1. A bacteria culture grows at a rate proportional to the number of bacteria, $P(t)$, at time t. If the initial amount of bacteria in the culture is P_0, and if the size doubles in two hours, how long does it take the culture to triple in size?

2. The half-life of radium is approximately 1700 years. If ten grams of radium is present initially, how long will it take for the amount of radium to reduce to nine grams? Assume that the decay rate is proportional to the amount.

3. A baked cake is taken out of a 350°F oven and placed on a table in a 70°F room. It cools according to Newton's Law. Ten minutes later the temperature is 315°F. What is the temperature of the cake after 30 minutes?

4. An object is taken out of a 70°F room and placed outside where the temperature is 82°F room. It warms according to Newton's Law. Twenty minutes later the temperature is 72°F. What is the temperature of the object after one hour?

5. A tank contains 50 gallons of water in which 20 pounds of salt is dissolved. A brine solution containing 2 pounds of salt per gallon of water is pumped into the tank at the rate of 3 gallons per minute, and the well-stirred mixture is pumped out at the same rate. Let $A(t)$ represent the amount of salt in the tank at time t. Write down the initial value problem for $A(t)$. How much salt will there be in the tank after a long period of time?

6. Solve the initial value problem in the previous problem.

7. Find the current in an L-R circuit if the inductance is 0.1 henry, the resistance is 20 ohms, the applied voltage is 50 volts, and the initial current is 0 amperes.

8. Find the charge on the capacitor in an R-C circuit if the the resistance is 20 ohms, the capacitance is 0.002 farad, the applied voltage is $12e^{-10t}$ volts, and the initial charge is 0 coulombs.

9. In the logistic model for population growth, $\dfrac{dP}{dt} = P(4-3P)$, what is the carrying capacity of the population $P(t)$?

10. Solve the logistic equation $\dfrac{dP}{dt} = P(4-3P)$ with initial condition $P(0) = 1/2$.

11. Solve the Gompertz differential equation $\dfrac{dP}{dt} = P(a - b\ln P)$.

12. Two chemicals, A and B, are combined, forming chemical C. The rate of the reaction is jointly proportional to the amounts of A and B not yet converted to C. Initially, there are 100 grams of A and 150 grams of B, and, during the reaction, for each gram of A used up in the conversion, there are two grams of B used up. An experiments shows that 40 grams of C are produced in the first ten minutes. After a long period of time, how much of A and of B remains, and how much of C has been produced?

13. In the previous problem, what is the amount, $X(t)$, of chemical C produced by time t?

14. The differential equation $y' = (-x + \sqrt{x^2 + y^2})/y$ describes the shape of a plane curve C that will reflect all incoming light rays to the same point. It could be used to model the mirror for a reflecting telescope. Solve the equation using the substitution $u = x^2 + y^2$.

15. Radioactive element X decays to element Y with decay constant -0.4. Y, in turn, decays to stable element Z with decay constant -0.5. Write down the system of differential equations and initial conditions for the amounts, $x(t)$, $y(t)$, $z(t)$ of the elements X, Y, Z, respectively, at time t, if, initially, there are 5 grams of X, but no Y or Z.

16. Solve the system in the previous problem. Hint: solve the first equation for x, then the second equation for y, then the third equation for z.

17. Tank A contains 100 gallons of water in which 20 pounds of salt has been dissolved. Tank B contains 70 gallons of water in which 10 pounds of salt has been dissolved. A brine mixture with a concentration of 0.5 pounds of salt per gallon of water is pumped into tank A at the rate of 8 gallons per minute. The well-mixed solution is then pumped from tank A to tank B at the rate of 10 gallons per minute. The solution from tank B is also pumped through another pipe into tank A at the rate of 2 gallons per minute, and the solution from tank B is also pumped out of the system at the rate of 8 gallons per minute. Write down the differential equations with initial conditions for the amounts, $x(t)$ and $y(t)$, of salt in tanks A and B, respectively, at time t.

18. In the previous problem, how much salt will there be in tanks A and B after a long period of time?

19. In the Lotka-Volterra predator-prey model $\frac{dx}{dt} = -ax + bxy$, $\frac{dy}{dt} = ey - cxy$, where $x(t)$ is the predator population and $y(t)$ is the prey population, the coefficient a represents which of the following: the predator die-off rate, the prey growth rate, the increase in the predator population due to interactions with the prey, the decrease in the prey population due to interactions with the predator, or none of the above.

20. In the competition model $\frac{dx}{dt} = ax - bxy$, $\frac{dy}{dt} = cy - dxy$, where $x(t)$ and $y(t)$ are the populations of the competing species, moose and deer, respectively, the coefficient a represents which of the following: the moose growth rate, the deer growth rate, the decrease in the moose population due to interactions with the deer, the decrease in the deer population due to interactions with the moose, or none of the above.

ANSWER KEY
Zill Differential Equations 9e Chapter 3 Form A

1. 3.17 hours
2. 258 years
3. 258°F
4. 75.1°F
5. $\dfrac{dA}{dt} = 6 - 3A/50,\ A(0) = 20$, 100 pounds
6. $A = 100 - 80e^{-3t/50}$
7. $i = 5(1 - e^{-200t})/2$ amperes
8. $q = 0.04(e^{-10t} - e^{-25t})$ coulombs
9. 4/3
10. $P = 4/(3 + 5e^{-4t})$
11. $a - b \ln P = ce^{-bt}$
12. 25 grams of A, 0 grams of B, 225 grams of C
13. $X = 900(1 - e^{-75kt})/(4 - 3e^{-75kt})$, where $k = \ln(780/740)/750$
14. $y = \sqrt{2cx + c^2}$
15. $\dfrac{dx}{dt} = -0.4x,\ \dfrac{dy}{dt} = 0.4x - 0.5y,\ \dfrac{dz}{dt} = 0.5y,\ x(0) = 5,\ y(0) = 0,\ z(0) = 0$
16. $x = 5e^{-0.4t},\ y = 20(e^{-0.4t} - e^{-0.5t}),\ z = 5 - 25e^{-0.4t} + 20e^{-0.5t}$
17. $\dfrac{dx}{dt} = 4 - x/10 + y/35,\ \dfrac{dy}{dt} = x/10 - y/7,\ x(0) = 20,\ y(0) = 10$
18. 50 pounds in A, 35 pounds in B
19. the predator die-off rate
20. the moose growth rate

1. The population of a town grows at a rate proportional to the number of people, $P(t)$, at time t. If the initial population is P_0, and if the size doubles in twelve years, how long does it take the population to triple in size?

2. In a breeder reactor, uranium 238 is converted to plutonium 239. After 15 years, 0.043% of the plutonium has decayed. Find the half life of plutonium 239, if the rate of decay is proportional to the amount present.

3. A chicken is taken out of the freezer (0°C) and placed on a table in a 20°C room. Five minutes later the temperature is 2°C. It warms according to Newton's Law. What is the temperature of the chicken after 30 minutes?

4. An object is taken out of a 65°F room and placed outside where the temperature is 35°F room. Fifteen minutes later the temperature is 60°F. It cools according to Newton's Law. What is the temperature of the object after one hour?

5. A tank contains 400 liters of water in which 200 grams of salt is dissolved. A brine solution containing 8 grams of salt per liter of water is pumped into the tank at the rate of 5 liters per minute, and the well-stirred mixture is pumped out at the same rate. Let $A(t)$ represent the amount of salt in the tank at time t. Write down the initial value problem for $A(t)$. How much salt will there be in the tank after a long period of time?

6. Solve the initial value problem in the previous problem.

7. Find the current in an L-R circuit if the inductance is 0.2 henry, the resistance is 50 ohms, the applied voltage is e^{-25t} volts, and the initial current is 0 amperes.

8. Find the charge on the capacitor in an R-C circuit if the the resistance is 50 ohms, the capacitance is 0.004 farad, the applied voltage is 20 volts, and the initial charge is 0 coulombs.

9. In the logistic model for population growth, $\dfrac{dP}{dt} = P(6 - 2P)$, what is the carrying capacity of the population $P(t)$?

10. Solve the logistic equation $\dfrac{dP}{dt} = P(6 - 2P)$ with initial condition $P(0) = 1$.

11. Two chemicals, A and B, are combined, forming chemical C. The rate of the reaction is jointly proportional to the amounts of A and B not yet converted to C. Initially, there are 500 grams of A and 250 grams of B, and, during the reaction, for each gram of A used up in the conversion, there are two grams of B used up. An experiments shows that 300 grams of C are produced in the first ten minutes. After a long period of time, how much of A and of B remains, and how much of C has been produced?

12. In the previous problem, what is the amount, $X(t)$, of chemical C produced by time t?

13. A simple model for the shape of a tsunami wave is $\dfrac{dW}{dx} = W\sqrt{4 - 2W}$, where $W(x) > 0$ is the height of the wave as a function of a point offshore. By inspection, find all constant solutions of the differential equation.

14. Solve the differential equation in the previous problem by using the substitution $u = \sqrt{4 - 2W}$.

15. Radioactive element X decays to element Y with decay constant -0.6. Y, in turn, decays to stable element Z with decay constant -0.8. Write down the system of differential equations and initial conditions for the amounts, $x(t)$, $y(t)$, $z(t)$ of the elements X, Y, Z, respectively, at time t, if, initially, there are 10 grams of X, but no Y or Z.

16. Solve the system in the previous problem.

17. Tank A contains 50 gallons of water in which 5 pounds of salt has been dissolved. Tank B contains 30 gallons of water in which 3 pounds of salt has been dissolved. A brine mixture with a concentration of 0.6 pounds of salt per gallon of water is pumped into tank A at the rate of 5 gallons per minute. The well-mixed solution is then pumped from tank A to tank B at the rate of 8 gallons per minute. The solution from tank B is also pumped through another pipe into tank A at the rate of 3 gallons per minute, and the solution from tank B is also pumped out of the system at the rate of 5 gallons per minute. Write down the differential equations with initial conditions for the amounts, $x(t)$ and $y(t)$, of salt in tanks A and B, respectively, at time t.

18. In the previous problem, how much salt will there be in tanks A and B after a long period of time?

19. In the Lotka-Volterra predator-prey model $\dfrac{dx}{dt} = -ax + bxy$, $\dfrac{dy}{dt} = ey - cxy$, where $x(t)$ is the predator population and $y(t)$ is the prey population, the coefficient b represents which of the following: the predator die-off rate, the prey growth rate, the increase in the predator population due to interactions with the prey, the decrease in the prey population due to interactions with the predator, or none of the above.

20. In the competition model $\dfrac{dx}{dt} = ax - bxy$, $\dfrac{dy}{dt} = cy - dxy$, where $x(t)$ and $y(t)$ are the populations of the competing species, moose and deer, respectively, the coefficient b represents which of the following: the moose growth rate, the deer growth rate, the decrease in the moose population due to interactions with the deer, the decrease in the deer population due to interactions with the moose, or none of the above.

ANSWER KEY
Zill Differential Equations 9e Chapter 3 Form B

1. 19.0 years

2. 24,200 years

3. 9.37°C

4. 49.5°F

5. $\dfrac{dA}{dt} = 40 - A/80$, $A(0) = 200$, 3200 grams

6. $A = 3200 - 3000e^{-t/80}$

7. $i = (e^{-25t} - e^{-250t})/45$

8. $q = 2(1 - e^{-5t})/25$

9. 3

10. $P = 3/(1 + 2e^{-6t})$

11. 375 grams of A, 0 grams of B, 375 grams of C

12. $X = 1500(1 - e^{-1125kt})/(4 - e^{-1125kt})$, where $k = \ln(4)/11250$

13. $W = 0$ and $W = 2$

14. $\sqrt{4 - 2W} = (2 + 2ce^{2x})/(1 - ce^{2x})$

15. $\dfrac{dx}{dt} = -0.6x$, $\dfrac{dy}{dt} = 0.6x - 0.08y$, $\dfrac{dz}{dt} = 0.8y$, $x(0) = 10$, $y(0) = 0$, $z(0) = 0$

16. $x = 10e^{-0.6t}$, $y = 30(e^{-0.6t} - e^{-0.8t})$, $z = 10 - 40e^{-0.6t} + 30e^{-0.8t}$

17. $\dfrac{dx}{dt} = 3 - 4x/25 + y/10$, $\dfrac{dy}{dt} = 4x/25 - 4y/15$, $x(0) = 5$, $y(0) = 3$

18. 30 pounds in A, 18 pounds in B

19. the increase in the predator population due to interactions with the prey

20. the decrease in the moose population due to interactions with the deer

1. The population of a certain town doubles in 14 years. How long will it take for the population to triple? Assume that the rate of increase of the population is proportional to the population.

   Select the correct answer.

   (a) 18.2 years
   (b) 20.2 years
   (c) 22.2 years
   (d) 23.2 years
   (e) 24.2 years

2. The half-life of radium is 1700 years. Assume that the decay rate is proportional to the amount. An initial amount of 5 grams of radium deca to 3 grams in

   Select the correct answer.

   (a) 850 years
   (b) 1050 years
   (c) 1150 years
   (d) 1250 years
   (e) 1350 years

3. An object is taken out of a 65°F room and placed outside where the temperature is 35°F room. Five minutes later the temperature is 63°F. It cools according to Newton's Law. The temperature of the object after one hour is

   Select the correct answer.

   (a) 50.5°F
   (b) 49.9°F
   (c) 49.3°F
   (d) 48.7°F
   (e) 48.1°F

4. A chicken is taken out of the freezer (0°C) and placed on a table in a 20°C room. Ten minutes later the temperature is 2°C. It warms according to Newton's Law. How long does it take before the temperature reaches 15°C?

   Select the correct answer.

   (a) 122 minutes
   (b) 127 minutes
   (c) 132 minutes
   (d) 137 minutes
   (e) 142 minutes

5. In Newton's Law of cooling, $\dfrac{dT}{dt} = k(T - T_m)$ the constant k is

   Select the correct answer.

   (a) a constant of integration evaluated from an initial condition
   (b) a constant of integration evaluated from another condition
   (c) a proportionality constant evaluated from an initial condition
   (d) a proportionality constant evaluated from another condition

6. A tank contains 50 gallons of water in which 2 pounds of salt is dissolved. A brine solution containing 1.5 pounds of salt per gallon of water is pumped into the tank at the rate of 4 gallons per minute, and the well-stirred mixture is pumped out at the same rate. Let $A(t)$ represent the amount of salt in the tank at time t. The correct initial value problem for $A(t)$ is

   Select the correct answer.

   (a) $\dfrac{dA}{dt} = 6 - 2A/25,\ A(0) = 2$
   (b) $\dfrac{dA}{dt} = 6 + 2A/25,\ A(0) = 2$
   (c) $\dfrac{dA}{dt} = 4 + 2A/25,\ A(0) = 2$
   (d) $\dfrac{dA}{dt} = 1.5 - 2A/25,\ A(0) = 0$
   (e) $\dfrac{dA}{dt} = 4 - 2A/25,\ A(0) = 0$

7. In the previous problem, how much salt will there be in the tank after a long period of time?

   Select the correct answer.

   (a) 2 pounds
   (b) 50 pounds
   (c) 75 pounds
   (d) 200 pounds
   (e) none of the above

8. In the previous two problems, the amount of salt in the tank at time t is

   Select the correct answer.

   (a) $A(t) = -75 + 77e^{2t/25}$
   (b) $A(t) = 75 - 73e^{-2t/25}$
   (c) $A(t) = 50 - 48e^{-2t/25}$
   (d) $A(t) = -50 + 52e^{2t/25}$
   (e) $A(t) = 75 - 73e^{2t/25}$

9. A ball is thrown upward from the top of a 200 foot tall building with a velocity of 40 feet per second. Take the positive direction upward and the origin of the coordinate system at ground level. What is the initial value problem for the position, $x(t)$, of the ball at time t?

   Select the correct answer.

   (a) $\frac{d^2x}{dt^2} = 40$, $x(0) = 200$, $\frac{dx}{dt}(0) = 40$
   (b) $\frac{d^2x}{dt^2} = -40$, $x(0) = 200$, $\frac{dx}{dt}(0) = 40$
   (c) $\frac{d^2x}{dt^2} = 32$, $x(0) = 200$, $\frac{dx}{dt}(0) = 40$
   (d) $\frac{d^2x}{dt^2} = -32$, $x(0) = 200$, $\frac{xA}{dt}(0) = 40$
   (e) $\frac{d^2x}{dt^2} = 200$, $x(0) = 32$, $\frac{dx}{dt}(0) = 40$

10. In the previous problem, the solution of the initial value problem is

    Select the correct answer.

    (a) $x = 16t^2 + 40t + 200$
    (b) $x = -16t^2 + 200t + 40$
    (c) $x = -16t^2 + 40t + 200$
    (d) $x = 32t^2 + 40t + 200$
    (e) $x = -32t^2 + 40t + 200$

11. In the logistic model for population growth, $\frac{dP}{dt} = P(8 - 2P)$, the carrying capacity of the population $P(t)$ is

    Select the correct answer.

    (a) 8
    (b) 2
    (c) 4
    (d) 1/4
    (e) 16

12. The solution of the logistic equation $\dfrac{dP}{dt} = P(8 - 2P)$ with initial condition $P(0) = 2$ is

    Select the correct answer.

    (a) $P = 4/(2 - e^{-8t})$
    (b) $P = 2/(8 + e^{-8t})$
    (c) $P = 8/(2 + e^{-8t})$
    (d) $P = 8/(2 - e^{-8t})$
    (e) $P = 4/(1 + e^{-8t})$

13. Two chemicals, A and B, are combined, forming chemical C. The rate of the reaction is jointly proportional to the amounts of A and B not yet converted to C. Initially, there are 200 grams of A and 300 grams of B, and, during the reaction, for each gram of A used up in the conversion, there are three grams of B used up. An experiments shows that 75 grams of C are produced in the first ten minutes. After a long period of time, how much of A and of B remains, and how much of C has been produced?

    Select the correct answer.

    (a) 200 grams of A, 0 grams of B, 300 grams of C
    (b) 0 grams of A, 0 grams of B, 500 grams of C
    (c) 100 grams of A, 0 grams of B, 400 grams of C
    (d) 0 grams of A, 100 grams of B, 400 grams of C
    (e) 0 grams of A, 200 grams of B, 300 grams of C

14. In the previous problem, the amount of chemical C, $X(t)$, produced by time t is

    Select the correct answer.

    (a) $X = 800(1 - e^{-400kt})/(2 - e^{-400kt})$, where $k = \ln(29/26)/4000$
    (b) $X = 800(1 - e^{-800kt})/(4 - e^{-800kt})$, where $k = \ln(29/20)/8000$
    (c) $X = 400(1 - e^{-800kt})/(4 - e^{-800kt})$, where $k = \ln(29/20)/8000$
    (d) $X = 400(1 - e^{-400kt})/(4 - e^{-400kt})$, where $k = \ln(29/26)/4000$
    (e) $X = 600(1 - e^{-800kt})/(4 - e^{-800kt})$, where $k = \ln(29/20)/8000$

15. Radioactive element X decays to element Y with decay constant -0.3. Y decays to stable element Z with decay constant -0.2. What is the system of differential equations for the amounts, $x(t)$, $y(t)$, $z(t)$ of the elements X, Y, Z, respectively, at time t.

    Select the correct answer.

    (a) $\dfrac{dx}{dt} = -0.2x,\ \dfrac{dy}{dt} = 0.2x - 0.3y,\ \dfrac{dz}{dt} = 0.2y$
    (b) $\dfrac{dx}{dt} = -0.2x,\ \dfrac{dy}{dt} = 0.2x - 0.3y,\ \dfrac{dz}{dt} = 0.3y$
    (c) $\dfrac{dx}{dt} = -0.3x,\ \dfrac{dy}{dt} = 0.3x - 0.2y,\ \dfrac{dz}{dt} = 0.2y$
    (d) $\dfrac{dx}{dt} = -0.3x,\ \dfrac{dy}{dt} = 0.3x - 0.2y,\ \dfrac{dz}{dt} = 0.3y$
    (e) $\dfrac{dx}{dt} = -0.3y,\ \dfrac{dy}{dt} = 0.3x - 0.2z,\ \dfrac{dz}{dt} = 0.2y$

16. The solution of the system of differential equations in the previous problem is

    Select the correct answer.

    (a) $x = c_1e^{-0.3t},\ y = -3c_1e^{-0.3t} + c_2e^{-0.2t},\ z = c_3 - 3c_1e^{-0.3t} - c_2e^{-0.2t}$
    (b) $x = c_1e^{-0.2t},\ y = -2c_1e^{-0.2t} + c_2e^{-0.3t},\ z = c_3 + 2c_1e^{-0.3t} - c_2e^{-0.2t}$
    (c) $x = c_1e^{-0.3t},\ y = -3c_1e^{-0.3t} + c_2e^{-0.2t},\ z = c_3 + 2c_1e^{-0.3t} + c_2e^{-0.2t}$
    (d) $x = c_1e^{-0.3t},\ y = -3c_1e^{-0.3t} + c_2e^{-0.2t},\ z = c_3 + 2c_1e^{-0.3t} - c_2e^{-0.2t}$
    (e) $x = c_1e^{-0.2t},\ y = -2c_1e^{-0.2t} + c_2e^{-0.3t},\ z = c_3 + 2c_1e^{-0.3t} - c_2e^{-0.2t}$

17. Tank A contains 80 gallons of water in which 20 pounds of salt has been dissolved. Tank B contains 30 gallons of water in which 5 pounds of salt has been dissolved. A brine mixture with a concentration of 0.5 pounds of salt per gallon of water is pumped into tank A at the rate of 4 gallons per minute. The well-mixed solution is then pumped from tank A to tank B at the rate of 6 gallons per minute. The solution from tank B is also pumped through another pipe into tank A at the rate of 2 gallons per minute, and the solution from tank B is also pumped out of the system at the rate of 4 gallons per minute. The correct differential equations with initial conditions for the amounts, $x(t)$ and $y(t)$, of salt in tanks A and B, respectively, at time t are

    Select the correct answer.

    (a) $\dfrac{dx}{dt} = 2 - x/40 + y/5,\ \dfrac{dy}{dt} = x/40 - y/3,\ x(0) = 20,\ y(0) = 5$
    (b) $\dfrac{dx}{dt} = 2 - 3x/40 + y/15,\ \dfrac{dy}{dt} = 3x/40 - y/5,\ x(0) = 20,\ y(0) = 5$
    (c) $\dfrac{dx}{dt} = 4 - 3x/40 + y/15,\ \dfrac{dy}{dt} = 3x/40 - y/5,\ x(0) = 20,\ y(0) = 5$
    (d) $\dfrac{dx}{dt} = 4 - x/40 + y/5,\ \dfrac{dy}{dt} = x/40 - y/3,\ x(0) = 20,\ y(0) = 5$
    (e) $\dfrac{dx}{dt} = 2 - 3x/40 + y/5,\ \dfrac{dy}{dt} = x/40 - y/5,\ x(0) = 20,\ y(0) = 5$

18. In the previous problem, how much salt will there be in tanks A and B after a long period of time?

    Select the correct answer.

    (a) 5 pounds in A, 20 pounds in B
    (b) 20 pounds in A, 5 pounds in B
    (c) 5 pounds in A, 40 pounds in B
    (d) 40 pounds in A, 15 pounds in B
    (e) none of the above

19. In the Lotka-Volterra predator-prey model $\frac{dx}{dt} = -ax + bxy$, $\frac{dy}{dt} = ey - cxy$, where $x(t)$ is the predator population and $y(t)$ is the prey population, the coefficient c represents which of the following:

    Select the correct answer.

    (a) the predator die-off rate
    (b) the prey growth rate
    (c) the increase in the predator population due to interactions with the prey
    (d) the decrease in the prey population due to interactions with the predator
    (e) none of the above

20. In the competition model $\frac{dx}{dt} = ax - bxy$, $\frac{dy}{dt} = cy - dxy$, where $x(t)$ and $y(t)$ are the populations of the competing species, moose and deer, respectively, the coefficient c represents which of the following:

    Select the correct answer.

    (a) the moose growth rate
    (b) the deer growth rate
    (c) the decrease in the moose population due to interactions with the deer
    (d) the decrease in the deer population due to interactions with the moose
    (e) none of the above

ANSWER KEY

Zill Differential Equations 9e Chapter 3 Form C

1. c
2. d
3. e
4. c
5. d
6. a
7. c
8. b
9. d
10. c
11. c
12. e
13. c
14. a
15. c
16. d
17. b
18. d
19. d
20. b

1. A bacteria culture doubles in size in 8 hours. How long will it take for the size to triple? Assume that the rate of increase of the culture is proportional to the size.

   Select the correct answer.

   (a) 12.7 hours
   (b) 13.1 hours
   (c) 13.5 hours
   (d) 13.9 hours
   (e) 14.3hours

2. The half-life of plutonium 239 is 24,200 years. Assume that the decay rate is proportional to the amount. An initial amount of 3 grams of radium would decay to 2 grams in approximately

   Select the correct answer.

   (a) 12200 years
   (b) 14200 years
   (c) 15200 years
   (d) 17200 years
   (e) 18200 years

3. An object is taken out of a 21°C room and placed outside where the temperature is 4°C room. Twenty-five minutes later the temperature is 17°C. It cools according to Newton's Law. The temperature of the object after one hour is

   Select the correct answer.

   (a) 12.2°C
   (b) 12.9°C
   (c) 13.6°C
   (d) 14.3°C
   (e) 15.0°C

4. A chicken is taken out of the freezer (0°C) and placed on a table in a 23°C room. Forty-five minutes later the temperature is 10°C. It warms according to Newton's Law. How long does it take before the temperature reaches 20°C?

   Select the correct answer.

   (a) 147 minutes
   (b) 153 minutes
   (c) 157 minutes
   (d) 161 minutes
   (e) 165 minutes

5. In Newton's Law of cooling, $\dfrac{dT}{dt} = k(T - T_m)$, T_m is

   Select the correct answer.

   (a) the temperature of the object
   (b) the temperature of the environment
   (c) the initial temperature
   (d) the temperature after a specified period of time
   (e) none of the above

6. A tank contains 200 liters of water in which 300 grams of salt is dissolved. A brine solution containing 0.4 kilograms of salt per liter of water is pumped into the tank at the rate of 5 liters per minute, and the well-stirred mixture is pumped out at the same rate. Let $A(t)$ represent the amount of salt in the tank at time t. The correct initial value problem for $A(t)$ is

   Select the correct answer.

   (a) $\dfrac{dA}{dt} = 2 + A/40,\ A(0) = 0.3$
   (b) $\dfrac{dA}{dt} = 2 - A/40,\ A(0) = 0.3$
   (c) $\dfrac{dA}{dt} = 5 + A/40,\ A(0) = 300$
   (d) $\dfrac{dA}{dt} = 5 - A/40,\ A(0) = 300$
   (e) $\dfrac{dA}{dt} = 0.4 - A/40,\ A(0) = 300$

7. In the previous problem, how much salt will there be in the tank after a long period of time?

   Select the correct answer.

   (a) 1000 kilograms
   (b) 300 kilograms
   (c) 120 kilograms
   (d) 80 kilograms
   (e) none of the above

8. The amount of salt in the tank at time t in the previous two problems is

   Select the correct answer.

   (a) $A(t) = -200 + 200.3e^{t/40}$
   (b) $A(t) = 200 - 199.7e^{-t/40}$
   (c) $A(t) = 80 - 79.7e^{-t/40}$
   (d) $A(t) = -80 + 80.3e^{t/25}$
   (e) $A(t) = 200 + 100e^{-t/40}$

9. The differential equation $\frac{dP}{dt} = (k\cos t)P$, where k is a positive constant, models a population that undergoes yearly fluctuations. The solution of the equation is

   Select the correct answer.

   (a) $P = e^{ck\sin t}$
   (b) $P = ce^{k\cos t}$
   (c) $P = ce^{-k\cos t}$
   (d) $P = ce^{-k\sin t}$
   (e) $P = ce^{k\sin t}$

10. In the logistic model for population growth, $\frac{dP}{dt} = P(12 - 3P)$, what is the carrying capacity of the population $P(t)$?

    Select the correct answer.

    (a) 4
    (b) 1/4
    (c) 12
    (d) 3

11. The solution of the equation $\frac{dP}{dt} = P(12 - 3P)$ with initial condition $P(0) = 3$ is

    Select the correct answer.

    (a) $P = 12/(3 + e^{-12t})$
    (b) $P = 4/(3 + e^{-12t})$
    (c) $P = 4/(3 - e^{-12t})$
    (d) $P = 3/(12 + e^{-12t})$
    (e) $P = 3/(4 + e^{-12t})$

12. Two chemicals, A and B, are combined, forming chemical C. The rate of the reaction is jointly proportional to the amounts of A and B not yet converted to C. Initially, there are 50 grams of A and 80 grams of B, and, during the reaction, for each two grams of A used up in the conversion, there are three grams of B used up. An experiments shows that 100 grams of C are produced in the first ten minutes. After a long period of time, how much of A and of B remains, and how much of C has been produced?

    Select the correct answer.

    (a) 30 grams of A, 0 grams of B, 100 grams of C
    (b) 0 grams of A, 30 grams of B, 100 grams of C
    (c) 10 grams of A, 0 grams of B, 120 grams of C
    (d) 0 grams of A, 5 grams of B, 125 grams of C
    (e) 0 grams of A, 0 grams of B, 130 grams of C

13. In the previous problem, the amount of chemical C, $X(t)$, produced by time t is

    Select the correct answer.

    (a) $X = 2000(1 - e^{-25kt/3})/(16 - 15e^{-25kt/3})$, where $k = 3\ln(5/4)/250$
    (b) $X = 2000(1 - e^{-125kt})/(4 - e^{-125kt})$, where $k = \ln(19/16)/1250$
    (c) $X = 400(1 - e^{-25kt/3})/(3 - e^{-25kt/3})$, where $k = 3\ln(3)/250$
    (d) $X = 400(1 - e^{-125kt})/(3 - e^{-125kt})$, where $k = \ln(3)/1250$
    (e) $X = 800(1 - e^{-25kt/3})/(4 - e^{-25kt/3})$, where $k = 3\ln(7/4)/250$

14. Radioactive element X decays to element Y with decay constant -0.5. Y, in turn, decays to stable element Z with decay constant -0.1. What is the system of differential equations for the amounts, $x(t)$, $y(t)$, $z(t)$ of the elements X, Y, Z, respectively, at time t, if the initial conditions are $x(0) = 10$, $y(0) = 0$, $z(0) = 0$.

    Select the correct answer.

    (a) $\dfrac{dx}{dt} = -0.5x,\ \dfrac{dy}{dt} = 0.1x - 0.5y,\ \dfrac{dz}{dt} = 0.2y$
    (b) $\dfrac{dx}{dt} = -0.5x,\ \dfrac{dy}{dt} = 0.5x - 0.1y,\ \dfrac{dz}{dt} = 0.5y$
    (c) $\dfrac{dx}{dt} = -0.5x,\ \dfrac{dy}{dt} = 0.5x - 0.1y,\ \dfrac{dz}{dt} = 0.1y$
    (d) $\dfrac{dx}{dt} = -0.1x,\ \dfrac{dy}{dt} = 0.1x - 0.5y,\ \dfrac{dz}{dt} = 0.1y$
    (e) $\dfrac{dx}{dt} = -0.1y,\ \dfrac{dy}{dt} = 0.5x - 0.1z,\ \dfrac{dz}{dt} = 0.5y$

15. In the previous problem, how much of X, Y, and Z are left after a long period of time?

    Select the correct answer.

    (a) $x = 0,\ y = 5,\ z = 5$
    (b) $x = 5,\ y = 5,\ z = 0$
    (c) $x = 5,\ y = 0,\ z = 5$
    (d) $x = 0,\ y = 0,\ z = 10$
    (e) none of the above

16. The solution of the system of differential equations in the two previous problems is

    Select the correct answer.

    (a) $x = 10e^{-0.1t}$, $y = 12.5(e^{-0.1t} - e^{-0.5t})$, $z = 10 - 12.5e^{-0.1t} + 2.5e^{-0.5t}$
    (b) $x = 10e^{-0.1t}$, $y = 12.5(e^{-0.5t} - e^{-0.1t})$, $z = 10 - 12.5e^{-0.5t} + 2.5e^{-0.1t}$
    (c) $x = 10e^{-0.5t}$, $y = 12.5(e^{-0.5t} - e^{-0.1t})$, $z = 10 - 12.5e^{-0.2t} + 2.5e^{-0.3t}$
    (d) $x = 10e^{-0.5t}$, $y = 12.5(e^{-0.1t} - e^{-0.5t})$, $z = 10 - 12.5e^{-0.5t} + 2.5e^{-0.1t}$
    (e) $x = 10e^{-0.5t}$, $y = 12.5(e^{-0.1t} - e^{-0.5t})$, $z = 10 - 12.5e^{-0.1t} + 2.5e^{-0.5t}$

17. Tank A contains 50 gallons of water in which 2 pounds of salt has been dissolved. Tank B contains 30 gallons of water in which 3 pounds of salt has been dissolved. A brine mixture with a concentration of 0.8 pounds of salt per gallon of water is pumped into tank A at the rate of 3 gallons per minute. The well-mixed solution is then pumped from tank A to tank B at the rate of 4 gallons per minute. The solution from tank B is also pumped through another pipe into tank A at the rate of 1 gallon per minute, and the solution from tank B is also pumped out of the system at the rate of 3 gallons per minute. The correct differential equations with initial conditions for the amounts, $x(t)$ and $y(t)$, of salt in tanks A and B, respectively, at time t are

    Select the correct answer.

    (a) $\frac{dx}{dt} = 3 - 2x/25 + y/5,\ \frac{dy}{dt} = x/25 - y/15,\ x(0) = 2,\ y(0) = 3$
    (b) $\frac{dx}{dt} = 3 - x/25 + y/15,\ \frac{dy}{dt} = 2x/25 - 2y/15,\ x(0) = 2,\ y(0) = 3$
    (c) $\frac{dx}{dt} = 2.4 - 2x/25 + y/30,\ \frac{dy}{dt} = 2x/25 - 2y/15,\ x(0) = 2,\ y(0) = 3$
    (d) $\frac{dx}{dt} = 2.4 - x/50 + y/30,\ \frac{dy}{dt} = x/40 - y/3,\ x(0) = 2,\ y(0) = 3$
    (e) $\frac{dx}{dt} = 2.4 - x/25 + y/15,\ \frac{dy}{dt} = x/50 - y/30,\ x(0) = 2,\ y(0) = 3$

18. In the previous problem, how much salt will there be in tanks A and B after a long period of time?

    Select the correct answer.

    (a) 3 pounds in A, 2 pounds in B
    (b) 40 pounds in A, 24 pounds in B
    (c) 0 pounds in A, 0 pounds in B
    (d) 40 pounds in A, 30 pounds in B
    (e) none of the above

19. In the Lotka-Volterra predator-prey model $\frac{dx}{dt} = -ax + bxy,\ \frac{dy}{dt} = ey - cxy$, where $x(t)$ is the predator population and $y(t)$ is the prey population, the coefficient e represents which of the following:

    Select the correct answer.

    (a) the predator die-off rate
    (b) the prey growth rate
    (c) the increase in the predator population due to interactions with the prey
    (d) the decrease in the prey population due to interactions with the predator
    (e) none of the above

20. In the competition model $\dfrac{dx}{dt} = ax - bxy$, $\dfrac{dy}{dt} = cy - dxy$, where $x(t)$ and $y(t)$ are the populations of the competing species, moose and deer, respectively, the coefficient d represents which of the following:

Select the correct answer.

(a) the moose growth rate

(b) the deer growth rate

(c) the decrease in the moose population due to interactions with the deer

(d) the decrease in the deer population due to interactions with the moose

(e) none of the above

ANSWER KEY

Zill Differential Equations 9e Chapter 3 Form D

1. a
2. b
3. b
4. d
5. b
6. b
7. d
8. c
9. e
10. a
11. a
12. d
13. a
14. c
15. d
16. e
17. c
18. b
19. b
20. d

1. A bacteria culture grows at a rate proportional to the number of bacteria, $P(t)$, at time t. If the initial amount of bacteria in the culture is P_0, and if the size doubles in three hours, how long does it take the culture to triple in size?

2. A fossilized tree is found. It contains 2% of the carbon-14 found in living matter. What is the age of the tree? Assume that the C-14 decays at a rate proportional to the amount present. The half-life of C-14 is approximately 5600 years.

3. The half-life of radium is 1700 years. Assume that the decay rate is proportional to the amount. An initial amount of 5 grams of radium would decay to 3 grams in approximately

   Select the correct answer.

   (a) 850 years
   (b) 1050 years
   (c) 1150 years
   (d) 1250 years
   (e) 1350 years

4. A baked cake is taken out of a 400°F oven and placed on a table in a 65°F room. Twenty minutes later the temperature is 320°F. It cools according to Newton's Law. The temperature of the cake after 30 minutes is

   Select the correct answer.

   (a) 287°F
   (b) 284°F
   (c) 281°F
   (d) 278°F
   (e) 275°F

5. A tank contains 100 gallons of water in which 20 pounds of salt is dissolved. A brine solution containing 3 pounds of salt per gallon of water is pumped into the tank at the rate of 4 gallons per minute, and the well-stirred mixture is pumped out at the same rate. Let $A(t)$ represent the amount of salt in the tank at time t. The correct initial value problem for $A(t)$ is

   Select the correct answer.

   (a) $\dfrac{dA}{dt} = 4 - A/25,\ A(0) = 0$
   (b) $\dfrac{dA}{dt} = 3 - A/25,\ A(0) = 0$
   (c) $\dfrac{dA}{dt} = 4 + A/25,\ A(0) = 20$
   (d) $\dfrac{dA}{dt} = 12 + A/25,\ A(0) = 20$
   (e) $\dfrac{dA}{dt} = 12 - A/25,\ A(0) = 20$

6. In the previous problem, how much salt will be present in the tank after a long period of time?

   Select the correct answer.

   (a) 20 pounds
   (b) 50 pounds
   (c) 300 pounds
   (d) 400 pounds
   (e) none of the above

7. Solve the initial value problem discussed in the previous two problems.

8. An object is taken out of the freezer at 32°F and placed in an oven where the temperature is 350°F room. Twenty minutes later the temperature is 83°F. It warms according to Newton's Law. What is the temperature of the object after one hour?

9. The solution of the logistic equation $\dfrac{dP}{dt} = P(6-3P)$ with initial condition $P(0) = 1/2$ is

   Select the correct answer.

   (a) $P = 6/(2 + e^{-6t})$
   (b) $P = 6/(3 - e^{-6t})$
   (c) $P = 3/(3 + 3e^{-6t})$
   (d) $P = 2/(1 + 3e^{-6t})$
   (e) $P = 2/(6 + 3e^{-6t})$

10. In the logistic model for population growth, $\dfrac{dP}{dt} = P(6 - 3P)$, what is the carrying capacity of the population $P(t)$?

11. Suppose that a fish population grows logistically and also that fish are harvested at a constant rate, h. Write down the differential equation for the fish population, $P(t)$, at time t.

12. Solve the differential equation in the previous problem, assuming that the natural growth rate of the population is $a = 4$, the coefficient due to competition is $b = 1$, and the harvesting rate is $h = 3$.

13. Radioactive element X decays to element Y with decay constant -0.3. Y, in turn, decays to stable element Z with decay constant -0.8. Write down the system of differential equations and initial conditions for the amounts, $x(t)$, $y(t)$, $z(t)$ of the elements X, Y, Z, respectively, at time t, if, initially, there are 5 grams of X, but no Y or Z.

14. Solve the system in the previous problem.

15. Two chemicals, A and B, are combined, forming chemical C. The rate of the reaction is jointly proportional to the amounts of A and B not yet converted to C. Initially, there are 50 grams of A and 80 grams of B, and, during the reaction, for each two grams of A used up in the conversion, there are three grams of B used up. An experiments shows that 100 grams of C are produced in the first ten minutes. After a long period of time, how much of A and of B remains, and how much of C has been produced?

    Select the correct answer.

    (a) 30 grams of A, 0 grams of B, 100 grams of C
    (b) 0 grams of A, 30 grams of B, 100 grams of C
    (c) 0 grams of A, 5 grams of B, 125 grams of C
    (d) 10 grams of A, 0 grams of B, 120 grams of C
    (e) 0 grams of A, 0 grams of B, 130 grams of C

16. In the previous problem, the amount of chemical C, $X(t)$, produced by time t is

    Select the correct answer.

    (a) $X = 2000(1 - e^{-125kt})/(4 - e^{-125kt})$, where $k = \ln(19/16)/1250$
    (b) $X = 2000(1 - e^{-25kt/3})/(16 - 15e^{-25kt/3})$, where $k = 3\ln(5/4)/250$
    (c) $X = 400(1 - e^{-25kt/3})/(3 - e^{-25kt/3})$, where $k = 3\ln(3)/250$
    (d) $X = 400(1 - e^{-125kt})/(3 - e^{-125kt})$, where $k = \ln(3)/1250$
    (e) $X = 800(1 - e^{-25kt/3})/(4 - e^{-25kt/3})$, where $k = 3\ln(7/4)/250$

17. Tank A contains 40 gallons of water in which 6 pounds of salt has been dissolved. Tank B contains 80 gallons of water in which 8 pounds of salt has been dissolved. A brine mixture with a concentration of 1.5 pounds of salt per gallon of water is pumped into tank A at the rate of 5 gallons per minute. The well-mixed solution is then pumped from tank A to tank B at the rate of 7 gallons per minute. The solution from tank B is also pumped through another pipe into tank A at the rate of 2 gallon per minute, and the solution from tank B is also pumped out of the system at the rate of 5 gallons per minute. The correct differential equations with initial conditions for the amounts, $x(t)$ and $y(t)$, of salt in tanks A and B, respectively, at time t are

    Select the correct answer.

    (a) $\dfrac{dx}{dt} = 7.5 - 7x/40 + y/40$, $\dfrac{dy}{dt} = 7x/40 - 7y/80$, $x(0) = 6$, $y(0) = 8$
    (b) $\dfrac{dx}{dt} = 7.5 - 7x/40 + y/80$, $\dfrac{dy}{dt} = 7x/40 - 7y/80$, $x(0) = 6$, $y(0) = 8$
    (c) $\dfrac{dx}{dt} = 7.5 - 7x/80 + y/40$, $\dfrac{dy}{dt} = 7x/40 - 7y/80$, $x(0) = 6$, $y(0) = 8$
    (d) $\dfrac{dx}{dt} = 5 - 7x/80 + y/40$, $\dfrac{dy}{dt} = 7x/80 - y/3$, $x(0) = 6$, $y(0) = 8$
    (e) $\dfrac{dx}{dt} = 5 - 7x/80 + y/80$, $\dfrac{dy}{dt} = 7x/80 - y/30$, $x(0) = 6$, $y(0) = 8$

18. In the previous problem, how much salt will there be in tanks A and B after a long period of time?

    Select the correct answer.

    (a) 8 pounds in A, 6 pounds in B
    (b) 12 pounds in A, 9 pounds in B
    (c) 60 pounds in A, 120 pounds in B
    (d) 0 pounds in A, 0 pounds in B
    (e) none of the above

19. In the competition model $\frac{dx}{dt} = ax - bxy$, $\frac{dy}{dt} = cy - dxy$, where $x(t)$ and $y(t)$ are the populations of the competing species, moose and deer, respectively, the coefficient a represents which of the following: the moose growth rate, the deer growth rate, the decrease in the moose population due to interactions with the deer, the decrease in the deer population due to interactions with the moose, or none of the above.

20. In the Lotka-Volterra predator-prey model $\frac{dx}{dt} = -ax + bxy$, $\frac{dy}{dt} = ey - cxy$, where $x(t)$ is the predator population and $y(t)$ is the prey population, the coefficient a represents which of the following:

    Select the correct answer.

    (a) the predator die-off rate
    (b) the prey growth rate
    (c) the increase in the predator population due to interactions with the prey
    (d) the decrease in the prey population due to interactions with the predator
    (e) none of the above

ANSWER KEY
Zill Differential Equations 9e Chapter 3 Form E

1. 4.75 hours
2. 31,600 years
3. d
4. a
5. e
6. c
7. $A = 300 - 280e^{-t/25}$
8. 162°F
9. d
10. 2
11. $\dfrac{dP}{dt} = P(a - bP) - h$
12. $P = (3 - ce^{-2t})/(1 - ce^{-2t})$
13. $\dfrac{dx}{dt} = -0.3x,\ \dfrac{dy}{dt} = 0.3x - 0.8y,\ \dfrac{dz}{dt} = 0.8y,\ x(0) = 5,\ y(0) = 0,\ z(0) = 0$
14. $x = 5e^{-0.3t},\ y = 3(e^{-0.3t} - e^{-0.8t}),\ z = 5 - 8e^{-0.3t} + 3e^{-0.8t}$
15. c
16. b
17. a
18. c
19. the moose growth rate
20. a

1. The population of a town grows at a rate proportional to the number of people, $P(t)$, at time t. If the initial population is P_0, and if the size doubles in fifteen years, how long does it take the population to triple in size?

2. The half-life of radium is approximately 1700 years. If five grams of radium is present initially, how long will it take for the amount of radium to reduce to four grams? Assume that the decay rate is proportional to the amount.

3. A chicken is taken out of the freezer (0°C) and placed on a table in a 22°C room. Fifteen minutes later the temperature is 4°C. It warms according to Newton's Law. The temperature of the chicken one hour later is

   Select the correct answer.

   (a) 12.1°C
   (b) 12.5°C
   (c) 12.9°C
   (d) 13.3°C
   (e) 13.7°C

4. In Newton's Law of cooling, $\dfrac{dT}{dt} = k(T - T_m)$, T_m is

   Select the correct answer.

   (a) the temperature of the object
   (b) the temperature of the environment
   (c) the initial temperature
   (d) the temperature after a specified period of time

5. An object is taken out of the oven at 375°F room and placed on a table where the temperature is 65°F room. Ten minutes later the temperature is 335°F. It cools according to Newton's Law. What is the temperature of the object one hour later?

6. A tank contains 1000 gallons of water in which 40 pounds of salt is dissolved. A brine solution containing 0.5 pounds of salt per gallon of water is pumped into the tank at the rate of 6 gallons per minute, and the well-stirred mixture is pumped out at the same rate. Let $A(t)$ represent the amount of salt in the tank at time t. The correct initial value problem for $A(t)$ is

   Select the correct answer.

   (a) $\dfrac{dA}{dt} = 6 - 3A/500,\ A(0) = 40$
   (b) $\dfrac{dA}{dt} = 0.5 - 3A/500,\ A(0) = 40$
   (c) $\dfrac{dA}{dt} = 6 + 3A/500,\ A(0) = 40$
   (d) $\dfrac{dA}{dt} = 3 - 3A/500,\ A(0) = 40$

7. In the previous problem, how much salt will be present in the tank after a long period of time?

   Select the correct answer.

   (a) 400 pounds
   (b) 240 pounds
   (c) 40 pounds
   (d) 20 pounds
   (e) none of the above

8. Solve the initial value problem discussed in the previous two problems.

9. A ball is thrown upward from the top of a 200 foot tall building with a velocity of 40 feet per second. Take the positive direction upward and the origin of the coordinate system at ground level. What is the initial value problem for the position, $x(t)$, of the ball at time t?

   Select the correct answer.

   (a) $\dfrac{d^2x}{dt^2} = 40,\ x(0) = 200,\ \dfrac{dx}{dt}(0) = 40$
   (b) $\dfrac{d^2x}{dt^2} = -40,\ x(0) = 200,\ \dfrac{dx}{dt}(0) = 40$
   (c) $\dfrac{d^2x}{dt^2} = 32,\ x(0) = 200,\ \dfrac{dx}{dt}(0) = 40$
   (d) $\dfrac{d^2x}{dt^2} = -32,\ x(0) = 200,\ \dfrac{dx}{dt}(0) = 40$
   (e) $= 200,\ x(0) = 32,\ \dfrac{dx}{dt}(0) = 40$

10. In the logistic model for population growth, $\dfrac{dP}{dt} = P(8 - 3P)$, the carrying capacity of the population $P(t)$ is

    Select the correct answer.

    (a) 8
    (b) 8/3
    (c) 3
    (d) 3/8
    (e) 24

11. Two chemicals, A and B, are combined, forming chemical C. The rate of the reaction is jointly proportional to the amounts of A and B not yet converted to C. Initially, there are 100 grams of A and 150 grams of B, and, during the reaction, for each gram of A used up in the conversion, there are two grams of B used up. An experiments shows that 40 grams of C are produced in the first ten minutes. After a long period of time, how much of A and of B remains, and how much of C has been produced?

12. In the previous problem, what is the amount, $X(t)$, of chemical C produced by time t?

13. A differential equation for the velocity, $v(t)$, of a falling mass, m, subject to air resistance proportional to the square of the velocity is $m\frac{dv}{dt} = mg - kv^2$, where k is a positive constant of proportionality and the positive direction is downward. What is the terminal velocity?

    Select the correct answer.

    (a) $\sqrt{gm/k}$
    (b) $\sqrt{gk/m}$
    (c) $\sqrt{m/(gk)}$
    (d) gk/m
    (e) gm/k

14. Radioactive element X decays to element Y with decay constant -0.6. Y, in turn, decays to stable element Z with decay constant -0.9. What is the system of differential equations for the amounts, $x(t)$, $y(t)$, $z(t)$ of the elements X, Y, Z, respectively, at time t.

    Select the correct answer.

    (a) $\frac{dx}{dt} = -0.9x,\ \frac{dy}{dt} = 0.9x - 0.6y,\ \frac{dz}{dt} = 0.2y$
    (b) $\frac{dx}{dt} = -0.9x,\ \frac{dy}{dt} = 0.6x - 0.9y,\ \frac{dz}{dt} = 0.9y$
    (c) $\frac{dx}{dt} = -0.9x,\ \frac{dy}{dt} = 0.6x - 0.9y,\ \frac{dz}{dt} = 0.6y$
    (d) $\frac{dx}{dt} = -0.6x,\ \frac{dy}{dt} = 0.9x - 0.6y,\ \frac{dz}{dt} = 0.6y$
    (e) $\frac{dx}{dt} = -0.6x,\ \frac{dy}{dt} = 0.6x - 0.9y,\ \frac{dz}{dt} = 0.9y$

15. The solution of the system of differential equations in the previous problem, with the initial conditions $x(0) = 10$, $y(0) = 0$, $z(0) = 0$, is

    Select the correct answer.

    (a) $x = 10e^{-0.6t}$, $y = 20(e^{-0.6t} - e^{-0.9t})$, $z = 10 - 30e^{-0.9t} + 20e^{-0.6t}$
    (b) $x = 10e^{-0.6t}$, $y = 20(e^{-0.9t} - e^{-0.6t})$, $z = 10 - 30e^{-0.6t} + 20e^{-0.9t}$
    (c) $x = 10e^{-0.6t}$, $y = 20(e^{-0.6t} - e^{-0.9t})$, $z = 10 - 30e^{-0.6t} + 20e^{-0.9t}$
    (d) $x = 10e^{-0.9t}$, $y = 20(e^{-0.6t} - e^{-0.9t})$, $z = 10 - 30e^{-0.6t} + 20e^{-0.9t}$
    (e) $x = 10e^{-0.9t}$, $y = 20(e^{-0.9t} - e^{-0.6t})$, $z = 10 - 30e^{-0.9t} + 20e^{-0.6t}$

16. In the Lotka-Volterra predator-prey model $\frac{dx}{dt} = -ax + bxy$, $\frac{dy}{dt} = ey - cxy$, where $x(t)$ is the predator population and $y(t)$ is the prey population, the coefficient b represents which of the following: the predator die-off rate, the prey growth rate, the increase in the predator population due to interactions with the prey, the decrease in the prey population due to interactions with the predator, or none of the above.

17. In the competition model $\dfrac{dx}{dt} = ax - bxy$, $\dfrac{dy}{dt} = cy - dxy$, where $x(t)$ and $y(t)$ are the populations of the competing species, moose and deer, respectively, the coefficient b represents which of the following: the moose growth rate, the deer growth rate, the increase in the moose population due to interactions with the deer, the increase in the deer population due to interactions with the moose, or none of the above.

18. Tank A contains 20 gallons of water in which 2 pounds of salt has been dissolved. Tank B contains 30 gallons of water in which 1 pound of salt has been dissolved. A brine mixture with a concentration of 2.5 pounds of salt per gallon of water is pumped into tank A at the rate of 6 gallons per minute. The well-mixed solution is then pumped from tank A to tank B at the rate of 8 gallons per minute. The solution from tank B is also pumped through another pipe into tank A at the rate of 2 gallons per minute, and the solution from tank B is also pumped out of the system at the rate of 6 gallons per minute. Write down the differential equations with initial conditions for the amounts, $x(t)$ and $y(t)$, of salt in tanks A and B, respectively, at time t.

19. In the previous problem, how much salt will there be in tanks A and B after a long period of time?

20. What is the solution of the logistic equation $\dfrac{dP}{dt} = P(8 - 3P)$ with initial condition $P(0) = 1$?

ANSWER KEY
Zill Differential Equations 9e Chapter 3 Form F

1. 23.8 years
2. 547 years
3. a
4. b
5. 200°F
6. d
7. e
8. $A = 500 - 460e^{-3t/500}$
9. d
10. b
11. 25 grams of A, 0 grams of B, 225 grams of C
12. $X = 900(1 - e^{-75kt})/(4 - 3e^{-75kt})$, where $k = \ln(780/740)/750$
13. a
14. e
15. c
16. the increase in the predator population due to interactions with the prey
17. none of the above
18. $\dfrac{dx}{dt} = 15 - 2x/5 + y/15$, $\dfrac{dy}{dt} = 2x/5 - 4y/15$, $x(0) = 2$, $y(0) = 1$
19. 50 pounds in A, 75 pounds in B
20. $P = 8/(3 + 5e^{-8t})$

1. A bacteria culture grows at a rate proportional to the number of bacteria, $P(t)$, at time t. If the initial amount of bacteria in the culture is P_0, and if the size doubles in four hours, how long does it take the culture to triple in size?

2. The half life of uranium 238 is approximately 4,500,000,000 years. What percentage of the uranium 238 that was present when the dinosaurs died (65,000,000 years ago) is still here today? Assume that the decay rate is proportional to the amount.

3. An object is taken out of a 68°F room and placed outside where the temperature is 22°F room. Fifteen minutes later the temperature is 58°F. It cools according to Newton's Law. The temperature of the object one hour later is

   Select the correct answer.

   (a) 38.9°F
   (b) 39.3°F
   (c) 39.7°F
   (d) 40.1°F
   (e) 40.5°F

4. A chicken is taken out of the freezer (32°F) and placed on a table in a 70°F room. Twelve minutes later the temperature is 38°F. It warms according to Newton's Law. How long does it take before the temperature reaches 55°F?

5. In Newton's Law of cooling, $\dfrac{dT}{dt} = k(T - T_m)$ the constant k is

   Select the correct answer.

   (a) a constant of integration evaluated from an initial condition
   (b) a constant of integration evaluated from another condition
   (c) a proportionality constant evaluated from an initial condition
   (d) a proportionality constant evaluated from another condition

6. A tank contains 1000 liters of water in which 2 kilograms of salt is dissolved. A brine solution containing 0.3 kilograms of salt per liter of water is pumped into the tank at the rate of 4 liters per minute, and the well-stirred mixture is pumped out at the same rate. Let $A(t)$ represent the amount of salt in the tank at time t. The correct initial value problem for $A(t)$ is

   Select the correct answer.

   (a) $\dfrac{dA}{dt} = 4 - A/250,\ A(0) = 0$
   (b) $\dfrac{dA}{dt} = 0.3 - A/250,\ A(0) = 0$
   (c) $\dfrac{dA}{dt} = 4 + A/250,\ A(0) = 2$
   (d) $\dfrac{dA}{dt} = 1.2 - A/250,\ A(0) = 2$

7. In the previous problem, how much salt will be present in the tank after a long period of time?

   Select the correct answer.

   (a) 400 kilograms
   (b) 300 kilograms
   (c) 20 kilograms
   (d) 2 kilograms

8. In the previous two problems, how much salt will be present in the tank at time t?

9. In the logistic model for population growth, $\frac{dP}{dt} = P(18 - 5P)$, what is the carrying capacity of the population $P(t)$?

10. The solution of the logistic equation $\frac{dP}{dt} = P(18 - 5P)$ with $P(0) = 2$ is

    Select the correct answer.

    (a) $P = 18/(10 - e^{-18t})$
    (b) $P = 18/(5 + 4e^{-18t})$
    (c) $P = 5/(18 - 8e^{-18t})$
    (d) $P = 5/(8 + 2e^{-18t})$
    (e) $P = 10/(18 + 2e^{-18t})$

11. Write down the differential equation for a population of fish that grows logistically and for which there is an annual harvest at rate h. Represent the population it time t by $P(t)$.

12. Solve the differential equation of the previous problem if the growth rate of the fish is $a = 4$, the competition coefficient is $b = 1$, and the harvest rate is $h = 3$.

13. Two chemicals, A and B, are combined, forming chemical C. The rate of the reaction is jointly proportional to the amounts of A and B not yet converted to C. Initially, there are 200 grams of A and 300 grams of B, and, during the reaction, for each two grams of A used up in the conversion, there are three grams of B used up. An experiments shows that 50 grams of C are produced in the first ten minutes. After a long period of time, how much of A and of B remains, and how much of C has been produced?

    Select the correct answer.

    (a) 200 grams of A, 0 grams of B, 300 grams of C
    (b) 200 grams of A, 300 grams of B, 0 grams of C
    (c) 100 grams of A, 0 grams of B, 400 grams of C
    (d) 0 grams of A, 100 grams of B, 400 grams of C
    (e) 0 grams of A, 0 grams of B, 500 grams of C

14. In the previous problem, the amount of chemical C, $X(t)$, produced by time t is

    Select the correct answer.

    (a) $X = 600(1 - e^{-100kt})/(3 - 2e^{-100kt})$, where $k = \ln(4/3)/1000$
    (b) $X = 300(1 - e^{-100kt})/(3 - e^{-100kt})$, where $k = \ln(5/3)/1000$
    (c) $X = 500(1 - 1/(500kt + 1))$, where $k = 1/45000$
    (d) $X = 250(1 - e^{-100kt})/(1 - e^{-100kt}/2)$, where $k = \ln(9/8)/1000$
    (e) $X = 500(1 - 1/(5000kt + 1))$, where $k = 1/450000$

15. Radioactive element X decays to element Y with decay constant -0.6. Y, in turn, decays to stable element Z with decay constant -0.7. Write down the system of differential equations and initial conditions for the amounts, $x(t)$, $y(t)$, $z(t)$ of the elements X, Y, Z, respectively, at time t, if, initially, there are 5 grams of X, but no Y or Z.

16. Solve the system in the previous problem.

17. Tank A contains 200 gallons of water in which 10 pounds of salt has been dissolved. Tank B contains 300 gallons of water in which 20 pounds of salt has been dissolved. A brine mixture with a concentration of 2 pounds of salt per gallon of water is pumped into tank A at the rate of 4 gallons per minute. The well-mixed solution is then pumped from tank A to tank B at the rate of 6 gallons per minute. The solution from tank B is also pumped through another pipe into tank A at the rate of 2 gallons per minute, and the solution from tank B is also pumped out of the system at the rate of 4 gallons per minute. The correct differential equations with initial conditions for the amounts, $x(t)$ and $y(t)$, of salt in tanks A and B, respectively, at time t are

    Select the correct answer.

    (a) $\dfrac{dx}{dt} = 4 - 3x/100 + y/5,\ \dfrac{dy}{dt} = 3x/200 - y/15,\ x(0) = 10,\ y(0) = 20$
    (b) $\dfrac{dx}{dt} = 4 - 6x/100 + y/15,\ \dfrac{dy}{dt} = 6x/200 - 2y/15,\ x(0) = 10,\ y(0) = 20$
    (c) $\dfrac{dx}{dt} = 8 - 6x/100 + y/30,\ \dfrac{dy}{dt} = 6x/200 - 2y/15,\ x(0) = 10,\ y(0) = 20$
    (d) $\dfrac{dx}{dt} = 8 - 3x/100 + y/150,\ \dfrac{dy}{dt} = x/100 - y/100,\ x(0) = 10,\ y(0) = 20$
    (e) $\dfrac{dx}{dt} = 8 - 3x/100 + y/150,\ \dfrac{dy}{dt} = 3x/100 - y/50,\ x(0) = 10,\ y(0) = 20$

18. In the previous problem, how much salt will there be in tanks A and B after a long period of time?

    Select the correct answer.

    (a) 400 pounds in A, 600 pounds in B
    (b) 20 pounds in A, 10 pounds in B
    (c) 0 pounds in A, 0 pounds in B
    (d) 400 pounds in A, 0 pounds in B
    (e) none of the above

19. In the Lotka-Volterra predator-prey model $\frac{dx}{dt} = -ax+bxy$, $\frac{dy}{dt} = ey-cxy$, where $x(t)$ is the predator population and $y(t)$ is the prey population, the coefficient c represents which of the following:

    Select the correct answer.

    (a) the predator die-off rate
    (b) the prey growth rate
    (c) the increase in the predator population due to interactions with the prey
    (d) the decrease in the prey population due to interactions with the predator
    (e) none of the above

20. In the competition model $\frac{dx}{dt} = ax - bxy$, $\frac{dy}{dt} = cy - dxy$, where $x(t)$ and $y(t)$ are the populations of the competing species, moose and deer, respectively, the coefficient c represents which of the following: the moose growth rate, the deer growth rate, the decrease in the moose population due to interactions with the deer, the decrease in the deer population due to interactions with the moose, or none of the above.

ANSWER KEY
Zill Differential Equations 9e Chapter 3 Form G

1. 6.34 hours
2. 99.0%
3. b
4. 64.9 minutes
5. d
6. d
7. b
8. $A = 300 - 298e^{-t/250}$
9. 18/5
10. b
11. $\dfrac{dP}{dt} = P(a - bP) - h$
12. $P = (3 - ce^{-2t})/(1 - ce^{-2t})$
13. e
14. c
15. $\dfrac{dx}{dt} = -0.6x$, $\dfrac{dy}{dt} = 0.6x - 0.7y$, $\dfrac{dz}{dt} = 0.7y$, $x(0) = 5$, $y(0) = 0$, $z(0) = 0$
16. $x = 5e^{-0.6t}$, $y = 30(e^{-0.6t} - e^{-0.7t})$, $z = 5 - 35e^{-0.6t} + 30e^{-0.7t}$
17. e
18. a
19. d
20. the deer growth rate

1. The population of a town grows at a rate proportional to the number of people, $P(t)$, at time t. If the initial population is P_0, and if the size doubles in eighteen years, how long does it take the population to triple in size?

2. A fossilized tree is found. It contains 3% of the carbon-14 found in living matter. What is the age of the tree? Assume that the C-14 decays at a rate proportional to the amount present. The half-life of C-14 is approximately 5600 years.

3. The half-life of plutonium 239 is 24,200 years. Assume that the decay rate is proportional to the amount. An initial amount of 3 grams of radium would decay to 2 grams in approximately

   Select the correct answer.

   (a) 12200 years
   (b) 14200 years
   (c) 15200 years
   (d) 17200 years
   (e) 18200 years

4. An object is taken out of a 65°F room and placed outside where the temperature is 85°F. Twelve minutes later the temperature is 68°F. It warms according to Newton's Law. The temperature of the object after one hour is

   Select the correct answer.

   (a) 74.5°F
   (b) 74.9°F
   (c) 75.3°F
   (d) 75.7°F
   (e) 76.1°F

5. In Newton's Law of cooling, $\dfrac{dT}{dt} = k(T - T_m)$ the constant k is

   Select the correct answer.

   (a) a constant of integration evaluated from an initial condition
   (b) a constant of integration evaluated from another condition
   (c) a proportionality constant evaluated from an initial condition
   (d) a proportionality constant evaluated from another condition
   (e) none of the above

6. An object is taken out the oven at 400°F and placed on a table where the temperature is 70°F. Twenty minutes later the temperature is 335°F. It cools according to Newton's Law. What is the temperature of the object after one hour?

7. A tank contains 100 liters of water in which 3 kilograms of salt is dissolved. A brine solution containing 0.2 kilograms of salt per liter of water is pumped into the tank at the rate of 8 liters per minute, and the well-stirred mixture is pumped out at the same rate. Let $A(t)$ represent the amount of salt in the tank at time t. The correct initial value problem for $A(t)$ is

   Select the correct answer.

   (a) $\dfrac{dA}{dt} = 1.6 - 2A/25,\ A(0) = 3$
   (b) $\dfrac{dA}{dt} = 1.6 + 2A/25,\ A(0) = 3$
   (c) $\dfrac{dA}{dt} = 8 + 2A/25,\ A(0) = 3$
   (d) $\dfrac{dA}{dt} = 8 - 2A/25,\ A(0) = 3$
   (e) $\dfrac{dA}{dt} = 0.2 - 2A/25,\ A(0) = 3$

8. In the previous problem, how much salt will be present in the tank after a long period of time?

   Select the correct answer.

   (a) 800 kilograms
   (b) 200 kilograms
   (c) 20 kilograms
   (d) 3 kilograms
   (e) none of the above

9. In the previous two problems, how much salt will be present at time t?

10. In the logistic model for population growth, $\dfrac{dP}{dt} = P(10 - 4P)$, the carrying capacity of the population $P(t)$ is

    Select the correct answer.

    (a) 40
    (b) 10
    (c) 4
    (d) 5/2
    (e) 2/5

11. What is the solution of the logistic equation $\dfrac{dP}{dt} = P(10 - 4P)$ with initial condition $P(0) = 2$?

12. Two chemicals, A and B, are combined, forming chemical C. The rate of the reaction is jointly proportional to the amounts of A and B not yet converted to C. Initially, there are 50 grams of A and 30 grams of B, and, during the reaction, for each two grams of A used up in the conversion, there are three grams of B used up. An experiments shows that 40 grams of C are produced in the first ten minutes. After a long period of time, how much of A and of B remains, and how much of C has been produced?

13. In the previous problem, what is the amount, $X(t)$, of chemical C produced by time t?

14. Radioactive element X decays to element Y with decay constant -0.3. Y, in turn, decays to stable element Z with decay constant -0.2. What is the system of differential equations for the amounts, $x(t)$, $y(t)$, $z(t)$ of the elements X, Y, Z, respectively, at time t, if the initial conditions are $x(0) = 5$, $y(0) = 0$, $z(0) = 0$.

    Select the correct answer.

    (a) $\dfrac{dx}{dt} = -0.2x,\ \dfrac{dy}{dt} = 0.2x - 0.3y,\ \frac{dz}{dt} = 0.2y$
    (b) $\dfrac{dx}{dt} = -0.2x,\ \dfrac{dy}{dt} = 0.2x - 0.3y,\ \frac{dz}{dt} = 0.3y$
    (c) $\dfrac{dx}{dt} = -0.3x,\ \dfrac{dy}{dt} = 0.3x - 0.2y,\ \frac{dz}{dt} = 0.2y$
    (d) $\dfrac{dx}{dt} = -0.3x,\ \dfrac{dy}{dt} = 0.3x - 0.2y,\ \frac{dz}{dt} = 0.3y$
    (e) $\dfrac{dx}{dt} = -0.3y,\ \dfrac{dy}{dt} = 0.3x - 0.2z,\ \frac{dz}{dt} = 0.2y$

15. In the previous problem, the amounts of X, Y, and Z after a long period of time are

    Select the correct answer.

    (a) $x = 0,\ y = 0,\ z = 5$
    (b) $x = 0,\ y = 5,\ z = 0$
    (c) $x = 2.5,\ y = 2.5,\ z = 0$
    (d) $x = 0,\ y = 2.5,\ z = 2.5$
    (e) none of the above

16. The solution of the system of differential equations in the previous two problems is

    Select the correct answer.

    (a) $x = 5e^{-0.3t},\ y = 15(e^{-0.2t} - e^{-0.3t}),\ z = 5 - 15e^{-0.3t} + 10e^{-0.2t}$
    (b) $x = 5e^{-0.2t},\ y = 15(e^{-0.3t} - e^{-0.2t}),\ z = 5 - 15e^{-0.2t} + 10e^{-0.3t}$
    (c) $x = 5e^{-0.3t},\ y = 15(e^{-0.3t} - e^{-0.2t}),\ z = 5 - 15e^{-0.3t} + 10e^{-0.2t}$
    (d) $x = 5e^{-0.2t},\ y = 15(e^{-0.2t} - e^{-0.3t}),\ z = 5 - 15e^{-0.2t} + 10e^{-0.3t}$
    (e) $x = 5e^{-0.3t},\ y = 15(e^{-0.2t} - e^{-0.3t}),\ z = 5 - 15e^{-0.2t} + 10e^{-0.3t}$

17. Tank A contains 30 gallons of water in which 8 pounds of salt has been dissolved. Tank B contains 50 gallons of water in which 4 pounds of salt has been dissolved. A brine mixture with a concentration of 1.5 pounds of salt per gallon of water is pumped into tank A at the rate of 6 gallons per minute. The well-mixed solution is then pumped from tank A to tank B at the rate of 10 gallons per minute. The solution from tank B is also pumped through another pipe into tank A at the rate of 4 gallons per minute, and the solution from tank B is also pumped out of the system at the rate of 6 gallons per minute. Write down the differential equations with initial conditions for the amounts, $x(t)$ and $y(t)$, of salt in tanks A and B, respectively, at time t.

18. In the previous problem, how much salt will there be in tanks A and B after a long period of time?

19. In the Lotka-Volterra predator-prey model $\frac{dx}{dt} = -ax+bxy$, $\frac{dy}{dt} = ey-cxy$, where $x(t)$ is the predator population and $y(t)$ is the prey population, the coefficient e represents which of the following: the predator die-off rate, the prey growth rate, the increase in the predator population due to interactions with the prey, the decrease in the prey population due to interactions with the predator, or none of the above.

20. In the competition model $\frac{dx}{dt} = ax - bxy$, $\frac{dy}{dt} = cy - dxy$, where $x(t)$ and $y(t)$ are the populations of the competing species, moose and deer, respectively, the coefficient d represents which of the following:

    Select the correct answer.

    (a) the moose growth rate
    (b) the deer growth rate
    (c) the decrease in the moose population due to interactions with the deer
    (d) the decrease in the deer population due to interactions with the moose
    (e) none of the above

ANSWER KEY
Zill Differential Equations 9e Chapter 3 Form H

1. 28.5 years
2. 28,300 years
3. b
4. e
5. d
6. 241°F
7. a
8. c
9. $A = 20 - 17e^{-2t/25}$
10. d
11. $P = 10/(4 + e^{-10t})$
12. 30 grams of A, 0 grams of B, 50 grams of C
13. $X = 250(1 - e^{-75kt})/(5 - 2e^{-75kt})$, where $k = \ln(17/5)/750$
14. c
15. a
16. e
17. $\dfrac{dx}{dt} = 9 - x/3 + 2y/25$, $\dfrac{dy}{dt} = x/3 - y/5$, $x(0) = 8$, $y(0) = 4$
18. 45 pounds in A, 75 pounds in B
19. the prey growth rate
20. d