1. For the differential equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, what is the order? Is it linear or nonlinear? If it is linear, is it homogeneous or non-homogeneous?

2. What two ordinary differential equations result when $u(x,y)=X(x)Y(y)$ is substituted into the partial differential equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$?

3. What is the solution of $y''+\lambda y=0$, $y(0)=0$, $y(\pi)=0$ if $\lambda>0$?

4. What is the solution of $y''+\lambda y=0$, $y(0)=0$, $y(\pi)=0$ if $\lambda<0$?

5. What is the solution of $y''+\lambda y=0$, $y(0)=0$, $y(\pi)=0$ if $\lambda=0$?

6. Classify the equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ as hyperbolic, parabolic, or elliptic.

7. Write down the one-dimensional heat equation and initial and boundary conditions for a rod with temperature A at the left end and temperature B at the right end. Define all terms.

8. Consider the equation $u_{xx}-u_{tt}=0$ with conditions $u(0,t)=0$, $u(1,t)=0$, $u(x,0)=f(x)$, $\frac{du}{dt}(x,0)=0$. Describe a physical situation having this problem as a mathematical model.

9. In the previous problem, separate variables. What are the differential equation and boundary conditions for the function of the spatial variable? What are the differential equation and initial conditions for the function of the time variable?

10. In the previous problem, what are the solutions of the resulting eigenvalue problem? What are the solutions of the problem involving the time variable?

11. In the previous problem, what are the product solutions of the original partial differential equation.

12. In the previous four problems, if $f(x)=\sin(\pi x)$, what is the Fourier series solution?

13. Consider the boundary value problem $ku_{xx}+r=u_t$, $u(0,t)=0$, $u(1,t)=u_0$, $u(x,0)=f(x)$, r and k are positive constants. Substitute $u(x,t)=v(x,t)+\psi(x)$, and write down the new equation.

14. In the previous problem, what condition on ψ makes the differential equation for v homogeneous? What conditions on ψ make the boundary conditions for v homogeneous?

15. In the previous two problems, solve for ψ, assuming the conditions you specified hold.

16. In the previous three problems, what is the resulting boundary value problem for v?

17. In the previous problem, separate variables in the partial differential equation for v, using $v(x,t)=X(x)T(t)$. What are the resulting differential equations and boundary conditions for X and T?

18. In the previous problem, solve the eigenvalue problem.

19. In the previous two problems, what are the product solutions for v?

20. In the previous seven problems, what is the solution of the problem posed for $u(x,t)$?

ANSWER KEY

Zill Differential Equations 9e Chapter 12 Form A

1. second order, linear, homogeneous
2. $X'' + \lambda X = 0$, $Y'' - \lambda Y = 0$
3. $\lambda = n^2$, $y = c_n \sin(nx)$, $n = 1, 2, 3, \ldots$
4. $y = 0$
5. $y = 0$
6. elliptic
7. $ku_{xx} - u_t = 0$, $u(0,t) = A$, $u(L,t) = B$, $u(x,0) = f(x)$, where $u(x,t)$ is the temperature at point x and time t in the rod, L is the length of the rod, T_1 and T_2 are given constant end temperatures, $f(x)$ is the given initial temperature distribution, and k is the thermal diffusivity.
8. Vibrations of a string, tightly stretched between two points, $x = 0$ and $x = 1$, along the x-axis, with initial position $f(x)$ and zero initial velocity.
9. $X'' + \lambda X = 0$, $X(0) = 0$, $X(1) = 0$, $T'' + \lambda T = 0$, $T'(0) = 0$
10. $\lambda = n^2\pi^2$, $X(x) = c_n \sin(n\pi x)$, $T(t) = \cos(n\pi t)$
11. $X(x)T(t) = c_n \sin(n\pi x)\cos(n\pi t)$
12. $u(x,t) = \sin(\pi x)\cos(\pi t)$
13. $kv_{xx} + k\psi'' + r = v_t$
14. $\psi'' + r/k = 0$, $\psi(0) = 0$, $\psi(1) = u_0$
15. $\psi = -rx^2/(2k) + (u_0 + r/(2k))x$
16. $kv_{xx} = v_t$, $v(0,t) = 0$, $v(1,t) = 0$, $v(x,0) = f(x) - \psi(x)$
17. $X'' + \lambda X = 0$, $X(0) = 0$, $X(1) = 0$, $T' + k\lambda T = 0$
18. $\lambda = n^2\pi^2$, $X = c_n \sin(n\pi x)$, $n = 1, 2, 3, \ldots$
19. $v = c_n \sin(n\pi x)e^{-kn^2\pi^2 t}$
20. $u = \sum_1^\infty c_n \sin(n\pi x)e^{-kn^2\pi^2 t} + \psi(x)$, where $c_n = 2\int_0^1 (f(x) - \psi(x))\sin(n\pi x)dx$

1. For the differential equation $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = 0$, what is the order? Is it linear or nonlinear? If it is linear, is it homogeneous or non-homogeneous?

2. What two ordinary differential equations result when $u(x,t) = X(x)T(t)$ is substituted into the partial differential equation $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = 0$?

3. What is the solution of $y'' + \lambda y = 0$, $y(0) = 0$, $y(1) = 0$ if $\lambda > 0$?

4. What is the solution of $y'' + \lambda y = 0$, $y(0) = 0$, $y(1) = 0$ if $\lambda < 0$?

5. What is the solution of $y'' + \lambda y = 0$, $y(0) = 0$, $y(1) = 0$ if $\lambda = 0$?

6. Classify the equation $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = 0$ as hyperbolic, parabolic, or elliptic.

7. Write down the equation and initial and boundary conditions for a vibrating string, fixed at both ends. Define all terms.

8. Consider the equation $u_{xx} - u_t = 0$ with conditions $u(0,t) = 0$, $u(1,t) = 0$, $u(x,0) = f(x)$. Describe a physical situation having this problem as a mathematical model.

9. In the previous problem, separate variables. What is the differential equation and boundary conditions for the function of the spatial variable? What is the differential equation for the function of the time variable?

10. In the previous problem, what are the solutions of the resulting eigenvalue problem? What are the solutions of the problem involving the time variable?

11. In the previous problem, what are the product solutions of the original partial differential equation.

12. In the previous four problems, if $f(x) = \sin(3\pi x)$, what is the Fourier series solution?

13. Consider the wave equation for in infinitely long string: $a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$, $-\infty < x < \infty$, $t > 0$, $u(x,0) = f(x)$, $u_t(x,0) = g(x)$. Make the change of variables $\xi = x + at$, $\eta = x - at$ and find the resulting differential equation for $u(\xi, \eta)$.

14. Solve the resulting differential equation that you derived in the previous problem.

15. Use the initial conditions in the previous two problems to exhibit the solution of the original boundary value problem for $u(x,t)$.

16. Explain how to solve the problem with non-homogeneous boundary values $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $u(0,y) = F(y)$, $u(1,y) = G(y)$, $u(x,0) = f(x)$, $u(x,1) = g(x)$.

17. Explain how to solve the non-homogeneous boundary value problem $k\frac{\partial^2 u}{\partial x^2} + F(x,t) = \frac{\partial u}{\partial t}$, $u(0,t) = 0$, $u(L,t) = 0$, $u(x,0) = f(x)$.

18. In the problem $\frac{\partial^2 u}{\partial x^2} + xe^t = \frac{\partial u}{\partial t}$, $u(0,t) = 0$, $u(L,t) = 0$, $u(x,0) = 0$, what are the eigenvalues and eigenfunctions of the underlying homogeneous problem.

19. In the previous problem, write the non-homogeneous term, xe^t, as an eigenfunction expansion in term of the eigenfunctions you found there.

20. Solve the problem posed in the previous two problems.

ANSWER KEY
Zill Differential Equations 9e Chapter 12 Form B

1. second order, linear, homogeneous

2. $X'' + \lambda X = 0$, $Y'' + \lambda Y = 0$

3. $\lambda = n^2\pi^2$, $y = c_n \sin(n\pi x)$

4. $y = 0$

5. $y = 0$

6. hyperbolic

7. $c^2 u_{xx} - u_{tt} = 0$, $u(0,t) = 0$, $u(L,t) = 0$, $u(x,0) = f(x)$, $u_t(x,0) = g(x)$, where $u(x,t)$ is the displacement from the x-axis at point x and time t, c is the speed of propagation, L is the length of the string, and $f(x)$ and $g(x)$ are the initial position and velocity.

8. This could represent the temperature distribution in a rod of length 1, with temperature fixed at a value of 0 at $x = 0$ and $x = 1$, initial temperature distribution $f(x)$, and thermal diffusivity $k = 1$.

9. $X'' + \lambda X = 0$, $X(0) = 0$, $X(1) = 0$, $T' + \lambda T = 0$

10. $\lambda = n^2\pi^2$, $X = c_n \sin(n\pi x)$, $n = 1, 2, 3, \ldots$, $T = e^{-n^2\pi^2 t}$

11. $u = c_n \sin(n\pi x)e^{-n^2\pi^2 t}$

12. $u = \sin(3\pi x)e^{-9\pi^2 t}$

13. $\frac{\partial^2 u}{\partial\xi\partial\eta} = 0$

14. $u(\xi,\eta) = F(\xi) + G(\eta) = F(x+at) + G(x-at)$

15. $u(x,t) = [f(x+at) + f(x-at)]/2 + \int_{x-at}^{x+at} g(s)ds/(2a)$

16. Split it into two problems, $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $u(0,y) = 0$, $u(1,y) = 0$, $u(x,0) = f(x)$, $u(x,1) = g(x)$ and $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $u(0,y) = F(y)$, $u(1,y) = G(y)$, $u(x,0) = 0$, $u(x,1) = 0$, and solve each separately by the method already outlined (separation of variables, solve an eigenvalue problem, find the solution as an infinite series, using the non-homogeneous boundary values to find the values of the constants). Finally, add the two solutions together.

17. Consider the homogeneous problem ($F(x,t) = 0$) and solve the eigenvalue problem. Expand both u and F in eigenfunction expansions, substitute into the differential equation, and solve a first order ordinary differential equation for the unknown coefficients in u.

18. $\lambda = n^2\pi^2/L^2$, $X = c_n \sin(n\pi x/L)$

19. $xe^t = e^t \sum_{n=1}^{\infty}(-1)^{n+1}\, 2L\sin(n\pi x/L)/(n\pi)$

20. $u(x,t) = \sum_{n=1}^{\infty} u_n(t)\sin(n\pi x/L)$, where $u_n(t) =$
    $(-1)^{n+1}\, 2L(e^t - e^{-n^2\pi^2 t/L^2})/(n\pi(1 + n^2\pi^2/L^2))$

1. The differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = u$ is

   Select the correct answer.

   (a) first order, linear, homogeneous
   (b) first order, linear, non-homogeneous
   (c) second order, nonlinear
   (d) second order, linear, homogeneous
   (e) second order, linear, non-homogeneous

2. When $u(x, y) = X(x)Y(y)$ is substituted into the equation $u_{xx} - u_{yy} = 0$, the resulting equations for X and Y are

   Select the correct answer.

   (a) $X'' + \lambda X = 0$, $Y'' + \lambda Y = 0$
   (b) $X'' + \lambda X = 0$, $Y'' - \lambda Y = 0$
   (c) $X'' - \lambda X = 0$, $Y'' + \lambda Y = 0$
   (d) $X' + \lambda X = 0$, $Y' + \lambda Y = 0$
   (e) $X' + \lambda X = 0$, $Y' - \lambda Y = 0$

3. The general solution of $y'' + n^2 y = 0$, $y(0) = 0$, $y(\pi) = 0$, $n = 1, 2, 3, \ldots$, is

   Select the correct answer.

   (a) $y = 0$
   (b) $y = c \cdot \sin(nx)$
   (c) $y = c \cdot \cos(nx)$
   (d) $y = c(e^{nx} - e^{-nx})$
   (e) $y = c(e^{nx} + e^{-nx})$

4. The solution of $y'' - n^2 y = 0$, $y(0) = 0$, $y(\pi) = 0$, $n = 1, 2, 3, \ldots$, is

   Select the correct answer.

   (a) $y = 0$
   (b) $y = c \cdot \sin(nx)$
   (c) $y = c \cdot \cos(nx)$
   (d) $y = c(e^{nx} - e^{-nx})$
   (e) $y = c(e^{nx} + e^{-nx})$

5. The solution of $y'' + \lambda y = 0$, $y(0) = 0$, $y(\pi) = 0$ if $\lambda = 0$ is
   Select the correct answer.
   (a) $y = ax + b$
   (b) $y = x - \pi$
   (c) $y = 0$
   (d) $y = \sin x$
   (e) $y = \cos x$

6. The differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = u$ is
   Select all that apply.
   (a) linear
   (b) nonlinear
   (c) hyperbolic
   (d) elliptic
   (e) parabolic

7. The differential equation $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = \sin u$ is
   Select all that apply.
   (a) linear
   (b) nonlinear
   (c) hyperbolic
   (d) elliptic
   (e) parabolic

8. The quantity of heat in an element of a rod of mass m is proportional to
   Select all that apply.
   (a) mass
   (b) thermal conductivity
   (c) specific heat
   (d) thermal diffusivity
   (e) temperature

9. Consider the equation $u_{xx} - u_t = 0$ with conditions $u(0,t) = 0$, $u(L,t) = 0$, $u(x,0) = f(x)$. When separating variables with $u(x,t) = X(x)T(t)$, the resulting problems for X, T are

   Select the correct answer.

   (a) $X'' + \lambda X = 0$, $X(0) = 0$, $X(L) = 0$, $T' + \lambda T = 0$, $T(0) = f(0)$
   (b) $X'' + \lambda X = 0$, $X(0) = 0$, $X(L) = 0$, $T' + \lambda T = 0$, $T(0) = f(x)$
   (c) $X'' - \lambda X = 0$, $X(0) = 0$, $X(L) = 0$, $T' + \lambda T = 0$
   (d) $X'' + \lambda X = 0$, $X(0) = 0$, $X(L) = 0$, $T' - \lambda T = 0$
   (e) $X'' + \lambda X = 0$, $X(0) = 0$, $X(L) = 0$, $T' + \lambda T = 0$

10. The solution of the eigenvalue problem from the previous problem is

    Select the correct answer.

    (a) $\lambda = n\pi/L$, $X(x) = \sin(n\pi x/L)$, $n = 1, 2, 3, \ldots$
    (b) $\lambda = n\pi/L$, $X(x) = \cos(n\pi x/L)$, $n = 1, 2, 3, \ldots$
    (c) $\lambda = (n\pi/L)^2$, $X(x) = \sin(n\pi x/L)$, $n = 1, 2, 3, \ldots$
    (d) $\lambda = (n\pi/L)^2$, $X(x) = \cos(n\pi x/L)$, $n = 0, 1, 2, \ldots$
    (e) $\lambda = (n\pi/L)$, $X(x) = \sin(n\pi x/L)$, $n = 0, 1, 2, \ldots$

11. In the previous two problems, the product solutions are

    Select the correct answer.

    (a) $\sin(n\pi x/L)e^{n\pi t/L}$
    (b) $\sin(n\pi x/L)e^{-n\pi t/L}$
    (c) $\cos(n\pi x/L)e^{(n\pi/L)^2 t}$
    (d) $\sin(n\pi x/L)e^{-(n\pi/L)^2 t}$
    (e) $\cos(n\pi x/L)e^{-n\pi t/L}$

12. In the previous three problems, the solution of the original problem is

    Select the correct answer.

    (a) $u(x,t) = \sum_{n=1}^{\infty} c_n \sin(n\pi x/L)e^{-(n\pi/L)^2 t}$, where $c_n = \int_0^L f(x)\cos(n\pi x/L)dx$
    (b) $u(x,t) = \sum_{n=1}^{\infty} c_n \cos(n\pi x/L)e^{-(n\pi/L)^2 t}$, where $c_n = \int_0^L f(x)\sin(n\pi x/L)dx$
    (c) $u(x,t) = \sum_{n=1}^{\infty} c_n \sin(n\pi x/L)e^{-(n\pi/L)^2 t}$, where $c_n = 2\int_0^L f(x)\cos(n\pi x/L)dx/L$
    (d) $u(x,t) = \sum_{n=1}^{\infty} c_n \sin(n\pi x/L)e^{-(n\pi/L)^2 t}$, where $c_n = 2\int_0^L f(x)\sin(n\pi x/L)dx/L$
    (e) $u(x,t) = \sum_{n=0}^{\infty} c_n \cos(n\pi x/L)e^{-(n\pi/L)^2 t}$, where $c_n = 2\int_0^L f(x)\cos(n\pi x/L)dx$

13. Consider the equation $u_{xx}+u_{yy}=0$ with conditions $u(0,y)=0$, $u(L,y)=0$, $u(x,0)=f(x)$, $u(x,H)=0$. When separating variables with $u(x,y)=X(x)Y(y)$, the resulting problems for X, Y are

    Select the correct answer.

    (a) $X''+\lambda X=0$, $X(0)=0$, $X(L)=0$, $Y''+\lambda Y=0$, $Y(H)=0$
    (b) $X''+\lambda X=0$, $X(0)=0$, $X(L)=0$, $Y''-\lambda Y=0$, $Y(H)=0$
    (c) $X''+\lambda X=0$, $X(0)=0$, $X(L)=0$, $Y''-\lambda Y=0$, $Y(0)=0$
    (d) $X''-\lambda X=0$, $X(0)=0$, $X(L)=0$, $Y''-\lambda Y=0$, $Y(H)=f(0)$
    (e) $X''+\lambda X=0$, $X(0)=0$, $X(L)=0$, $Y''+\lambda Y=0$, $Y(0)=f(x)$

14. The solution of the eigenvalue problem from the previous problem is

    Select the correct answer.

    (a) $\lambda=n\pi/L$, $X(x)=\sin(n\pi x/L)$, $n=1,2,3,\ldots$
    (b) $\lambda=n\pi/L$, $X(x)=\cos(n\pi x/L)$, $n=1,2,3,\ldots$
    (c) $\lambda=(n\pi/L)^2$, $X(x)=\sin(n\pi x/L)$, $n=1,2,3,\ldots$
    (d) $\lambda=(n\pi/L)^2$, $X(x)=\cos(n\pi x/L)$, $n=0,1,2,\ldots$
    (e) $\lambda=(n\pi/L)$, $X(x)=\sin(n\pi x/L)$, $n=0,1,2,\ldots$

15. In the previous two problems, the product solutions are

    Select the correct answer.

    (a) $\sin(n\pi x/L)\sinh(n\pi(y-H)/L)$
    (b) $\cos(n\pi x/L)\sinh(-n\pi(y-H)/L)$
    (c) $\cos(n\pi x/L)\cosh((n\pi/L)^2(y-H))$
    (d) $\sin(n\pi x/L)\cosh(-(n\pi/L)^2(y-H))$
    (e) $\cos(n\pi x/L)\cosh(-n\pi(y-H)/L)$

16. In the previous three problems, the solution of the original problem is

    Select the correct answer.

    (a) $u(x,t)=\sum_{n=1}^{\infty} c_n \sin(n\pi x/L)\sinh(n\pi(y-H)/L)$, where
    $c_n=\int_0^L f(x)\cos(n\pi x/L)dx/\sinh(-n\pi H/L)$
    (b) $u(x,t)=\sum_{n=1}^{\infty} c_n \sin(n\pi x/L)\sinh(n\pi(y-H)/L)$, where
    $c_n=2\int_0^L f(x)\sin(n\pi x/L)dx/(L\sinh(-n\pi H/L))$
    (c) $u(x,t)=\sum_{n=1}^{\infty} c_n \sin(n\pi x/L)\cosh(n\pi(y-H)/L)$, where
    $c_n=2\int_0^L f(x)\cos(n\pi x/L)dx/(L\sinh(-n\pi H/L))$
    (d) $u(x,t)=\sum_{n=1}^{\infty} c_n \sin(n\pi x/L)\cosh((n\pi/L)^2(y-H))$, where
    $c_n=2\int_0^L f(x)\sin(n\pi x/L)dx/(L\sinh(-n\pi H/L))$
    (e) $u(x,t)=\sum_{n=0}^{\infty} c_n \cos(n\pi x/L)\sinh((n\pi/L)^2(y-H))$, where
    $c_n=\int_0^L f(x)\cos(n\pi x/L)dx/\sinh(-n\pi H/L)$

17. In the problem $\frac{\partial^2 u}{\partial x^2} + xe^t = \frac{\partial u}{\partial t}$, $u(0,t) = 0$, $u(L,t) = 0$, $u(x,0) = 0$, the eigenvalues and eigenfunctions of the underlying homogeneous problem are

    Select the correct answer.

    (a) $\lambda = n^2$, $X = \sin(nx)$
    (b) $\lambda = n^2$, $X = \cos(nx)$
    (c) $\lambda = n^2\pi^2/L^2$, $X = \sin(n\pi x/L)$
    (d) $\lambda = n^2\pi^2/L^2$, $X = \cos(n\pi x/L)$
    (e) $\lambda = n^2\pi^2$, $X = \sin(n\pi x)$

18. In the previous problem, the eigenfunction expansion if xe^t is

    Select the correct answer.

    (a) $e^t \sum_{n=1}^{\infty} (-1)^n \, 2L \cos(n\pi x/L)/(n\pi)$
    (b) $e^t \sum_{n=1}^{\infty} (-1)^n \, 2L \sin(n\pi x/L)/(n^2\pi^2)$
    (c) $e^t \sum_{n=1}^{\infty} (-1)^n \, 2L \cos(n\pi x/L)/(n^2\pi^2)$
    (d) $e^t \sum_{n=1}^{\infty} (-1)^{n+1} \, 2L \cos(n\pi x/L)/(n\pi)$
    (e) $e^t \sum_{n=1}^{\infty} (-1)^{n+1} \, 2L \sin(n\pi x/L)/(n\pi)$

19. In the previous two problems, the solution for u takes the form

    Select the correct answer.

    (a) $u(x,t) = \sum_{n=1}^{\infty} u_n(t) \sin(n\pi x/L)$
    (b) $u(x,t) = \sum_{n=1}^{\infty} u_n(t) \cos(n\pi x)$
    (c) $u(x,t) = \sum_{n=1}^{\infty} u_n(t) \sin(n^2\pi^2 x/L^2)$
    (d) $u(x,t) = \sum_{n=1}^{\infty} u_n(t) \cos(n^2\pi^2 x/L^2)$
    (e) $u(x,t) = \sum_{n=1}^{\infty} u_n(t) \sin(n\pi x)$

20. In the previous three problems, the solution for $u_n(t)$ is

    Select the correct answer.

    (a) $u_n(t) = (-1)^n \, 2L(e^t - e^{-n^2\pi^2 t/L^2})/(n\pi(1 + n\pi/L))$
    (b) $u_n(t) = (-1)^n \, 2L(e^t + e^{-n^2\pi^2 t/L^2})/(n\pi(1 + n^2\pi^2/L^2))$
    (c) $u_n(t) = (-1)^{n+1} \, 2L(e^t + e^{-n^2\pi^2 t/L^2})/(n\pi(1 + n^2\pi^2/L^2))$
    (d) $u_n(t) = (-1)^{n+1} \, 2L(e^t - e^{-n^2\pi^2 t/L^2})/(n\pi(1 + n^2\pi^2/L^2))$
    (e) $u_n(t) = 2L(e^t - e^{-n^2\pi^2 t/L^2})/(n\pi(1 + n^2\pi^2/L^2))$

ANSWER KEY

Zill Differential Equations 9e Chapter 12 Form C

1. d
2. a
3. b
4. a
5. c
6. a, d
7. b, c
8. a, c, e
9. e
10. c
11. d
12. d
13. b
14. c
15. a
16. b
17. c
18. e
19. a
20. d

1. The differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is

   Select the correct answer.

   (a) first order, linear, homogeneous
   (b) first order, linear, non-homogeneous
   (c) second order, nonlinear
   (d) second order, linear, homogeneous
   (e) second order, linear, non-homogeneous

2. When $u(x, y) = X(x)Y(y)$ is substituted into the equation $u_{xx} + u_{yy} = 0$, the resulting equations for X and Y are

   Select the correct answer.

   (a) $X' + \lambda X = 0$, $Y' + \lambda Y = 0$
   (b) $X' + \lambda X = 0$, $Y' - \lambda Y = 0$
   (c) $X'' + \lambda X = 0$, $Y'' + \lambda Y = 0$
   (d) $X'' + \lambda X = 0$, $Y'' - \lambda Y = 0$
   (e) $X'' - \lambda X = 0$, $Y'' + \lambda Y = 0$

3. The general solution of $y'' + n^2\pi^2 y = 0$, $y(0) = 0$, $y(1) = 0$, $n = 1, 2, 3, \ldots$, is

   Select the correct answer.

   (a) $y = 0$
   (b) $y = c \cdot \sin(n\pi x)$
   (c) $y = c \cdot \cos(n\pi x)$
   (d) $c(e^{n\pi x} - e^{-n\pi x})$
   (e) $c(e^{n\pi x} + e^{-n\pi x})$

4. The solution of $y'' - n^2\pi^2 y = 0$, $y(0) = 0$, $y(1) = 0$, $n = 1, 2, 3, \ldots$, is

   Select the correct answer.

   (a) $y = 0$
   (b) $y = c \cdot \sin(n\pi x)$
   (c) $y = c \cdot \cos(n\pi x)$
   (d) $c(e^{n\pi x} - e^{-n\pi x})$
   (e) $c(e^{n\pi x} + e^{-n\pi x})$

5. The solution of $y'' + \lambda y = 0$, $y(0) = 0$, $y(\pi) = 0$ if $\lambda = 0$ is

   Select the correct answer.

   (a) $y = \sin x$
   (b) $y = \cos x$
   (c) $y = 0$
   (d) $y = ax + b$
   (e) $y = x - \pi$

6. The differential equation $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = u$ is

   Select all that apply.

   (a) nonlinear
   (b) linear
   (c) hyperbolic
   (d) elliptic
   (e) parabolic

7. The differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = u$ is

   Select all that apply.

   (a) linear
   (b) nonlinear
   (c) hyperbolic
   (d) elliptic
   (e) parabolic

8. The wave equation for a vibrating string is derived using the assumptions

   Select all that apply.

   (a) the string is perfectly flexible.
   (b) the displacements may be large.
   (c) the tension acts perpendicular to the string.
   (d) the tension is large compared with gravity.
   (e) the string is homogeneous.

9. Consider the equation $u_{xx} - u_{tt} = 0$ with conditions $\frac{du}{dx}(0,t) = 0$, $\frac{du}{dx}(L,t) = 0$, $u(x,0) = f(x)$, $\frac{du}{dt}(0) = 0$. When separating variables with $u(x,t) = X(x)T(t)$, the resulting problems for X, T are

   Select the correct answer.

   (a) $X'' + \lambda X = 0$, $X'(0) = 0$, $X'(L) = 0$, $T'' + \lambda T = 0$, $T(0) = 0$
   (b) $X'' - \lambda X = 0$, $X'(0) = 0$, $X'(L) = 0$, $T'' + \lambda T = 0$, $T'(0) = 0$
   (c) $X'' + \lambda X = 0$, $X'(0) = 0$, $X'(L) = 0$, $T'' + \lambda T = 0$, $T'(0) = 0$
   (d) $X'' + \lambda X = 0$, $X'(0) = 0$, $X'(L) = 0$, $T'' - \lambda T = 0$, $T'(0) = 0$
   (e) $X'' + \lambda X = 0$, $X'(0) = 0$, $X'(L) = 0$, $T'' + \lambda T = 0$, $T(0) = f(x)$

10. The solution of the eigenvalue problem from the previous problem is

    Select the correct answer.

    (a) $\lambda = n\pi/L$, $X(x) = \sin(n\pi x/L)$, $n = 1, 2, 3, \ldots$
    (b) $\lambda = n\pi/L$, $X(x) = \cos(n\pi x/L)$, $n = 0, 1, 2, \ldots$
    (c) $\lambda = (n\pi/L)$, $X(x) = \sin(n\pi x/L)$, $n = 0, 1, 2, \ldots$
    (d) $\lambda = (n\pi/L)^2$, $X(x) = \sin(n\pi x/L)$, $n = 1, 2, 3, \ldots$
    (e) $\lambda = (n\pi/L)^2$, $X(x) = \cos(n\pi x/L)$, $n = 0, 1, 2, \ldots$

11. In the previous two problems, the product solutions are

    Select the correct answer.

    (a) $\sin(n\pi x/L)\sin(n\pi t/L)$
    (b) $\sin(n\pi x/L)\cos(n\pi t/L)$
    (c) $\cos(n\pi x/L)\cos(n\pi t/L)$
    (d) $\sin(n\pi x/L)e^{n\pi t/L}$
    (e) $\cos(n\pi x/L)e^{-n\pi t/L}$

12. In the previous three problems, the solution of the original problem is

    Select the correct answer.

    (a) $u(x,t) = \sum_{n=1}^{\infty} c_n \sin(n\pi x/L)e^{-(n\pi/L)t}$, where $c_n = \int_0^L f(x)\cos(n\pi x/L)dx$
    (b) $u(x,t) = \sum_{n=0}^{\infty} c_n \cos(n\pi x/L)\cos(n\pi t/L)$ where $c_0 = \int_0^L f(x)dx/L$, $c_n = 2\int_0^L f(x)\cos(n\pi x/L)dx/L$
    (c) $u(x,t) = \sum_{n=1}^{\infty} c_n \sin(n\pi x/L)\sin(n\pi t/L)$, where $c_n = 2\int_0^L f(x)\cos(n\pi x/L)dx/L$
    (d) $u(x,t) = \sum_{n=1}^{\infty} c_n \sin(n\pi x/L)e^{-n\pi t/L}$, where $c_n = 2\int_0^L f(x)\sin(n\pi x/L)dx/L$
    (e) $u(x,t) = \sum_{n=0}^{\infty} c_n \cos(n\pi x/L)e^{-n\pi t/L}$, where $c_n = \int_0^L f(x)\cos(n\pi x/L)dx$

13. Consider the equation $u_{xx} - u_t = 0$ with conditions $u(0,t) = 0$, $u_x(L,t) = 0$, $u(x,0) = f(x)$. When separating variables with $u(x,t) = X(x)T(t)$, the resulting problems for X, T are

    Select the correct answer.

    (a) $X'' + \lambda X = 0$, $X(0) = 0$, $X'(L) = 0$, $T' - \lambda T = 0$, $T(0) = f(x)$
    (b) $X'' + \lambda X = 0$, $X(0) = 0$, $X'(L) = 0$, $T' + \lambda T = 0$, $T(0) = f(x)$
    (c) $X'' - \lambda X = 0$, $X(0) = 0$, $X'(L) = 0$, $T' + \lambda T = 0$
    (d) $X'' + \lambda X = 0$, $X(0) = 0$, $X'(L) = 0$, $T' - \lambda T = 0$
    (e) $X'' + \lambda X = 0$, $X(0) = 0$, $X'(L) = 0$, $T' + \lambda T = 0$

14. The solution of the eigenvalue problem from the previous problem is

    Select the correct answer.

    (a) $\lambda = n\pi/L$, $X(x) = \sin(n\pi x/L)$, $n = 1, 2, 3, \ldots$
    (b) $\lambda = n\pi/L$, $X(x) = \cos(n\pi x/L)$, $n = 0, 1, 2, \ldots$
    (c) $\lambda = (n - 1/2)\pi/L$, $X(x) = \sin((n - 1/2)\pi x/L)$, $n = 1, 2, 3, \ldots$
    (d) $\lambda = ((n - 1/2)\pi/L)^2$, $X(x) = \sin((n - 1/2)\pi x/L)$, $n = 1, 2, 3, \ldots$
    (e) $\lambda = ((n - 1/2)\pi/L)^2$, $X(x) = \cos((n - 1/2)\pi x/L)$, $n = 1, 2, 3, \ldots$

15. In the previous two problems, the product solutions are

    Select the correct answer.

    (a) $\sin((n - 1/2)\pi x/L)e^{-(n-1/2)^2\pi^2 t/L^2}$
    (b) $\cos((n - 1/2)\pi x/L)e^{-(n-1/2)^2\pi^2 t/L^2}$
    (c) $\sin((n - 1/2)\pi x/L)e^{-(n-1/2)\pi t/L}$
    (d) $\sin((n - 1/2)\pi x/L)e^{(n-1/2)^2\pi^2 t/L^2}$
    (e) $\cos((n - 1/2)\pi x/L)e^{(n-1/2)^2\pi^2 t/L^2}$

16. In the previous three problems, the solution of the original problem is

    Select the correct answer.

    (a) $u(x,t) = \sum_{n=1}^{\infty} c_n \cos((n - 1/2)\pi x/L)e^{-(n-1/2)^2\pi^2 t/L^2}$, where $c_n = 2\int_0^L f(x)\cos((n - 1/2)\pi x/L)dx/L$
    (b) $u(x,t) = \sum_{n=1}^{\infty} c_n \sin((n - 1/2)\pi x/L)e^{-(n-1/2)^2\pi^2 t/L^2}$, where $c_n = 2\int_0^L f(x)\sin((n - 1/2)\pi x/L)dx/L$
    (c) $u(x,t) = \sum_{n=1}^{\infty} c_n \cos((n - 1/2)\pi x/L)e^{(n-1/2)^2\pi^2 t/L^2}$, where $c_n = \int_0^L f(x)\cos((n - 1/2)\pi x/L)dx$
    (d) $u(x,t) = \sum_{n=0}^{\infty} c_n \sin((n - 1/2)\pi x/L)e^{(n-1/2)^2\pi^2 t/L^2}$, where $c_n = 2\int_0^L f(x)\sin((n - 1/2)\pi x/L)dx/L$
    (e) $u(x,t) = \sum_{n=0}^{\infty} c_n \sin((n - 1/2)\pi x/L)e^{-(n-1/2)^2\pi^2 t/L^2}$, where $c_n = \int_0^L f(x)\sin((n - 1/2)\pi x/L)dx$

17. The model describing the temperature in a rod where the temperature at the left end is zero and where there is heat transfer from the right boundary into the external medium is

    Select the correct answer.

    (a) $k\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$, $u(0,t) = 0$, $\frac{\partial u}{\partial x}(L,t) = -hu(L,t)$, $h > 0$, $u(x,0) = f(x)$
    (b) $k\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$, $u(0,t) = 0$, $\frac{\partial u}{\partial x}(L,t) = hu(L,t)$, $h > 0$, $u(x,0) = f(x)$
    (c) $k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$, $u(0,t) = 0$, $\frac{\partial u}{\partial x}(L,t) = -hu(L,t)$, $h > 0$, $u(x,0) = f(x)$
    (d) $k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$, $u(0,t) = 0$, $\frac{\partial u}{\partial x}(L,t) = hu(L,t)$, $h > 0$, $u(x,0) = f(x)$
    (e) $k\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial t^2} = 0$, $u(0,t) = 0$, $\frac{\partial u}{\partial x}(L,t) = -hu(L,t)$, $h > 0$, $u(x,0) = f(x)$

18. After separating variables in the previous problem, the eigenvalue problem becomes

    Select the correct answer.

    (a) $X'' + \lambda X = 0$, $X(0) = 0$, $X(L) = 0$
    (b) $X'' + \lambda X = 0$, $X(0) = 0$, $X'(L) = -hX(L)$
    (c) $X'' + \lambda X = 0$, $X(0) = 0$, $X(L) = -hX'(0)$
    (d) $X'' + \lambda X = 0$, $X(0) = 0$, $X'(L) = 0$
    (e) $X'' + \lambda X = 0$, $X(0) = 0$, $X'(L) = hX(L)$

19. The solution of the eigenvalue problem in the previous problem is

    Select the correct answer.

    (a) $\lambda = ((n-1/2)\pi/L)^2$, $X = \sin((n-1/2)\pi x/L)$
    (b) $\sqrt{\lambda}/h = \tan(\sqrt{\lambda}L)$, $X = \sin(\sqrt{\lambda}x)$
    (c) $\lambda = (n\pi/L)^2$, $X = \sin(n\pi x/L)$
    (d) $\sqrt{\lambda}/h = -\tan(\sqrt{\lambda}L)$, $X = \sin(\sqrt{\lambda}x)$
    (e) $\sqrt{\lambda}h = -\tan(\sqrt{\lambda}L)$, $X = \sin(\sqrt{\lambda}x)$

20. The solution of the previous three problems is $\sum_{n=1}^{\infty} c_n X_n(x) e^{-\lambda_n kt}$, where X_n and λ_n are given in the previous problem and

    Select the correct answer.

    (a) $c_n = \int_0^L f(x)dx$
    (b) $c_n = \int_0^L f(x)X_n(x)dx$
    (c) $c_n = \int_0^L f(x)X_n(x)dx / \int_0^L X_n(x)dx$
    (d) $c_n = \int_0^L X_n^2(x)dx / \int_0^L f(x)X_n(x)dx$
    (e) $c_n = \int_0^L f(x)X_n(x)dx / \int_0^L X_n^2(x)dx$

ANSWER KEY

Zill Differential Equations 9e Chapter 12 Form D

1. d
2. d or e
3. b
4. a
5. c
6. b, e
7. a, d
8. a, d, e
9. c
10. e
11. c
12. b
13. e
14. d
15. a
16. b
17. c
18. b
19. d
20. e

1. For the differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = u$, what is the order? Is it linear or nonlinear? If it is linear, is it homogeneous or non-homogeneous.

2. What two ordinary differential equations result when $u(x, y) = X(x)Y(y)$ is substituted into the partial differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = u$?

3. The general solution of $y'' + n^2\pi^2 y = 0$, $y'(0) = 0$, $y'(1) = 0$, $n = 1, 2, 3, \ldots$, is

   Select the correct answer.

   (a) $y = 0$
   (b) $y = c \cdot \sin(n\pi x)$
   (c) $y = c \cdot \cos(n\pi x)$
   (d) $c(e^{n\pi x} - e^{-n\pi x})$
   (e) $c(e^{n\pi x} + e^{-n\pi x})$

4. The solution of $y'' - n^2\pi^2 y = 0$, $y'(0) = 0$, $y'(1) = 0$, $n = 1, 2, 3, \ldots$, is

   Select the correct answer.

   (a) $y = 0$
   (b) $y = c \cdot \sin(n\pi x)$
   (c) $y = c \cdot \cos(n\pi x)$
   (d) $c(e^{n\pi x} - e^{-n\pi x})$
   (e) $c(e^{n\pi x} + e^{-n\pi x})$

5. The solution of $y'' + \lambda y = 0$, $y'(0) = 0$, $y'(1) = 0$ if $\lambda = 0$ is

   Select the correct answer.

   (a) $y = \sin x$
   (b) $y = \cos x$
   (c) $y = ax + b$
   (d) $y = 0$
   (e) $y = c$

6. Classify the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial t} = 0$ as hyperbolic, parabolic, or elliptic.

7. Laplace's equation can be a model for

   Select all that apply.

   (a) steady state temperature distribution in a one-dimensional rod
   (b) steady state temperature distribution in a two-dimensional plate
   (c) electrostatic potentials
   (d) heat conduction in a rod
   (e) velocity in fluid flows

8. Write down the equation and initial and boundary conditions for a vibrating string of length L, fixed at both ends. Suppose that the middle is pulled up a distance $L/10$ and released. Define all terms.

9. The condition $\frac{du}{dx}(x_0, t) + hu(x_0, t) = 0$ in a heat equation problem is referred to as

   Select the correct answer.

   (a) a Dirichlet condition
   (b) a Neumann condition
   (c) a Robin condition
   (d) none of the above

10. Consider the equation $u_{xx} + u_{yy} = 0$ with conditions $u(0, y) = 0$, $u(1, y) = 0$, $u(x, 0) = f(x)$, $\frac{du}{dy}(x, 1) = 0$. Describe a physical situation having this problem as a mathematical model.

11. In the previous problem, separate variables. What is the differential equation and boundary conditions for the function of the x variable? What is the differential equation and boundary conditions for the function of the y variable?

12. In the previous problem, what are the solutions of the resulting eigenvalue problem? What are the solutions of the problem involving the other variable?

13. In the previous problem, what are the product solutions of the original partial differential equation.

14. In the previous four problems, if $f(x) = \sin(\pi x)$, what is the Fourier series solution?

15. Consider the equation $u_{xx} - u_t = 0$ with conditions $u(0, t) = 0$, $u(1, t) = 0$, $u(x, 0) = f(x)$. When separating variables with $u(x, t) = X(x)T(t)$, the resulting problems for X, T are

   Select the correct answer.

   (a) $X'' + \lambda X = 0$, $X(0) = 0$, $X(1) = 0$, $T' + \lambda T = 0$
   (b) $X'' - \lambda X = 0$, $X(0) = 0$, $X(1) = 0$, $T' + \lambda T = 0$
   (c) $X'' + \lambda X = 0$, $X(0) = 0$, $X(1) = 0$, $T' - \lambda T = 0$
   (d) $X'' + \lambda X = 0$, $X(0) = 0$, $X(1) = 0$, $T' + \lambda T = 0$, $T(0) = f(0)$
   (e) $X'' + \lambda X = 0$, $X(0) = 0$, $X(1) = 0$, $T' + \lambda T = 0$, $T(0) = f(x)$

16. The solution of the eigenvalue problem from the previous problem is

   Select the correct answer.

   (a) $\lambda = n\pi$, $X(x) = \sin(n\pi x)$, $n = 1, 2, 3, \ldots$
   (b) $\lambda = n\pi$, $X(x) = \cos(n\pi x)$, $n = 1, 2, 3, \ldots$
   (c) $\lambda = (n\pi)^2$, $X(x) = \sin(n\pi x)$, $n = 1, 2, 3, \ldots$
   (d) $\lambda = (n\pi)^2$, $X(x) = \cos(n\pi x)$, $n = 0, 1, 2, \ldots$
   (e) $\lambda = (n\pi)$, $X(x) = \sin(n\pi x)$, $n = 0, 1, 2, \ldots$

17. In the previous two problems, the product solutions are

    Select the correct answer.

    (a) $\sin(n\pi x)e^{n\pi t}$
    (b) $\sin(n\pi x)e^{-n\pi t}$
    (c) $\cos(n\pi x)e^{-(n\pi)^2 t}$
    (d) $\sin(n\pi x)e^{-(n\pi)^2 t}$
    (e) $\cos(n\pi x)e^{-n\pi t}$

18. In the previous three problems, the solution of the original problem is

    Select the correct answer.

    (a) $u(x,t) = \sum_{n=1}^{\infty} c_n \sin(n\pi x)e^{-(n\pi)^2 t}$, where $c_n = \int_0^1 f(x)\sin(n\pi x)dx$
    (b) $u(x,t) = \sum_{n=1}^{\infty} c_n \sin(n\pi x)e^{-(n\pi)^2 t}$, where $c_n = 2\int_0^1 f(x)\sin(n\pi x)dx/L$
    (c) $u(x,t) = \sum_{n=0}^{\infty} c_n \cos(n\pi x)e^{-(n\pi)^2 t}$, where $c_n = \int_0^1 f(x)\cos(n\pi x)dx$
    (d) $u(x,t) = \sum_{n=0}^{\infty} c_n \cos(n\pi x)e^{-(n\pi)^2 t}$, where $c_n = \int_0^1 f(x)\sin(n\pi x)dx$
    (e) $u(x,t) = \sum_{n=1}^{\infty} c_n \sin(n\pi x)e^{-(n\pi)^2 t}$, where $c_n = 2\int_0^1 f(x)\cos(n\pi x)dx$

19. Solve the eigenvalue problem $X'' + \lambda X = 0$, $X(0) = 0$, $X'(1) = -hX(1)$.

20. Describe the eigenvalue solutions of the previous problem.

ANSWER KEY

Zill Differential Equations 9e Chapter 12 Form E

1. second order, linear, homogeneous
2. $X'' + \lambda X = 0$, $Y'' - (\lambda + 1)Y = 0$
3. c
4. a
5. e
6. parabolic
7. b, c, e
8. $c^2 u_{xx} - u_{tt} = 0$, $u(0,t) = 0$, $u(L,t) = 0$, $u(x,0) = \left\{ \begin{array}{ccc} x/5 & \text{if} & 0 \le x \le L/2 \\ (L-x)/5 & \text{if} & L/2 \le x \le L \end{array} \right\}$, $u_t(x,0) = 0$, where $u(x,t)$ is the deflection from the x-axis at point x and time t, and c is the speed of propagation of the wave.
9. c
10. Steady state temperature distribution in a plate (length of side is 1), with zero temperature along the sides $x = 0$ and $x = 1$, insulated along the side $y = 1$, and with a specified temperature of $f(x)$ along the side $y = 0$.
11. $X'' + \lambda X = 0$, $X(0) = 0$, $X(1) = 0$, $Y'' - \lambda Y = 0$, $Y'(1) = 0$
12. $\lambda = n^2\pi^2$, $X = c_n \sin(n\pi x)$, $Y = \cosh(n\pi(y-1))$
13. $u = c_n \sin(n\pi x)\cosh(n\pi(y-1))$
14. $u = \sin(\pi x)\cosh(\pi(y-1))/\cosh(\pi)$
15. a
16. c
17. d
18. b
19. $\sqrt{\lambda}/h = -\tan(\sqrt{\lambda})$, $X = \sin(\sqrt{\lambda}x)$
20. The eigenvalues occur along the positive λ-axis where the graphs of $y = -\tan(\sqrt{\lambda})$ and $y = \sqrt{\lambda}/h$ intersect.

1. The differential equation $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = \sin(x+y)u$ is

   Select the correct answer.

   (a) first order, linear, homogeneous
   (b) first order, linear, non-homogeneous
   (c) second order, nonlinear
   (d) second order, linear, homogeneous
   (e) second order, linear, non-homogeneous

2. When $u(x, y) = X(x)T(t)$ is substituted into the equation $u_{xx} - u_t = 0$, the resulting equations for X and T are

   Select the correct answer.

   (a) $X'' + \lambda X = 0$, $T' + \lambda T = 0$
   (b) $X'' + \lambda X = 0$, $T' - \lambda T = 0$
   (c) $X'' - \lambda X = 0$, $T' + \lambda T = 0$
   (d) $X' + \lambda X = 0$, $T' + \lambda T = 0$
   (e) $X' + \lambda X = 0$, $T' - \lambda T = 0$

3. What is the solution of $y'' + \lambda y = 0$, $y'(0) = 0$, $y'(\pi) = 0$ if $\lambda > 0$?

4. What is the solution of $y'' + \lambda y = 0$, $y'(0) = 0$, $y'(\pi) = 0$ if $\lambda < 0$?

5. What is the solution of $y'' + \lambda y = 0$, $y'(0) = 0$, $y'(\pi) = 0$ if $\lambda = 0$?

6. The differential equation $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = u$ is

   Select all that apply.

   (a) linear
   (b) nonlinear
   (c) hyperbolic
   (d) elliptic
   (e) parabolic

7. The differential equation $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = u\frac{\partial u}{\partial x}$ is

   Select all that apply.

   (a) linear
   (b) nonlinear
   (c) hyperbolic
   (d) elliptic
   (e) parabolic

8. Consider the problem $\frac{d^2u}{dx^2} - \frac{du}{dt} = 0$, $u(0,t) = 0$, $u(1,t) = 0$, $u(x,0) = 1 - x^2$. Describe a physical situation having this as a mathematical model.

9. In the previous problem, separate variables using $u(x,t) = X(x)T(t)$. What are the new problems for X and T?

10. What are the eigenvalues and eigenfunctions for the previous problem?

11. Using the results of the previous two problems, what is the solution of the problem $\frac{d^2u}{dx^2} - \frac{du}{dt} = 0$, $u(0,t) = 0$, $u(1,t) = 0$, $u(x,0) = 1 - x^2$?

12. Consider the equation $u_{xx} + u_{yy} = 0$ with conditions $\frac{du}{dx}(0,y) = 0$, $\frac{du}{dx}(1,y) = 0$, $u(x,0) = 0$, $u(x,2) = f(x)$. When separating variables with $u(x,t) = X(x)Y(y)$, the resulting problems for X, Y are

    Select the correct answer.

    (a) $X'' + \lambda X = 0$, $X'(0) = 0$, $X'(1) = 0$, $Y'' + \lambda Y = 0$, $Y(0) = 0$
    (b) $X'' - \lambda X = 0$, $X'(0) = 0$, $X'(1) = 0$, $Y'' + \lambda Y = 0$, $Y(0) = f(0)$
    (c) $X'' + \lambda X = 0$, $X'(0) = 0$, $X'(1) = 0$, $Y'' - \lambda Y = 0$, $Y(0) = 0$
    (d) $X'' + \lambda X = 0$, $X'(0) = 0$, $X'(1) = 0$, $Y'' - \lambda Y = 0$, $Y(0) = f(0)$
    (e) $X'' + \lambda X = 0$, $X'(0) = 0$, $X'(1) = 0$, $Y'' + \lambda Y = 0$, $Y(0) = f(x)$

13. The solution of the eigenvalue problem from the previous question is

    Select the correct answer.

    (a) $\lambda = n\pi$, $X(x) = \sin(n\pi x)$, $n = 1, 2, 3, \ldots$
    (b) $\lambda = n\pi$, $X(x) = \cos(n\pi x)$, $n = 0, 1, 2, \ldots$
    (c) $\lambda = (n\pi)^2$, $X(x) = \sin(n\pi x)$, $n = 1, 2, 3, \ldots$
    (d) $\lambda = (n\pi)^2$, $X(x) = \cos(n\pi x)$, $n = 0, 1, 2, \ldots$
    (e) $\lambda = (n\pi)$, $X(x) = \sin(n\pi x)$, $n = 0, 1, 2, 3, \ldots$

14. In the previous two problems, the product solutions are

    Select the correct answer.

    (a) $\cos(n\pi x)\sinh(n\pi y)$
    (b) $\sin(n\pi x)\cosh(n\pi y)$
    (c) $\sin(n\pi x)\sinh(n\pi y)$
    (d) $\cos(n\pi x)\cosh(n\pi y)$
    (e) $\cos(n\pi x)\sinh((n^2\pi^2 y)$

15. In the previous three problems, the solution of the original problem is

    Select the correct answer.

    (a) $u(x,t) = \sum_{n=1}^{\infty} c_n \sin(n\pi x) \cosh h(n\pi y)$, where $c_n = \int_0^2 f(x) \sin(n\pi x)\, dx / \cosh(2n\pi)$
    (b) $u(x,t) = \sum_{n=1}^{\infty} c_n \cos(n\pi x) \sinh(n\pi y)$, where $c_n = 2\int_0^1 f(x) \cos(n\pi x)\, dx / \sinh(2n\pi)$
    (c) $u(x,t) = \sum_{n=1}^{\infty} c_n \sin(n\pi x) e^{-n\pi y}$, where $c_n = 2\int_0^2 f(x) \sin(n\pi x)\, dx / e^{-2n\pi}$
    (d) $u(x,t) = \sum_{n=1}^{\infty} c_n \sin(n\pi x) \sinh(n\pi y)$, where $c_n = 2\int_0^2 f(x) \sin(n\pi x)\, dx / \sinh(2n\pi)$
    (e) $u(x,t) = \sum_{n=0}^{\infty} c_n \cos(n\pi x) \cosh(n\pi y)$, where $c_0 = \int_0^2 f(x) \cos(n\pi x)\, dx / (2\cosh(2n\pi))$,

    $c_n = \int_0^2 f(x) \cos(n\pi x) dx / \cosh(2n\pi)$

16. Consider the problem $u_{xx} - u_{tt} = 0$, $u_x(0,t) = hu(0,t)$, $u(1,t) = 0$, $u(x,0) = f(x)$, $u_t(x,0) = 0$. Describe a physical situation modeled by this problem.

17. What are the resulting problems when you separate variables in the previous problem using $u(x,t) = X(x)T(t)$?

18. Solve the eigenvalue problem you derived from the previous problem.

19. What are the product solutions for the previous three problems?

20. Combine the results of the previous three problems to solve the problem $u_{xx} - u_{tt} = 0$, $u_x(0,t) = hu(0,t)$, $u(1,t) = 0$, $u(x,0) = f(x)$, $u_t(x,0) = 0$.

ANSWER KEY

Zill Differential Equations 9e Chapter 12 Form F

1. d
2. a
3. $\lambda = n^2$, $y = c_n \cos(nx)$, $n = 1, 2, 3, \ldots$
4. $y = 0$
5. $y = c$
6. a, e
7. b, c
8. Temperature distribution in a rod of length one, with temperature fixed at zero at both ends and an initial temperature distribution of $f(x) = 1 - x^2$.
9. $X'' + \lambda X = 0$, $X(0) = 0$, $X(1) = 0$, $T' + \lambda T = 0$
10. $\lambda = n^2\pi^2$, $X = \sin(n\pi x)$
11. $u(x,t) = \sum_{n=1}^{\infty} c_n \sin(n\pi x) e^{-n^2\pi^2 t}$, where $c_n = 2/(n\pi) + 4(1 - \cos(n\pi))/(n\pi)^3$
12. c
13. d
14. a
15. b
16. Vibrations of a string, tightly stretched between $x = 0$ and $x = 1$. It is fixed at $x = 1$ and attached to a vertically oscillating mechanism at $x = 0$. Its initial position is $f(x)$, and its initial velocity is zero.
17. $X'' + \lambda X = 0$, $X'(0) = hX(0)$, $X(1) = 0$, $T'' + \lambda T = 0$, $T'(0) = 0$
18. $\sqrt{\lambda}/h = -\tan(\sqrt{\lambda})$, $X = \sin(\sqrt{\lambda}(x-1))$, $n = 1, 2, 3, \ldots$
19. $u = \sin(\sqrt{\lambda}(x-1))\cos(\sqrt{\lambda}t)$
20. $u = \sum_{n=1}^{\infty} a_n u_n(x,t)$, where u_n is found from the previous problem and
    $a_n = \int_0^1 f(x)\sin(\sqrt{\lambda}(x-1))dx / \int_0^1 \sin^2(\sqrt{\lambda}(x-1))dx$

1. For the differential equation $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = e^t u$, what is the order? Is it linear or nonlinear? If it is linear, is it homogeneous or non-homogeneous.

2. What two ordinary differential equations result when $u(x,t) = X(x)T(t)$ is substituted into the partial differential equation $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$?

3. The solution of $y'' + n^2 y = 0$, $y'(0) = 0$, $y'(\pi) = 0$, $n = 1, 2, 3, \ldots$, is

   Select the correct answer.

   (a) $y = 0$
   (b) $y = c \cdot \sin(nx)$
   (c) $y = c \cdot \cos(nx)$
   (d) $c(e^{nx} - e^{-nx})$
   (e) $c(e^{nx} + e^{-nx})$

4. The solution of $y'' - n^2 y = 0$, $y'(0) = 0$, $y'(\pi) = 0$, $n = 1, 2, 3, \ldots$, is

   Select the correct answer.

   (a) $y = 0$
   (b) $y = c \cdot \sin(nx)$
   (c) $y = c \cdot \cos(nx)$
   (d) $c(e^{nx} - e^{-nx})$
   (e) $c(e^{nx} + e^{-nx})$

5. The solution of $y'' + \lambda y = 0$, $y'(0) = 0$, $y'(\pi) = 0$ if $\lambda = 0$ is

   Select the correct answer.

   (a) $y = 0$
   (b) $y = \sin x$
   (c) $y = \cos x$
   (d) $y = b$
   (e) $y = x - \pi$

6. Classify the equation $\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} = 0$ as hyperbolic, parabolic, or elliptic.

7. The condition $u(x_0, t) = 0$ in a heat equation problem is referred to as

   Select the correct answer.

   (a) a Dirichlet condition
   (b) a Neumann condition
   (c) a Robin condition
   (d) none of the above

8. The condition $\frac{du}{dx}(x_0, t) = 0$ in a heat equation problem is referred to as

   Select the correct answer.

   (a) a Dirichlet condition
   (b) a Neumann condition
   (c) a Robin condition
   (d) none of the above

9. Write down the one dimensional heat equation and initial and boundary conditions for a rod with insulation at both ends and a given initial temperature distribution. Let $u(x, t)$ denote the temperature at position x and time t.

10. Consider the equation $u_{xx} - u_{tt} = 0$ with conditions $u(0, t) = 0$, $u(1, t) = 0$, $u(x, 0) = f(x)$, $\frac{du}{dt}(0) = 0$. Describe a physical situation having this problem as a mathematical model.

11. In the previous problem, separate variables. What is the differential equation and boundary conditions for the function of the spatial variable? What is the differential equation and initial conditions for the function of the time variable?

12. In the previous problem, what are the solutions of the resulting eigenvalue problem? What are the solutions of the problem involving the time variable?

13. In the previous problem, what are the product solutions of the original partial differential equation.

14. In the previous four problems, if $f(x) = \sin(\pi x)$, what is the Fourier series solution?

15. Consider the problem $\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = \frac{du}{dt}$, $u(0, y, t) = 0$, $u(1, y, t) = 0$, $u_y(x, 0, t) = 0$, $u_y(x, 2, t) = 0$, $u(x, y, 0) = f(x, y)$. Describe a physical situation having this as a mathematical model.

16. In the previous problem, separate variables using $u(x, y, t) = X(x)Y(y)T(t)$. What are the problems for X, Y, and T?

17. In the previous problem, the eigenvalue problems for X and Y are

    Select all that apply.

    (a) $X'' + \lambda X = 0$, $X(0) = 0$, $X(1) = 0$
    (b) $X'' + \lambda X = 0$, $X'(0) = 0$, $X'(1) = 0$
    (c) $X'' + \lambda X = 0$, $X(0) = 0$, $X'(1) = 0$
    (d) $Y'' + \mu Y = 0$, $Y(0) = 0$, $Y(1) = 0$
    (e) $Y'' + \mu Y = 0$, $Y'(0) = 0$, $Y'(1) = 0$

18. In the previous problem, the solutions of the eigenvalue problems are

    Select all that apply.

    (a) $\lambda = n^2\pi^2$, $X = \cos(n\pi x)$, $n = 0, 1, 2, \ldots$
    (b) $\lambda = n^2\pi^2$, $X = \sin(n\pi x)$, $n = 1, 2, 3, \ldots$
    (c) $\lambda = n\pi$, $X = \sin(n\pi x)$, $n = 1, 2, 3, \ldots$
    (d) $\mu = m^2\pi^2/4$, $Y = \cos(m\pi y/2)$, $m = 0, 1, 2, \ldots$
    (e) $\mu = m^2\pi^2/4$, $Y = \sin(m\pi y/2)$, $m = 1, 2, 3, \ldots$

19. In the previous four problems, what are the product solutions?

20. Combine the results of the previous four problems to write a series solution of the problem $\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = \frac{du}{dt}$, $u(0, y, t) = 0$, $u(1, y, t) = 0$, $u_y(x, 0, t) = 0$, $u_y(x, 2, t) = 0$, $u(x, y, 0) = f(x, y)$.

ANSWER KEY
Zill Differential Equations 9e Chapter 12 Form G

1. second order, linear, homogeneous
2. $X'' + \lambda X = 0$, $T' + \lambda T = 0$
3. c
4. a
5. d
6. parabolic
7. a
8. b
9. $ku_{xx} - u_t = 0$, $u_x(0,t) = 0$, $u_x(L,t) = 0$, $u(x,0) = f(x)$, where $u(x,t)$ is the temperature of a rod at point x and time t, $f(x)$ is the given initial temperature distribution in the rod, and k is the thermal diffusivity.
10. Vibration of a stretched string, fixed along the x-axis at $x = 0$ and $x = 1$, with initial deflection $f(x)$ and initial velocity zero.
11. $X'' + \lambda X = 0$, $X(0) = 0$, $X(1) = 0$, $T'' + \lambda T = 0$, $T'(0) = 0$
12. $\lambda = n^2\pi^2$, $X = c_n \sin(n\pi x)$, $T = \cos(n\pi t)$
13. $u = c_n \sin(n\pi x)\cos(n\pi t)$
14. $u = \sin(\pi x)\cos(\pi t)$
15. Temperature distribution in a rectangular plate ($0 \le x \le 1$, $0 \le y \le 2$) having zero temperature on the boundaries $x = 0$ and $x = 1$, insulated along $y = 0$ and $y = 2$, and having initial temperature distribution $f(x,y)$.
16. $X''+\lambda X = 0$, $X(0) = 0$, $X(1) = 0$, $Y''+\mu Y = 0$, $Y'(0) = 0$, $Y'(2) = 0$, $T'+(\lambda+\mu)T = 0$
17. a, e
18. b, d
19. $u = \sin(n\pi x)\cos(m\pi y/2)e^{-(n^2+m^2/4)\pi^2 t}$
20. $u = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} c_{nm} \sin(n\pi x)\cos(m\pi y/2)e^{-(n^2+m^2/4)\pi^2 t}$, where
$c_{nm} = 2\int_0^1\int_0^2 f(x,y)\sin(n\pi x)\cos(m\pi y/2)dydx$, with a similar formula for c_{n0}

1. The differential equation $\frac{\partial^3 u}{\partial x^3} + u\frac{\partial^3 u}{\partial y^3} = x + y$ is

   Select the correct answer.

   (a) second order, linear, homogeneous
   (b) second order, linear, non-homogeneous
   (c) third order, linear, homogeneous
   (d) third order, linear, non-homogeneous
   (e) third order, nonlinear

2. When $u(x,t) = X(x)T(t)$ is substituted into the equation $u_{xx} + u_t = 0$, the resulting equations for X and T are

   Select the correct answer.

   (a) $X'' + \lambda X = 0$, $T' + \lambda T = 0$
   (b) $X'' + \lambda X = 0$, $T' - \lambda T = 0$
   (c) $X'' - \lambda X = 0$, $T'' + \lambda T = 0$
   (d) $X' + \lambda X = 0$, $T'' + \lambda T = 0$
   (e) $X' + \lambda X = 0$, $T'' - \lambda T = 0$

3. What is the solution of $y'' + \lambda y = 0$, $y(0) = 0$, $y'(1) = 0$ if $\lambda > 0$?

4. What is the solution of $y'' + \lambda y = 0$, $y'(0) = 0$, $y'(1) = 0$ if $\lambda > 0$?

5. What is the solution of $y'' + \lambda y = 0$, $y'(0) = 0$, $y'(1) = 0$ if $\lambda < 0$?

6. What is the solution of $y'' + \lambda y = 0$, $y'(0) = 0$, $y'(1) = 0$ if $\lambda = 0$?

7. The differential equation $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}$ is

   Select all that apply.

   (a) linear
   (b) nonlinear
   (c) hyperbolic
   (d) elliptic
   (e) parabolic

8. The differential equation $3\frac{\partial^2 u}{\partial x^2} + 5\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = u$ is

   Select all that apply.

   (a) linear
   (b) nonlinear
   (c) hyperbolic
   (d) elliptic
   (e) parabolic

9. Consider the wave equation $u_{xx} - u_{tt} = 0$ with the conditions $u(0,t) = 0$, $u(L,t) = 0$, $u(x,0) = f(x)$, $\frac{du}{dt}(x,0) = g(x)$. The solution can be considered as a superposition of
   Select all that apply.
   (a) fundamental frequencies
   (b) standing waves
   (c) normal modes
   (d) overtones
   (e) product solutions

10. Standing waves of the problem in the previous problem are of the form
    Select all that apply.
    (a) $c_n \cos(n\pi t/L)\sin(n\pi x/L)$
    (b) $c_n \sin(n\pi t/L)\cos(n\pi x/L)$
    (c) $c_n \cos(n\pi t/L + \phi_n)\cos(n\pi x/L)$ for a certain ϕ_n
    (d) $c_n \sin(n\pi t/L + \phi_n)\sin(n\pi x/L)$ for a certain ϕ_n
    (e) $c_n \sin(n\pi t/L + \phi_n)\sin(n\pi x/L)$ for all ϕ_n

11. Consider the equation $ku_{xx} - u_t = 0$ with conditions $u(0,t) = 0$, $u(1,t) = 0$, $u(x,0) = f(x)$. Describe a physical situation having this problem as a mathematical model.

12. In the previous problem, separate variables. What is the differential equation and boundary conditions for the function of the spatial variable? What is the differential equation for the function of the time variable?

13. In the previous problem, what are the solutions of the resulting eigenvalue problem? What are the solutions of the problem involving the time variable?

14. In the previous three problems, what are the product solutions of the original partial differential equation.

15. In the previous four problems, if $f(x) = \left\{ \begin{array}{ccc} x/10 & \text{if} & 0 \le x \le 1/2 \\ (1-x)/10 & \text{if} & 1/2 \le x \le 1 \end{array} \right\}$, what is the Fourier series solution?

16. Consider a mathematical model for a vibrating rectangular plate ($0 \le x \le 1$, $0 \le y \le \pi$), fixed along the boundary, with initial position $f(x, y)$ and initial velocity zero. The boundary conditions are $u(0, y, t) = 0$, $u(1, y, t) = 0$, $u(x, 0, t) = 0$, $u(x, \pi, t) = 0$. The correct differential equation and initial conditions are

    Select the correct answer.

    (a) $\frac{d^2u}{dx^2} - \frac{d^2u}{dy^2} = \frac{d^2u}{dt^2}$, $u(x, y, 0) = f(x, y)$, $u_t(x, y, 0) = 0$
    (b) $\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dt^2} = 0$, $u(x, y, 0) = f(x, y)$, $u_t(x, y, 0) = 0$
    (c) $\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = \frac{d^2u}{dt^2}$, $u(x, y, 0) = f(x, y)$, $u_t(x, y, 0) = 0$
    (d) $\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = \frac{d^2u}{dt^2}$, $u(x, y, 0) = 0$, $u_t(x, y, 0) = f(x, y)$
    (e) $\frac{d^2u}{dx^2} + \frac{du}{dy} = \frac{d^2u}{dt^2}$, $u(x, y, 0) = f(x, y)$, $u_t(x, y, 0) = 0$

17. When separating variables in the previous problem using $u(x, y, t) = X(x)Y(y)T(t)$, the resulting problems for X, Y, and T are

    Select all that apply.

    (a) $X'' + \lambda X = 0$, $X(0) = 0$, $X(1) = 0$
    (b) $X'' + \lambda X = 0$, $X'(0) = 0$, $X'(1) = 0$
    (c) $Y'' + \mu Y = 0$, $Y(0) = 0$, $Y(\pi) = 0$
    (d) $Y'' + \mu Y = 0$, $Y'(0) = 0$, $Y'(\pi) = 0$
    (e) $T'' + (\lambda + \mu)T = 0$, $T'(0) = 0$

18. The solutions of the eigenvalue problems from the previous problem are

    Select all that apply.

    (a) $\lambda = n\pi$, $X = \sin(n\pi x)$, $n = 1, 2, 3, \ldots$
    (b) $\lambda = n^2\pi^2$, $X = \sin(n\pi x)$, $n = 1, 2, 3, \ldots$
    (c) $\lambda = n\pi$, $X = \cos(n\pi x)$, $n = 0, 1, 2, \ldots$
    (d) $\mu = m^2$, $Y = \sin(my)$, $m = 1, 2, 3, \ldots$
    (e) $\mu = m^2$, $Y = \cos(my)$, $m = 0, 1, 2, \ldots$

19. The product solutions of the previous three problems are

    Select the correct answer.

    (a) $u = \sin(n\pi x)\cos(my)\cos((n^2\pi^2 + m^2)t)$
    (b) $u = \sin(n\pi x)\sin(my)\cos((n^2\pi^2 + m^2)t)$
    (c) $u = \cos(n\pi x)\cos(my)\cos((n^2\pi^2 + m^2)t)$
    (d) $u = \cos(n\pi x)\cos(my)\sin((n^2\pi^2 + m^2)t)$
    (e) $u = \sin(n\pi x)\cos(my)\sin((n^2\pi^2 + m^2)t)$

20. The infinite series solution for the previous four problems is

    Select the correct answer.

    (a) $u = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} c_{nm}\sin(n\pi x)\sin(my)\cos((n^2\pi^2 + m^2)t)$, where $c_{nm} = 4\int_0^1\int_0^{\pi} f(x,y)\sin(n\pi x)\sin(my)dydx/\pi$
    (b) $u = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} c_{nm}\cos(n\pi x)\cos(my)\sin((n^2\pi^2 + m^2)t)$, where $c_{nm} = 4\int_0^1\int_0^{\pi} f(x,y)\cos(n\pi x)\cos(my)dydx/\pi$
    (c) $u = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} c_{nm}\sin(n\pi x)\cos(my)\sin((n^2\pi^2 + m^2)t)$, where $c_{nm} = 4\int_0^1\int_0^{\pi} f(x,y)\sin(n\pi x)\cos(my)dydx/\pi$
    (d) $u = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} c_{nm}\sin(n\pi x)\cos(my)\cos((n^2\pi^2 + m^2)t)$, where $c_{nm} = 4\int_0^1\int_0^{\pi} f(x,y)\sin(n\pi x)\cos(my)dydx/\pi$
    (e) $u = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} c_{nm}\cos(n\pi x)\cos(my)\cos((n^2\pi^2 + m^2)t)$, where $c_{nm} = 4\int_0^1\int_0^{\pi} f(x,y)\cos(n\pi x)\cos(my)dydx/\pi$

ANSWER KEY
Zill Differential Equations 9e Chapter 12 Form H

1. e
2. b
3. $\lambda = (2n-1)^2\pi^2/4$, $y = c_n \sin((2n-1)\pi x/2)$, $n = 1, 2, 3, \ldots$
4. $\lambda = n^2\pi^2$, $y = c_n \cos(n\pi x)$, $n = 0, 1, 2, \ldots$
5. $y = 0$
6. $y = c$
7. a, e
8. a, c
9. b, c, e
10. d
11. Temperature distribution in a rod of length 1, zero temperature at both ends, initial temperature distribution of $f(x)$, and thermal diffusivity k.
12. $X'' + \lambda X = 0$, $X(0) = 0$, $X(1) = 0$, $T' + k\lambda T = 0$
13. $\lambda = n^2\pi^2$, $X = c_n \sin(n\pi x)$, $T = e^{-kn^2\pi^2 t}$
14. $u = \sin(n\pi x)e^{-kn^2\pi^2 t}$
15. $u = \sum_0^\infty c_n \sin(n\pi x)e^{-kn^2\pi^2 t}$ where $c_n = 2\int_0^1 f(x)\sin(n\pi x)dx = 2\sin(n\pi/2)/(5n^2\pi^2)$
16. c
17. a, c, e
18. b, d
19. b
20. a