (e) charge on the capacitor

19. In the LRC circuit problem in the text, the units for R are

    Select the correct answer.

    (a) farads
    (b) ohms
    (c) amperes
    (d) henrys
    (e) coulombs

20. In the falling body problem, the units of acceleration might be
Select the correct answer.

(a) meters per second
(b) feet per second
(c) meters per second per second
(d) kilograms per meter
(e) kilograms per meter per second

ANSWER KEY
Zill Differential Equations 9e Chapter 1 Form E

1. a
2. c
3. $y = e^{2x}$
   $y' = 2e^{2x} = 2y$
4. $y = x^2 - 2 + \cos x - \sin x$
   $y' = 2x - \sin x - \cos x$
   $y'' = 2 - \cos x + \sin x$
   $y'' + y = x^2$
5. $y = \tan(2x)$
   $y' = 2\sec^2(2x) = 2(1 + \tan^2(2x)) = 2(1 + y^2) = 2y^2 + 2$
6. c
7. $y = cx, y(0) = 0$
8. $y = \frac{1}{x^2 - 11/3}$
9. $\{(x, y) : x > 1, y > 2\}$ or three other possible regions
10. c
11. b
12. d
13. $4x^2 - 3y^2 = c$
    $8x - 6yy' = 0$
    $yy' = 8x/6 = 4x/3$
14. No, the function $\sqrt{1 - y^2}$ is not continuous or differentiable at the point $(1, 1)$.
15. $\frac{dx}{dt} = kx(10000 - x)$
16. b
17. e
18. a
19. b
20. c

1. Is the differential equation $y'' + 2\sin y\ y' + 3e^x\ y = 0$ linear or nonlinear? If linear, is it homogeneous? What is its order?

2. The differential equation $y'' + y^2 = \sin x$ is

   Select the correct answer.

   (a) first order linear
   (b) second order linear
   (c) third order linear
   (d) first order nonlinear
   (e) second order nonlinear

3. Verify that $y = e^{4x} + 4$ is a solution of $y'' - 4y' = 0$.

4. Verify that $y = x + 3\cos x + 5\sin x$ is a solution of $y'' + y = x$.

5. Verify that $y = \ln x$ is a solution of $xy'' + y' = 0$.

6. The values of m for which $y = e^{mx}$ is a solution of $y'' - 3y' - 4y = 0$ are

   Select the correct answer.

   (a) 1 and -3
   (b) -1 and -3
   (c) 1 and -4
   (d) 1 and 3
   (e) -1 and 4

7. By inspection, find a one parameter family of solutions of the differential equation $xy' = 4y$. Verify that each member of this family also satisfies the initial condition $y(0) = 0$.

8. The values of m for which $y = x^m$ is a solution of $x^2y'' + 5xy' - 5y = 0$ are

   Select the correct answer.

   (a) 2 and 3
   (b) -1 and 5
   (c) 2 and -3
   (d) -2 and 3
   (e) 1 and -5

9. $y = \frac{1}{1+ce^{-x}}$ is a one parameter family of solutions of $y' = y - y^2$. Find a solution of this differential equation that also satisfies the initial condition $y(1) = 5$.

10. Consider the differential equation $y' = \sqrt{y-2}/(x^2 - 9)$. Determine a region in the $xy-$plane for which it has a unique solution whose graph passes through the point (x_0, y_0) in the region.

11. The solution of the initial value problem $y' = 3y + x^2, y(0) = 1$ is $y = -x^2/3 - 2x/9 - 2/27 + ce^{3x}$, where $c =$

    Select the correct answer.

    (a) $1/9$
    (b) $8/9$
    (c) $25/27$
    (d) $29/27$
    (e) 1

12. The initial value problem $y' = \sqrt{y^2 - 25}, y(x_0) = y_0$ has a unique solution guaranteed by Theorem 1.1 if

    Select the correct answer.

    (a) $y_0 = 5$
    (b) $y_0 = -5$
    (c) $y_0 = 0$
    (d) $y_0 = 8$
    (e) $y_0 = 4$

13. Verify that $6x^2 - y^2 = c$ is a one parameter family of solutions of the differential equation $ydy/dx = 6x$.

14. Determine whether Theorem 1.1 guarantees a unique solution of the initial value problem $y' = \sqrt{12 - y^2}, y(1) = 3$.

15. A disease is introduced into a community of 5000 individuals. If the rate at which the disease spreads through the community is jointly proportional to the number of people who have the disease and the number of individuals who have not caught it, write down the differential equation for the number of people, $x(t)$ who have caught the disease by time t.

16. Write down the differential equation for the charge on the capacitor in an LRC circuit where the inductance is .1 henry, the resistance is 50 ohms, and the capacitance is .001 farad.

17. A cup of coffee initially is at temperature 90°C. It is placed in a room where the temperature is 20°C. If the temperature obeys Newtons Law of cooling, the correct differential equation for the temperature, T, as a function of time, t, is

    Select the correct answer.

    (a) $\frac{dT}{dt} = k(T - 90)$
    (b) $\frac{dT}{dt} = k(t - 20)$
    (c) $\frac{dT}{dt} = T(T - 90)$
    (d) $\frac{dT}{dt} = k(T - 20)$
    (e) $\frac{dT}{dt} = T(T - 20)$

18. In the previous problem, over a long period of time, the temperature of the coffee will approach

    Select the correct answer.

    (a) 90°C
    (b) 100°C
    (c) 20°C
    (d) 55°C
    (e) 0°C

19. Assume that an object of mass m falls toward earth under the influence of gravity, and that it is also acted upon by the force of air resistance that is proportional to the square of the velocity. Take the positive direction downward. The correct differential equation for the velocity of the object is

    Select the correct answer.

    (a) $\frac{dv}{dt} = mg - kv$
    (b) $\frac{dv}{dt} = mg - kv^2$
    (c) $m\frac{dv}{dt} = mg - kv$
    (d) $m\frac{dv}{dt} = mg - kv^2$
    (e) $m\frac{dv}{dt} = mg + kv^2$

20. Write down the initial value problem for the position of a falling body above ground level, if it starts from a point 200 feet above ground level with an upward velocity of 40 feet per second.

ANSWER KEY
Zill Differential Equations 9e Chapter 1 Form F

1. second order nonlinear

2. e

3. $y = e^{4x} + 4 \qquad y' = 4e^{4x} \qquad y'' = 16e^{4x}$
   $y'' - 4y' = 16e^{4x} - 4(4e^{4x}) = 0$

4. $y = x + 3\cos x + 5\sin x$
   $y' = 1 - 3\sin x + 5\cos x$
   $y'' = -3\cos x - 5\sin x$
   $y'' + y = x$

5. $y = \ln x \qquad y' = 1/x \qquad y'' = -1/x^2$
   $xy'' + y' = -1/x + 1/x = 0$

6. e

7. $y = cx^4, y(0) = 0$

8. e

9. $y = 1/(1 - 4e^{1-x}/5)$

10. $\{(x, y) : x > 3, y > 2\}$ or $\{(x, y) : x < 3, y > 2\}$

11. d

12. d

13. $6x^2 - y^2 = c \qquad 12x - 2yy' = 0 \qquad yy' = 6x$

14. Yes

15. $\frac{dx}{dt} = kx(5000 - x)$

16. $.1\frac{d^2q}{dt^2} + 50\frac{dq}{dt} + 1000q = 0$

17. d

18. c

19. d

20. $\frac{d^2s}{dt^2} = -32, s(0) = 200, \frac{ds}{dt}(0) = 40$, where $s(t)$ is the position of the body relative to the ground at time t.

1. Is the differential equation $y''' + 2\sin x\ y'' + 3e^x\ y' - 5\ln x\ y = 0$ linear or nonlinear? If linear, is it homogeneous? What is its order?

2. The differential equation $y' + y^2 = e^x$ is

   Select the correct answer.

   (a) first order linear
   (b) second order linear
   (c) third order linear
   (d) first order nonlinear
   (e) second order nonlinear

3. Verify that $y = e^{3x} + 4$ is a solution of $y'' - 3y' = 0$.

4. Verify that $y = x^3 - 6x + 3\cos x + 5\sin x$ is a solution of $y'' + y = x^3$.

5. Verify that $y = \sec x$ is a solution of $y' = y\sqrt{y^2 - 1}$.

6. The values of m for which $y = e^{mx}$ is a solution of $y'' - 5y' + 6y = 0$ are

   Select the correct answer.

   (a) 2 and -3
   (b) -2 and -3
   (c) 1 and 2
   (d) 2 and 3
   (e) -1 and -2

7. By inspection, find a one parameter family of solutions of the differential equation $xy' = 3y$. Verify that each member of this family also satisfies the initial condition $y(0) = 0$.

8. The values of m for which $y = x^m$ is a solution of $x^2y'' + 4xy' - 4y = 0$ are

   Select the correct answer.

   (a) 1 and 2
   (b) -1 and 4
   (c) 1 and -3
   (d) -1 and 3
   (e) 1 and -4

9. $y = \frac{1}{x^2+c}$ is a one parameter family of solutions of $y' + 2xy^2 = 0$. Find a solution of this differential equation that also satisfies the initial condition $y(3) = 5$.

10. Consider the differential equation $y' = \sqrt{y^2 - 9}/(x - 3)$. Determine a region in the $xy-$plane for which it has a unique solution whose graph passes through the point (x_0, y_0) in the region.

11. The solution of the initial value problem $y' = 2y + x, y(-1) = 1/2$ is $y = -x/2 - 1/4 + ce^{2x}$, where $c =$

    Select the correct answer.

    (a) 2
    (b) $e^2/4$
    (c) e^2
    (d) $e^2/2$
    (e) 1

12. The initial value problem $y' = \sqrt{y^2 - 4}, y(x_0) = y_0$ has a unique solution guaranteed by Theorem 1.1 if

    Select the correct answer.

    (a) $y_0 = 2$
    (b) $y_0 = -2$
    (c) $y_0 = 0$
    (d) $y_0 = 6$
    (e) $y_0 = 1$

13. Verify that $6x^3 + y^2 = c$ is a one parameter family of solutions of the differential equation $ydy/dx = -9x^2$.

14. Determine whether Theorem 1.1 guarantees a unique solution of the initial value problem $y = \sqrt{4 - y^2}, y(1) = 2$.

15. A disease is introduced into a community of 1000 individuals. If the rate at which the disease spreads through the community is jointly proportional to the number of people who have the disease and the number of individuals who have not caught it, write down the differential equation for the number of people, $x(t)$ who have caught the disease by time t.

16. A large mixing tank initially contains 500 gallons of water in which 40 pounds of salt have been dissolved. Another brine solution is pumped into the tank at the rate of 4 gallons per minute, and the resulting mixture is pumped out at the same rate. The concentration of the incoming brine solution is 2 pounds of salt per gallon. If $A(t)$ represents the amount of salt in the tank at time t, the correct differential equation for A is

    Select the correct answer.

    (a) $\frac{dA}{dt} = 4 - .008A$
    (b) $\frac{dA}{dt} = 8 - .008A$
    (c) $\frac{dA}{dt} = 4 - .08A$
    (d) $\frac{dA}{dt} = 4 + .08A$
    (e) $\frac{dA}{dt} = 8 + .08A$

17. In the previous problem, over a long period of time, the total amount of salt in the tank will approach

    Select the correct answer.

    (a) 100 pounds
    (b) 300 pounds
    (c) 500 pounds
    (d) 800 pounds
    (e) 1000 pounds

18. In the LRC circuit problem in the text, L stands for

    Select the correct answer.

    (a) capacitance
    (b) resistance
    (c) current
    (d) inductance
    (e) charge on the capacitor

19. In the LRC circuit problem in the text, R stands for

    Select the correct answer.

    (a) capacitance
    (b) resistance
    (c) current
    (d) inductance
    (e) charge on the capacitor

20. In the falling body problem, the units of acceleration might be

    Select the correct answer.

    (a) meters per second
    (b) feet per second
    (c) meters per second per second
    (d) kilograms per meter
    (e) kilograms per meter per second

ANSWER KEY
Zill Differential Equations 9e Chapter 1 Form G

1. third order, linear, homogeneous
2. d
3. $y = e^{3x} + 4 \qquad y' = 3e^{3x} \qquad y'' = 9e^{3x}$
   $y'' - 3y' = 9e^{3x} - 3(3e^{3x}) = 0$
4. $y = x^3 - 6x + 3\cos x + 5\sin x$
   $y' = 3x^2 - 6 - 3\sin x + 5\cos x$
   $y'' = 6x - 3\cos x - 5\sin x$
   $y'' + y = x^3$
5. $y = \sec x \qquad y' = \sec x \tan x = \sec x\sqrt{\sec^2 x - 1} = y\sqrt{y^2 - 1}$
6. d
7. $y = cx^3, y(0) = 0$
8. e
9. $y = 1/(x^2 - 44/5)$
10. $\{(x, y) : x > 3, y > 3\}$ or 3 other regions
11. b
12. d
13. $6x^3 + y^2 = c \qquad 18x^2 + 2yy' = 0 \qquad yy' = -9x^2$
14. No, since $\sqrt{4 - y^2}$ is neither continuous nor differentiable there.
15. $\frac{dx}{dt} = kx(1000 - x)$
16. b
17. e
18. d
19. b
20. c

1. Is the differential equation $y''' + 2\sin x\ y'' + 3e^y\ y' - 5\ln x\ y = 0$ linear or nonlinear? If linear, is it homogeneous? What is its order?

2. The differential equation $y'' + y' = e^x$ is

   Select the correct answer.

   (a) first order linear
   (b) second order linear
   (c) third order linear
   (d) first order nonlinear
   (e) second order nonlinear

3. Verify that $y = e^{-5x} + 4$ is a solution of $y'' + 5y' = 0$.

4. Verify that $y = x^4 - 12x^2 + 24 - \cos x + \sin x$ is a solution of $y'' + y = x^4$.

5. Verify that $y = e^{x^2}$ is a solution of $y' = 2xy$.

6. The values of m for which $y = e^{mx}$ is a solution of $y'' - 4y' + 3y = 0$ are

   Select the correct answer.

   (a) 1 and -3
   (b) -1 and -3
   (c) 1 and 2
   (d) 1 and 3
   (e) -1 and -2

7. By inspection, find a one parameter family of solutions of the differential equation $xy' = y$. Verify that each member of this family also satisfies the initial condition $y(0) = 0$.

8. The values of m for which $y = x^m$ is a solution of $x^2y'' + 5xy' - 5y = 0$ are

   Select the correct answer.

   (a) 1 and 4
   (b) -1 and 5
   (c) 1 and -4
   (d) -1 and 4
   (e) 1 and -5

9. $y = \frac{1}{x^2+c}$ is a one parameter family of solutions of $y' + 2xy^2 = 0$. Find a solution of this differential equation that also satisfies the initial condition $y(-1) = 2$.

10. Determine a region in the $xy-$plane for which the differential equation $y' = \sqrt{y^2 - 9}/(x-1)$ has a unique solution whose graph passes through the point (x_0, y_0) in the region.

11. The solution of the initial value problem $y' = 2y + x, y(-1) = -1/2$ is $y = -x/2 - 1/4 + ce^{2x}$, where $c =$

    Select the correct answer.

    (a) 2
    (b) $-3e^2/4$
    (c) $5e^2/4$
    (d) $-e^2/2$
    (e) 1

12. The initial value problem $y' = \sqrt{y^2 - 1}, y(x_0) = y_0$ has a unique solution guaranteed by Theorem 1.1 if

    Select the correct answer.

    (a) $y_0 = 1$
    (b) $y_0 = -1$
    (c) $y_0 = 0$
    (d) $y_0 = 2$
    (e) $y_0 = 1/2$

13. Verify that $4x^4 + y^2 = c$ is a one parameter family of solutions of the differential equation $ydy/dx = -8x^3$.

14. Determine whether Theorem 1.1 guarantees a unique solution of the initial value problem $y = \sqrt{9 - y^2}, y(1) = -3$.

15. A new technology is introduced into a community of 5000 individuals. If the rate at which the technology spreads through the community is jointly proportional to the number of people who use the technology and the number of individuals who do not use it, write down the differential equation for the number of people, $x(t)$ who use the technology by time t.

16. A large mixing tank initially contains 50 gallons of water in which 3 pounds of salt have been dissolved. Another brine solution is pumped into the tank at the rate of 3 gallons per minute, and the resulting mixture is pumped out at the same rate. The concentration of the incoming brine solution is 3 pounds of salt per gallon. If $A(t)$ represents the amount of salt in the tank at time t, the correct differential equation for A is

    Select the correct answer.

    (a) $\frac{dA}{dt} = 9 - .06A$
    (b) $\frac{dA}{dt} = 3 - .6A$
    (c) $\frac{dA}{dt} = 3 - .06A$
    (d) $\frac{dA}{dt} = 9 - .6A$
    (e) $\frac{dA}{dt} = 9 + .6A$

17. In the previous problem, over a long period of time, the total amount of salt in the tank will approach

    Select the correct answer.

    (a) 10 pounds
    (b) 30 pounds
    (c) 50 pounds
    (d) 150 pounds
    (e) 200 pounds

18. In the LRC circuit problem in the text, L stands for

    Select the correct answer.

    (a) capacitance
    (b) resistance
    (c) current
    (d) inductance
    (e) charge on the capacitor

19. In the LRC circuit problem in the text, R stands for

    Select the correct answer.

    (a) capacitance
    (b) resistance
    (c) current
    (d) inductance
    (e) charge on the capacitor

20. In the falling body problem, the units of acceleration might be

    Select the correct answer.

    (a) meters per second
    (b) feet per second
    (c) meters per second per second
    (d) kilograms per meter
    (e) kilograms per meter per second

ANSWER KEY
Zill Differential Equations 9e Chapter 1 Form H

1. third order, nonlinear
2. b
3. $y = e^{-5x} + 4 \qquad y' = -5e^{-5x} \qquad y'' = 25e^{-5x}$
   $y'' + 5y' = 25e^{-5x} + 5(-5e^{-5x}) = 0$
4. $y = x^4 - 12x^2 + 24 - \cos x + \sin x$
   $y' = 4x^3 - 24x + \sin x + \cos x$
   $y'' = 12x^2 - 24 + \cos x - \sin x$
   $y'' + y = x^4$
5. $y = e^{x^2} \qquad y' = 2xe^{x^2} = 2xy$
6. d
7. $y = cx, y(0) = 0$
8. e
9. $y = 1/(x^2 - 1/2)$
10. $\{(x, y) : x > 1, y > 3\}$ or 3 other regions
11. b
12. d
13. $4x^4 + y^2 = c \qquad 16x^3 + 2yy' = 0 \qquad yy' = -8x^3$
14. No, since $\sqrt{9 - y^2}$ is neither continuous nor differentiable there.
15. $\frac{dx}{dt} = kx(5000 - x)$
16. a
17. d
18. d
19. b
20. c