1. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} x^n/n$
2. Find the interval of convergence of the power series in the previous problem.
3. Write down the power series expansion about $x = 0$ for the function $f(x) = e^x$.
4. Name the singular points of the differential equation $xy'' - y = 0$.
5. Find a power series solution about $x = 0$ of the differential equation $y'' + y = 0$.
6. Find a power series solution about $x = 0$ of the differential equation $y'' - 4y = 0$.
7. Find the recurrence relation for the terms in the power series solution about $x = 0$ of the differential equation $y'' + xy = 0$.
8. Find the first four nonzero terms of a power series solution about $x = 0$ of the differential equation $y'' + xy = 0$.
9. What is the radius of convergence of the power series solution of $y'' + xy = 0$?
10. Determine the singular points of $x^3y'' - 2xy' + y = 0$ and classify them as regular or irregular.
11. Determine the singular points of $(x^2 - 16)^2y'' - 2xy' + y = 0$ and classify them as regular or irregular.
12. Determine the singular points of $(x^3 + x)^2(x - 1)y'' - 2xy' + y = 0$ and classify them as regular or irregular.
13. The point $x = 0$ is a critical point of the differential equation $2xy'' + y = 0$. What is the indicial equation?
14. For the previous problem, find the recurrence relation for the series solution about $x = 0$.
15. Find a series solution of $2xy'' + y = 0$.
16. The point $x = 0$ is a critical point of the differential equation $xy'' + y' + 2y = 0$. What is the indicial equation?
17. For the previous problem, find the recurrence relation for the series solution about $x = 0$.
18. Find a series solution of $xy'' + y' + 2y = 0$.
19. What is the name (including the order) of the differential equation $x^2y'' + xy' + (x^2 - 1/4)y = 0$? Write the solution using the special function notation.
20. What is the name (including the order) of the differential equation $(1 - x^2)y'' - 2xy' + 6y = 0$? Write the solution using the special function notation.

ANSWER KEY

Zill Differential Equations 9e Chapter 6 Form A

1. $R = 1$
2. $[-1, 1)$
3. $\sum_{k=0}^{\infty} x^k/k!$
4. $x = 0$
5. $\sum_{k=0}^{\infty}(-1)^k x^{2k}/(2k)!$ or $\sum_{k=0}^{\infty}(-1)^k x^{2k+1}/(2k+1)!$
6. $\sum_{k=0}^{\infty}(2x)^{2k}/(2k)!$ or $\sum_{k=0}^{\infty}(-2x)^{2k+1}/(2k+1)!$
7. $c_{k+3}(k+3)(k+2) + c_k = 0,\ k = 0, 1, 2, \ldots$
8. $1 - x^3/6 + x^6/180 - x^9/12960$ or $x - x^4/12 + x^7/504 - x^{10}/45360$
9. $R = \infty$
10. $x = 0$ is an irregular singular point
11. $x = \pm 4$ are both regular singular points
12. $x = 0,\ 1,\ \pm i$ are all regular singular points
13. $r^2 - r = 0$
14. $2c_{k+1}(k+r+1)(k+r) + c_k = 0,\ k = 0, 1, 2, \ldots$
15. $x\sum_{k=}^{\infty}(-1)^k (x/2)^k/(k!(k+1)!)$
16. $r^2 = 0$
17. $c_{k+1}(k+r+1)^2 + 2c_k = 0,\ k = 0, 1, 2, \ldots$
18. $\sum_{k=0}^{\infty}(-2)^k x^k/(k!)^2$
19. Bessel's equation of order 1/2, $y = c_1 J_{1/2}(x) + Y_{1/2}(x)$
20. Legendre's equation of order 2, $y = c_1 P_2(x) + c_2 Q_2(x)$, where $Q_2(x)$ is given by an infinite series

1. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} x^n/n^2$
2. Find the interval of convergence of the power series in the previous problem.
3. Write down the power series expansion about $x = 0$ for the function $f(x) = \cos x$.
4. What are the singular points of the differential equation $x(1-x)y'' + y' = 0$.
5. Find a power series solution about $x = 0$ of the differential equation $y'' - y = 0$.
6. Find a power series solution about $x = 0$ of the differential equation $y''+4y = 0$.
7. Find the recurrence relation for the terms in the power series solution about $x = 0$ of the differential equation $y'' - xy = 0$.
8. Find the first four nonzero terms of a power series solution about $x = 0$ of the differential equation $y'' - xy = 0$.
9. What is the radius of convergence of the power series solution of $y'' - xy = 0$?
10. Determine the singular points of $x^2y''-2xy'+y = 0$ and classify them as regular or irregular.
11. Determine the singular points of $(x^2 - 4)^2y'' - 2xy' + y = 0$ and classify them as regular or irregular.
12. Determine the singular points of $x^3(x - 1)y'' - 2xy' + y = 0$ and classify them as regular or irregular.
13. The point $x = 0$ is a critical point of the differential equation $xy'' + y = 0$. What is the indicial equation?
14. For the previous problem, find the recurrence relation for the series solution about $x = 0$.
15. Find a series solution of $xy'' + y = 0$.
16. The point $x = 0$ is a critical point of the differential equation $2xy''-y'+2y = 0$. What is the indicial equation?
17. For the previous problem, find the recurrence relation for the series solution about $x = 0$.
18. Find the series solution of $2xy'' - y' + 2y = 0$.
19. What is the name (including the order) of the differential equation $x^2y''+xy' + (x^2 - 1/16)y = 0$? Write the solution using the special function notation.
20. What is the name (including the order) of the differential equation $(1-x^2)y'' - 2xy' + 12y = 0$? Write the solution using the special function notation.

ANSWER KEY

Zill Differential Equations 9e Chapter 6 Form B

1. $R = 1$
2. $[-1, 1]$
3. $\sum_{k=0}^{\infty}(-1)^k x^{2k}/(2k)!$
4. $x = 0,\ 1$
5. $\sum_{k=0}^{\infty} x^{2k}/(2k)!$ or $\sum_{k=0}^{\infty}(-x)^{2k+1}/(2k+1)!$
6. $\sum_{k=0}^{\infty}(-1)^k(2x)^{2k}/(2k)!$ or $\sum_{k=0}^{\infty}(-1)^k(2x)^{2k+1}/(2k+1)!$
7. $c_{k+3}(k+3)(k+2) - c_k = 0,\ k = 0, 1, 2, \ldots$
8. $1 + x^3/6 + x^6/180 + x^9/12960$ or $x + x^4/12 + x^7/504 + x^{10}/45360$
9. $R = \infty$
10. $x = 0$ is a regular singular point
11. $x = \pm 2$ are both regular singular points
12. $x = 0$ is an irregular singular point, $x = 1$ is a regular singular point
13. $r^2 - r = 0$
14. $c_{k+1}(k+r+1)(k+r) + c_k = 0,\ k = 0, 1, 2, \ldots$
15. $x\sum_{k=0}^{\infty}(-1)^k x^k/(k!(k+1)!)$
16. $2r^2 - 3r = 0$
17. $c_{k+1}(k+r+1)(2k+2r-1) + 2c_k = 0,\ k = 0, 1, 2, \ldots$
18. $c_1\left[1 + \sum_{k=1}^{\infty}(-2)^k x^k/(k!(-1)\cdot 1\cdot 3\ldots(2k-3))\right] + c_2 x^{3/2}\left[1 + \sum_{k=1}^{\infty}(-2)^k x^k/(k!5\cdot 7\cdot 9\ldots(2k+3))\right]$
19. Bessel's equation of order 1/4, $y = c_1 J_{1/4}(x) + c_2 J_{-1/4}(x)$
20. Legendre's equation of order 3, $y = c_1 P_3(x) + c_2 Q_3(x)$, where $Q_3(x)$ is given as an infinite series

1. The radius of convergence of the power series $\sum_{n=1}^{\infty} x^n/n$ is

   Select the correct answer.

   (a) 0
   (b) 1
   (c) 2
   (d) ∞
   (e) none of the above

2. The interval of convergence of the power series in the previous problem is

   Select the correct answer.

   (a) $\{0\}$
   (b) $(-1, 1)$
   (c) $[-1, 1]$
   (d) $[-1, 1)$
   (e) $(-\infty, \infty)$

3. The first four terms in the power series expansion of the function $f(x) = e^{2x}$ about $x = 0$ are

   Select the correct answer.

   (a) $1 + x + x^2 + x^3$
   (b) $1 + 2x + 2x^2 + 2x^3$
   (c) $1 + 2x + 2x^2 + 4x^3/3$
   (d) $1 + 2x + 2x^2 + 2x^3/3$
   (e) $1 + 2x + 4x^2 + 8x^3$

4. The singular points of the differential equation $y'' + y'/x + y(x-2)/(x-3) = 0$ are

   Select the correct answer.

   (a) none
   (b) 0
   (c) 0, 2
   (d) 0, 3
   (e) 0, 2, 3

5. The recurrence relation for the power series solution about $x = 0$ of the differential equation $y'' - y = 0$ is (for $k = 0, 1, 2, \ldots$)

   Select the correct answer.

   (a) $(k+2)(k+1)c_{k+2} = c_k$
   (b) $(k+2)(k+1)c_k = c_{k-2}$
   (c) $(k+1)kc_{k+2} = c_k$
   (d) $(k+1)kc_k = c_{k-2}$
   (e) $(k-2)(k-1)c_{k-2} = c_k$

6. The solution of the recurrence relation in the previous problem is

   Select the correct answer.

   (a) $c_{2k} = c_0/(2k)$, $c_{2k+1} = c_1/(2k+1)$
   (b) $c_{2k} = c_0/(2k)^2$, $c_{2k+1} = c_1/(2k+1)^2$
   (c) $c_{2k} = c_0/(2k)!$, $c_{2k+1} = c_1/(2k+1)!$
   (d) $c_{2k} = c_0/(2k+2)!$, $c_{2k+1} = c_1/(2k+3)!$
   (e) $c_{2k} = c_0/(2k-1)!$, $c_{2k+1} = c_1/(2k)!$

7. A power series solution about $x = 0$ of the differential equation $y'' - y = 0$ is

   Select the correct answer.

   (a) $y = c_0 \sum_{k=0}^{\infty} x^{2k}/(2k)! + c_1 \sum_{k=0}^{\infty} x^{2k+1}/(2k+1)!$
   (b) $y = c_0 \sum_{k=0}^{\infty} x^{2k}/(2k) + c_1 \sum_{k=0}^{\infty} x^{2k+1}/(2k+1)$
   (c) $y = c_0 \sum_{k=0}^{\infty} x^{2k}/(2k)^2 + c_1 \sum_{k=0}^{\infty} x^{2k+1}/(2k+1)^2$
   (d) $y = c_0 \sum_{k=0}^{\infty} x^{2k}/(2k)! + c_1 \sum_{k=0}^{\infty} x^{2k-1}/(2k-1)!$
   (e) $y = c_0 \sum_{k=0}^{\infty} x^{2k}/(2k) + c_1 \sum_{k=0}^{\infty} x^{2k-1}/(2k-1)$

8. The radius of convergence of the power series solution of $y'' - y = 0$ about $x = 0$ is

   Select the correct answer.

   (a) 0
   (b) 1
   (c) 2
   (d) ∞
   (e) none of the above

9. The singular points of $x^2(x-1)y'' - 2xy' + y = 0$ are $x =$
   Select all that apply.
   (a) 2
   (b) -1
   (c) 0
   (d) 1
   (e) none of the above

10. For the differential equation $(x^2-4)^2y'' - 2xy' + y = 0$, the point $x = 0$ is
    Select the correct answer.
    (a) an ordinary point
    (b) a regular singular point
    (c) an irregular singular point
    (d) a special point
    (e) none of the above

11. For the differential equation $(x^2-4)^2y'' - 2xy' + y = 0$, the point $x = 2$ is
    Select the correct answer.
    (a) an ordinary point
    (b) a regular singular point
    (c) an irregular singular point
    (d) a special point
    (e) none of the above

12. For the differential equation $(x^2-4)^3y'' - 2xy' + y = 0$, the point $x = -2$ is
    Select the correct answer.
    (a) an ordinary point
    (b) a regular singular point
    (c) an irregular singular point
    (d) a special point
    (e) none of the above

13. The indicial equation for the differential equation $2xy'' - y' + 2y = 0$ is

    Select the correct answer.

    (a) $r(2r - 1) = 0$
    (b) $r(2r - 3) = 0$
    (c) $r(2r - 2) = 0$
    (d) $r(r - 3) = 0$
    (e) $r(r - 2) = 0$

14. The recurrence relation for the differential equation $2xy'' - y' + 2y = 0$ is

    Select the correct answer.

    (a) $c_{k+1}(k + r)(2k + 2r - 1) + 2c_k = 0$
    (b) $c_{k+1}(k + r)(k + r - 1) + 2c_k = 0$
    (c) $c_{k+1}(k + r + 1)(2k + 2r - 1) - 2c_k = 0$
    (d) $c_{k+1}(k + r + 1)(2k + 2r - 1) + 2c_k = 0$
    (e) $c_{k+1}(k + r + 1)(2k + 2r) + 2c_k = 0$

15. The differential equation $x^2y'' + xy' + (x^2 - 1/16)y = 0$ is

    Select the correct answer.

    (a) Bessel's equation of order n
    (b) Bessel's equation of order 1/16
    (c) Bessel's equation of order 1/4
    (d) Legendre's equation of order 1/16
    (e) Legendre's equation of order 1/4

16. The solution of the previous problem is

    Select the correct answer.

    (a) $y = c_1P_{1/4}(x) + c_2P_{-1/4}(x)$
    (b) $y = c_1P_4(x) + c_2P_{-4}(x)$
    (c) $y = c_1J_4(x) + c_2Y_4(x)$
    (d) $y = c_1J_{1/4}(x) + c_2J_{-1/4}(x)$
    (e) $y = c_1J_{1/16}(x) + c_2J_{-1/16}(x)$

17. Consider the differential equation $2x^2y'' + 3xy' + (2x - 1)y = 0$. The indicial equation is $2r^2 + r - 1 = 0$. The recurrence relation is $c_k[2(k + r)(k + r - 1) + 3(k + r) - 1] + 2c_{k-1} = 0$. A series solution corresponding to the indicial root $r = -1$ is $y = x^{-1}\left[1 + \sum_{k=1}^{\infty} c_k x^k\right]$, where

    Select the correct answer.

    (a) $c_k = (-2)^k/[k!(-1) \cdot 1 \cdot 3 \cdots (2k - 3)]$
    (b) $c_k = -2^k/[k!1 \cdot 3 \cdots (2k - 3)]$
    (c) $c_k = (-2)^k/[k!(-1) \cdot 1 \cdot 3 \cdots (2k - 1)]$
    (d) $c_k = (-2)^k/[k!(-1)(2k - 3)!]$
    (e) $c_k = (-2)^k/[k!1 \cdot 3 \cdots (2k - 5)]$

18. In the previous problem, a series solution corresponding to the indicial root $r = 1/2$ is $y = x^{1/2}\{1 + \sum_{k=1}^{\infty} c_k x^k\}$, where

    Select the correct answer.

    (a) $c_k = (-2)^k/[k!3 \cdot 5 \cdot 7 \cdots (2k - 3)]$
    (b) $c_k = (-2)^k/[k!1 \cdot 3 \cdot 5 \cdots (2k - 3)]$
    (c) $c_k = -2^k/[k!5 \cdot 7 \cdot 9 \cdots (2k + 1)]$
    (d) $c_k = (-2)^k/[k!(2k + 3)!]$
    (e) $c_k = (-2)^k/[k!5 \cdot 7 \cdot 9 \cdots (2k + 3)]$

19. The differential equation $(1 - x^2)y'' - 2xy' + 20y = 0$ is

    Select the correct answer.

    (a) Bessel's equation of order 20
    (b) Bessel's equation of order 4
    (c) Legendre's equation of order n
    (d) Legendre's equation of order 20
    (e) Legendre's equation of order 4

20. The solution of the previous problem is

    Select the correct answer.

    (a) $y = c_1P_{20}(x) + c_2P_{-20}(x)$
    (b) $y = c_1P_4(x) + c_2Q_4(x)$, where $Q_4(x)$ is given by an infinite series
    (c) $y = c_1J_4(x) + c_2Y_4(x)$
    (d) $y = c_1J_{1/4}(x) + c_2J_{-1/4}(x)$
    (e) $y = c_1J_{20}(x) + c_2Y_{20}(x)$

ANSWER KEY
Zill Differential Equations 9e Chapter 6 Form C

1. b
2. d
3. c
4. d
5. a
6. c
7. a
8. d
9. c, d
10. a
11. b
12. b
13. b
14. d
15. c
16. d
17. a
18. e
19. e
20. b

1. The radius of convergence of the power series $\sum_{n=1}^{\infty} x^n/n!$ is

   Select the correct answer.

   (a) 0
   (b) 1
   (c) 2
   (d) ∞
   (e) none of the above

2. The interval of convergence of the power series in the previous problem is

   Select the correct answer.

   (a) $\{0\}$
   (b) $(-1, 1)$
   (c) $[-1, 1]$
   (d) $(-1, 1]$
   (e) $(-\infty, \infty)$

3. The first four nonzero terms in the power series expansion of the function $f(x) = \sin x$ about $x = 0$ are

   Select the correct answer.

   (a) $1 - x + x^2/2 - x^3/3$
   (b) $x - x^3/6 + x^5/120 - x^7/5040$
   (c) $x + x^3 + x^5 + x^7$
   (d) $1 + x^2/2 + x^4/4 + x^6/6$
   (e) $1 - x^2/2 + x^4/24 - x^6/720$

4. The singular points of the differential equation $xy'' + y' + y(x+2)/(x-4) = 0$ are

   Select the correct answer.

   (a) none
   (b) 0
   (c) 0, -2
   (d) 0, 4
   (e) 0, -2, 4

5. The recurrence relation for the power series solution about $x = 0$ of the differential equation $y'' + y = 0$ is

   Select the correct answer.

   (a) $(k+2)(k+1)c_{k+2} + c_k = 0$
   (b) $(k+2)(k+1)c_k + c_{k-2} = 0$
   (c) $(k+1)kc_{k+2} + c_k = 0$
   (d) $(k+1)kc_k + c_{k-2} = 0$
   (e) $(k-2)(k-1)c_{k-2} + c_k = 0$

6. The solution of the recurrence relation in the previous problem is

   Select the correct answer.

   (a) $c_{2k} = c_0(-1)^k/(2k)$, $c_{2k+1} = c_1(-1)^k/(2k+1)$
   (b) $c_{2k} = c_0(-1)^k/(2k)^2$, $c_{2k+1} = c_1(-1)^k/(2k+1)^2$
   (c) $c_{2k} = c_0(-1)^k/(2k)!$, $c_{2k+1} = c_1(-1)^k/(2k+1)!$
   (d) $c_{2k} = c_0(-1)^k/(2k+2)!$, $c_{2k+1} = c_1(-1)^k/(2k+3)!$
   (e) $c_{2k} = c_0(-1)^k/(2k-1)!$, $c_{2k+1} = c_1(-1)^k/(2k)!$

7. A power series solution about $x = 0$ of the differential equation $y'' + y = 0$ is

   Select the correct answer.

   (a) $y = c_0 \sum_{k=0}^{\infty}(-1)^k x^{2k}/(2k)! + c_1 \sum_{k=0}^{\infty}(-1)^k x^{2k+1}/(2k+1)!$
   (b) $y = c_0 \sum_{k=0}^{\infty}(-1)^k x^{2k}/(2k) + c_1 \sum_{k=0}^{\infty}(-1)^k x^{2k+1}/(2k+1)$
   (c) $y = c_0 \sum_{k=0}^{\infty}(-1)^k x^{2k}/(2k)^2 + c_1 \sum_{k=0}^{\infty}(-1)^k x^{2k+1}/(2k+1)^2$
   (d) $y = c_0 \sum_{k=0}^{\infty}(-1)^k x^{2k}/(2k)! + c_1 \sum_{k=0}^{\infty}(-1)^k x^{2k-1}/(2k-1)!$
   (e) $y = c_0 \sum_{k=0}^{\infty}(-1)^k x^{2k}/(2k) + c_1 \sum_{k=0}^{\infty}(-1)^k x^{2k-1}/(2k-1)$

8. The radius of convergence of the power series solution of $y'' + y = 0$ about $x = 0$ is

   Select the correct answer.

   (a) 0
   (b) 1
   (c) 2
   (d) ∞
   (e) none of the above

9. For the equation $(x^2-16)^3(x-1)y''-2xy'+y=0$, the point $x=0$ is
   Select the correct answer.

   (a) an ordinary point
   (b) a regular singular point
   (c) an irregular singular point
   (d) a special point
   (e) none of the above

10. For the equation $(x^2-16)^3(x-1)y''-2xy'+y=0$, the point $x=1$ is
    Select the correct answer.

    (a) an ordinary point
    (b) a regular singular point
    (c) an irregular singular point
    (d) a special point
    (e) none of the above

11. For the equation $(x^2-16)^3(x-1)y''-2xy'+y=0$, the point $x=4$ is
    Select the correct answer.

    (a) an ordinary point
    (b) a regular singular point
    (c) an irregular singular point
    (d) a special point
    (e) none of the above

12. The indicial equation for the differential equation $xy''+2y'-xy=0$ is
    Select the correct answer.

    (a) $r(r-1)=0$
    (b) $r(r+2)=0$
    (c) $r(2r+1)=0$
    (d) $r(2r-1)=0$
    (e) $r(r+1)=0$

13. The recurrence relation for the differential equation $xy'' + 2y' - xy = 0$ is

    Select the correct answer.

    (a) $c_k(k+r)(k+r-1) + c_{k-2} = 0$
    (b) $c_k(k+r)(k+r-1) - c_{k-2} = 0$
    (c) $c_k(k+r+1)^2 - c_{k-2} = 0$
    (d) $c_k(k+r+2)(k+r+1) + c_{k-2} = 0$
    (e) $c_k(k+r)(k+r+1) - c_{k-2} = 0$

14. Consider the differential equation $xy'' - xy' + y = 0$. The indicial equation is $r(r-1) = 0$. The recurrence relation is $c_{k+1}(k+r+1)(k+r) - c_k(k+r-1) = 0$. A series solution corresponding to the indicial root $r = 0$ is

    Select the correct answer.

    (a) $y_1 = x$
    (b) $y_1 = x^2$
    (c) $y_1 = \sum_{k=0}^{\infty}(-2x)^k/[k!(-1)\cdot 1\cdot 3\cdots(2k-1)]$
    (d) $y_1 = \sum_{k=0}^{\infty}(-2x)^k/[k!(2k-3)!]$
    (e) $y_1 = \sum_{k=0}^{\infty}(-2x)^k/[k!1\cdot 3\cdots(2k-3)]$

15. In the previous problem, a second solution is

    Select the correct answer.

    (a) $y_2 = e^x$
    (b) $y_2 = x\int e^x/x^2 dx$
    (c) $y_2 = 1 + \sum_{k=1}^{\infty} c_k x^k$, where $c_k = (k-1)/(k(k+1))$
    (d) $y_2 = 1 + \sum_{k=1}^{\infty} c_k x^k$, where $c_k = 1/k^2$
    (e) none of the above

16. The differential equation $x^2y'' + xy' + (x^2 - 1/25)y = 0$ is

    Select the correct answer.

    (a) Bessel's equation of order n
    (b) Bessel's equation of order 1/25
    (c) Bessel's equation of order 1/5
    (d) Legendre's equation of order 1/25
    (e) Legendre's equation of order 1/5

17. The solution of the previous problem is

    Select the correct answer.

    (a) $y = c_1 P_{1/5}(x) + c_2 P_{-1/5}(x)$
    (b) $y = c_1 P_5(x) + c_2 P_{-5}(x)$
    (c) $y = c_1 J_5(x) + c_2 Y_5(x)$
    (d) $y = c_1 J_{1/5}(x) + c_2 J_{-1/5}(x)$
    (e) $y = c_1 J_{1/25}(x) + c_2 Y_{1/25}(x)$

18. The differential equation $(1 - x^2)y'' - 2xy' + 12y = 0$ is

    Select the correct answer.

    (a) Bessel's equation of order 12
    (b) Bessel's equation of order 3
    (c) Legendre's equation of order 12
    (d) Legendre's equation of order 3
    (e) Legendre's equation of order 4

19. The solution of the previous problem is

    Select the correct answer.

    (a) $y = c_1 P_3(x) + c_2 P_{-3}(x)$
    (b) $y = c_1 P_3(x) + c_2 Q_3(x)$, where $Q_3(x)$ is given by an infinite series
    (c) $y = c_1 J_4(x) + c_2 Y_4(x)$
    (d) $y = c_1 J_3(x) + c_2 Y_3(x)$
    (e) $y = c_1 J_{12}(x) + c_2 Y_{12}(x)$

20. Find three positive values of λ for which the differential equation $(1 - x^2)y'' - 2xy' + \lambda y = 0$ has polynomial solutions.

    Select the correct answer.

    (a) 2, 6, 12
    (b) 1, 2, 3
    (c) 1, 4, 9
    (d) 2, 4, 6
    (e) 2, 6, 10

ANSWER KEY
Zill Differential Equations 9e Chapter 6 Form D

1. d
2. e
3. b
4. d
5. a
6. c
7. a
8. d
9. a
10. b
11. c
12. e
13. e
14. a
15. b
16. c
17. d
18. d
19. b
20. a

1. The radius of convergence of the power series $\sum_{n=1}^{\infty} x^n/n^{3/2}$ is

   Select the correct answer.

   (a) 0
   (b) 1
   (c) 2
   (d) ∞
   (e) none of the above

2. The interval of convergence of the power series in the previous problem is

   Select the correct answer.

   (a) $(-1, 1)$
   (b) $[-1, 1]$
   (c) $[-1, 1)$
   (d) $(-\infty, \infty)$
   (e) none of the above

3. Write down the power series expansion about $x = 0$ for the function $f(x) = e^{3x}$.

4. What are the singular points of the differential equation $\sin x\; y'' - y = 0$?

5. The recurrence relation for the power series solution about $x = 0$ of the differential equation $y'' + y' = 0$ is

   Select the correct answer.

   (a) $(k+2)(k+1)c_{k+2} + (k+1)c_{k+1} = 0$
   (b) $(k+2)(k+1)c_{k+2} + kc_k = 0$
   (c) $(k+1)kc_{k+2} + kc_k = 0$
   (d) $(k+1)kc_k + (k-2)c_{k-2} = 0$
   (e) $(k-2)(k-1)c_{k-2} + kc_k = 0$

6. The solution of the recurrence relation in the previous problem is (let $c_1 = 1$)

   Select the correct answer.

   (a) $c_k = (-1)^k/k$
   (b) $c_k = (-1)^k/k^2$
   (c) $c_k = (-1)^k/k!$
   (d) $c_k = (-1)^k/((k+2)(k+1))$
   (e) $c_k = (-1)^k/((k-1)(k-2))$

7. A power series solution about $x = 0$ of the differential equation $y'' + y' = 0$ is

   Select the correct answer.

   (a) $y = c_0 \sum_{k=0}^{\infty} (-1)^k x^{2k}/(2k)^2 + c_1 \sum_{k=0}^{\infty} (-1)^k x^{2k+1}/(2k+1)^2$
   (b) $y = c_0 \sum_{k=0}^{\infty} (-1)^k x^{2k}/(2k)! + c_1 \sum_{k=0}^{\infty} (-1)^k x^{2k+1}/(2k+1)!$
   (c) $y = c_0 \sum_{k=0}^{\infty} (-1)^k x^{2k}/(2k) + c_1 \sum_{k=0}^{\infty} (-1)^k x^{2k+1}/(2k+1)$
   (d) $y = c_0 + c_1 \sum_{k=0}^{\infty} (-1)^k x^k/k!$
   (e) $y = c_0 \sum_{k=0}^{\infty} x^k/k! + c_1 \sum_{k=0}^{\infty} (-1)^k x^k/k!$

8. The radius of convergence of the power series solution of $y''+y' = 0$ about $x = 0$ is

   Select the correct answer.

   (a) 0
   (b) 1
   (c) 2
   (d) ∞
   (e) none of the above

9. Determine the singular points of $x^3y'' - 2xy' + y = 0$ and classify them as regular or irregular.

10. Determine the singular points of $(x^2-1)^2y'' - 2xy' + y = 0$ and classify them as regular or irregular.

11. Determine the singular points of $(x^3+x)^2(x-1)^3y'' - 2xy' + y = 0$ and classify them as regular or irregular.

12. The indicial equation for the differential equation $x^2y'' - (x - 2/9)y = 0$ is
    Select the correct answer.

    (a) $r^2 - 2/9 = 0$
    (b) $r^2 + r + 2/9 = 0$
    (c) $r^2 - r - 2/9 = 0$
    (d) $r^2 + r - 2/9 = 0$
    (e) $r^2 - r + 2/9 = 0$

13. The recurrence relation for the differential equation $x^2y'' - (x - 2/9)y = 0$ is
    Select the correct answer.

    (a) $c_{k+1}((k + r + 1)(k + r) + 2/9) + c_k = 0$
    (b) $c_{k+1}((k + r)(k + r - 1) + 2/9) + c_k = 0$
    (c) $c_{k+1}((k + r + 1)(k + r) + 2/9) - c_k = 0$
    (d) $c_{k+1}((k + r)(k + r - 1) + 2/9) - c_k = 0$
    (e) $c_{k+1}((k + r + 1)(k + r) - 2/9) + 2c_k = 0$

14. Consider the differential equation $x^2y'' + xy' + (x^2 - 1/4)y = 0$. The indicial equation is $r^2 - 1/4 = 0$. The recurrence relation is $c_k[(k+r)^2 - 1/4] + c_{k-2} = 0$. One series solution corresponding to the indicial root $r = -1/2$ is $y = x^{-1/2}(1 + \sum_{k=1}^{\infty} c_k x^{2k})$, where
    Select the correct answer.

    (a) $c_k = (-1)^k/(2k)!$
    (b) $c_k = -1^k/k!$
    (c) $c_k = (-1)^k/k!$
    (d) $c_k = -1^k/(2k)!$
    (e) $c_k = (-1)^k/k!^2$

15. The point $x = 0$ is a critical point of the differential equation $3xy'' + (2 - x)y' - y = 0$. What is the indicial equation?

16. For the previous problem, find the recurrence relation for the series solution about $x = 0$.

17. Find the series solution of $3xy'' + (2 - x)y' - y = 0$.

18. The differential equation $(1-x^2)y'' - 2xy' + 2y = 0$ is

    Select the correct answer.

    (a) Bessel's equation of order 2
    (b) Bessel's equation of order 1
    (c) Legendre's equation of order 2
    (d) Legendre's equation of order 1
    (e) Legendre's equation of order n

19. The solution of the previous problem is

    Select the correct answer.

    (a) $y = c_1 P_1(x) + c_2 P_{-1}(x)$
    (b) $y = c_1 P_1(x) + c_2 Q_1(x)$, where $Q_1(x)$ is given by an infinite series
    (c) $y = c_1 J_1(x) + c_2 Y_1(x)$
    (d) $y = c_1 J_2(x) + c_2 Y_2(x)$
    (e) $y = c_1 J_1(x) + c_2 J_{-1}(x)$

20. In the previous two problems, what is the polynomial solution?

ANSWER KEY
Zill Differential Equations 9e Chapter 6 Form E

1. b
2. b
3. $\sum_{k=0}^{\infty}(3x)^k/k!$
4. $x = n\pi$, n is an integer
5. a
6. c
7. d
8. d
9. $x = 0$ is an irregular singular point
10. $x = \pm 1$ are both regular singular points
11. $x = 0, \pm i$ are regular singular points, $x = 1$ is an irregular singular point
12. e
13. c
14. a
15. $3r^2 - r = 0$
16. $(3k + 3r + 2)c_{k+1} - c_k = 0$
17. $c_0 x^{1/3} \sum_{k=0}^{\infty}(x/3)^k/k! + c_1[1 + \sum_{k=1}^{\infty} x^k/(2 \cdot 5 \cdot 8 \cdots (3k-1))]$
18. d
19. b
20. $P_1(x) = x$

1. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} x^n/n^{1/2}$.

2. Find the interval of convergence of the power series in the previous problem.

3. The first four nonzero terms in the power series expansion of the function $f(x) = \sin x$ about $x = 0$ are

   Select the correct answer.

   (a) $1 - x + x^2/2 - x^3/3$
   (b) $x - x^3/6 + x^5/120 - x^7/5040$
   (c) $x + x^3 + x^5 + x^7$
   (d) $1 + x^2/2 + x^4/4 + x^6/6$
   (e) $1 - x^2/2 + x^4/24 - x^6/720$

4. The singular points of the differential equation $y'' + y'/x + y(x-2)/(x-3) = 0$ are

   Select the correct answer.

   (a) none
   (b) 0
   (c) 0, 2
   (d) 0, 3
   (e) 0, 2, 3

5. Find a power series solution about $x = 0$ of the differential equation $y'' - y' = 0$.

6. Find a power series solution about $x = 0$ of the differential equation $y'' + 4y = 0$.

7. Find the recurrence relation for the terms in the power series solution about $x = 0$ of the differential equation $y'' + x^2 y = 0$.

8. Find the first four nonzero terms of a power series solution about $x = 0$ of the differential equation $y'' + x^2 y = 0$.

9. What is the radius of convergence of the power series solution of $y'' + x^2 y = 0$?

10. For the differential equation $(x^2-1)^3(x+2)y''-2xy'+y=0$, the point $x=1$ is

    Select the correct answer.

    (a) an ordinary point
    (b) a regular singular point
    (c) an irregular singular point
    (d) a special point
    (e) none of the above

11. For the differential equation $(x^2-1)^3(x+2)y''-2xy'+y=0$, the point $x=2$ is

    Select the correct answer.

    (a) an ordinary point
    (b) a regular singular point
    (c) an irregular singular point
    (d) a special point
    (e) none of the above

12. For the differential equation $(x^2-1)^3(x+2)y''-2xy'+y=0$, the point $x=-2$ is

    Select the correct answer.

    (a) an ordinary point
    (b) a regular singular point
    (c) an irregular singular point
    (d) a special point
    (e) none of the above

13. The point $x=0$ is a critical point of the differential equation $4xy''+y'/2+y=0$. What is the indicial equation?

14. For the previous problem, find the recurrence relation for the series solution about $x=0$.

15. Find the series solution of $4xy''+y'/2+y=0$.

16. The differential equation $x^2y'' + xy' + (x^2 - 1/9)y = 0$ is

    Select the correct answer.

    (a) Bessel's equation of order n
    (b) Bessel's equation of order $1/9$
    (c) Bessel's equation of order $1/3$
    (d) Legendre's equation of order $1/9$
    (e) Legendre's equation of order $1/3$

17. The solution of the previous problem is

    Select the correct answer.

    (a) $y = c_1P_{1/3}(x) + c_2P_{-1/3}(x)$
    (b) $y = c_1P_3(x) + c_2P_{-3}(x)$
    (c) $y = c_1J_3(x) + c_2Y_3(x)$
    (d) $y = c_1J_{1/3}(x) + c_2J_{-1/3}(x)$
    (e) $y = c_1J_{1/9}(x) + c_2Y_{1/9}(x)$

18. What is the name (including the order) of the differential equation $(1-x^2)y'' - 2xy' + 12y = 0$

19. In the previous problem, what is the recurrence relation for the series solution about $x = 0$?

20. What is the solution of the differential equation in the previous two problems?

ANSWER KEY
Zill Differential Equations 9e Chapter 6 Form F

1. $R = 1$
2. $[-1, 1)$
3. b
4. d
5. $c_0 + c_1 \sum_{k=1}^{\infty} x^k/k!$
6. $c_0 \sum_{k=0}^{\infty}(-1)^k(2x)^{2k}/(2k)! + c_1 \sum_{k=0}^{\infty}(-1)^k(2x)^{2k+1}/(2k+1)!$
7. $c_k k(k-1) + c_{k-4} = 0$
8. $1 - x^4/12 + x^8/672 - x^{12}/88704$ or $x - x^5/20 + x^9/1440 - x^{13}/224640$
9. $R = \infty$
10. c
11. a
12. b
13. $4r^2 - 7r/2 = 0$
14. $c_{k+1}(k+r+1)(4k+4r+1/2) + c_k = 0$
15. $c_0[1+\sum_{k=1}^{\infty}(-2x)^k/(k!1\cdot 9\cdots(8k-7)]+c_1x^{7/8}[1+\sum_{k=1}^{\infty}(-2x)^k/(k!15\cdot 23\cdots(8k+7)]$
16. c
17. d
18. Legendre's equation of order 3
19. $c_{k+2}(k+2)(k+1) - c_k(k+4)(k-3) = 0$
20. $c_1(x - 5x^3/3) + c_0[1 + \sum_{k=1}^{\infty} 2^k x^{2k}(-3)(-1)1\cdot 3\cdots(2k-5)/(2k)!]$

1. The radius of convergence of the power series $\sum_{n=1}^{\infty} x^n/n^3$ is

   Select the correct answer.

   (a) 0
   (b) 1
   (c) 2
   (d) ∞
   (e) none of the above

2. The interval of convergence of the power series in the previous problem is

   Select the correct answer.

   (a) $(-1, 1)$
   (b) $[-1, 1]$
   (c) $[-1, 1)$
   (d) $(-\infty, \infty)$
   (e) none of the above

3. Write down the power series expansion about $x = 0$ for the function $f(x) = \sin x$.

4. Identify the singular points of the differential equation $\ln x \cdot y'' + y' = 0$.

5. The recurrence relation for the power series solution about $x = 0$ of the differential equation $y'' - y' = 0$ is

   Select the correct answer.

   (a) $(k+1)c_{k+1} = c_k k$
   (b) $(k+1)c_k = c_{k-1}(k-1)$
   (c) $(k+2)(k+1)c_{k+2} = c_k k$
   (d) $(k+1)kc_k = c_{k-2}(k-2)$
   (e) $(k-2)(k-1)c_{k-2} = c_k k$

6. The solution of the recurrence relation in the previous problem is

   Select the correct answer.

   (a) $c_k = c_0/k$
   (b) $c_k = c_0/k^2$
   (c) $c_k = c_0/k!$
   (d) $c_k = c_0(-1)^k/(2k)!$
   (e) $c_k = c_0(-1)^k/(2k+1)!$

7. A power series solution about $x = 0$ of the differential equation $y'' - y' = 0$ is

   Select the correct answer.

   (a) $y = c_0 \sum_{k=0}^{\infty} x^{2k}/(2k)! + c_1 \sum_{k=0}^{\infty} x^{2k+1}/(2k+1)!$
   (b) $y = c_0 \sum_{k=0}^{\infty} x^{2k}/(2k) + c_1 \sum_{k=0}^{\infty} x^{2k+1}/(2k+1)$
   (c) $y = c_0 + c_1 \sum_{k=1}^{\infty} x^k/k!$
   (d) $y = c_0 \sum_{k=0}^{\infty} x^k/k! + c_1 \sum_{k=0}^{\infty}(-x)^k/k!$
   (e) none of the above

8. The radius of convergence of the power series solution of $y'' - y' = 0$ about $x = 0$ is

   Select the correct answer.

   (a) 0
   (b) 1
   (c) 2
   (d) ∞
   (e) none of the above

9. Determine the singular points of $x^3y'' + 5xy' + y = 0$ and classify them as regular or irregular.

10. Determine the singular points of $(1 - \cos x)y'' - 2xy' + y = 0$ and classify them as regular or irregular.

11. Determine the singular points of $(x - \sin x)(x - 1)^2y'' - 2xy' + y = 0$ and classify them as regular or irregular.

12. The indicial equation for the differential equation $xy'' + (1-x)y' - y = 0$ is
    Select the correct answer.

    (a) $r(r+1) = 0$
    (b) $r(r-1) = 0$
    (c) $r^2 - 1 = 0$
    (d) $r^2 + 1 = 0$
    (e) $r^2 = 0$

13. The recurrence relation for the differential equation $xy'' + (1-x)y' - y = 0$ is
    Select the correct answer.

    (a) $c_{k+1}(k+r) + c_k = 0$
    (b) $c_{k+1}(k+r) - c_k = 0$
    (c) $c_{k+1}(k+r+1) - c_k = 0$
    (d) $c_{k+1}(k+r+1) + c_k = 0$
    (e) $c_{k+1}(k+r+1)(k+r) - 2c_k = 0$

14. The solution of $xy'' + (1-x)y' - y = 0$ is
    Select the correct answer.

    (a) $c_1e^x + c_2e^x(\ln x + \sum_{k=1}^{\infty}(-1)^k x^k/(k \cdot k!)$
    (b) $c_1e^{-x} + c_2e^{-x}(\ln x + \sum_{k=1}^{\infty}(-1)^k x^k/(k \cdot k!)$
    (c) $c_1e^x + c_2e^x(\ln x + \sum_{k=1}^{\infty} x^k/(k \cdot k!)$
    (d) $c_1e^{-x} + c_2e^{-x}(\ln x + \sum_{k=1}^{\infty} x^k/(k \cdot k!)$
    (e) $c_1\sum_{k=0}^{\infty}(-1)^k x^k/(k \cdot k!) + c_2e^x(\ln x + \sum_{k=1}^{\infty}(-1)^k x^k/(k \cdot k!)$

15. The point $x = 0$ is a critical point of the differential equation $x^2y'' + xy' + (x^2 - 1/9)y = 0$. What is the indicial equation?

16. For the previous problem, find the recurrence relation for the series solution about $x = 0$.

17. Find the series solution of $x^2y'' + xy' + (x^2 - 1/9)y = 0$.

18. What is the name of the differential equation in the previous problem?

19. Write the solution of the previous four problems in terms of the special function notation.

20. Use the recurrence relation $(k+1)P_{k+1}(x) - (2k+1)xP_k(x) + kP_{k-1}(x) = 0$ and $P_0(x) = 1$, $P_1(x) = x$ to generate the next two Legendre polynomials.

ANSWER KEY
Zill Differential Equations 9e Chapter 6 Form G

1. b
2. b
3. $\sum_{k=0}^{\infty}(-1)^k x^{2k+1}/(2k+1)!$
4. all $x \le 0$
5. a
6. c
7. c
8. d
9. $x = 0$ is an irregular singular point
10. $x = 2n\pi$, n is an integer, are all regular singular points
11. $x = 0$ is an irregular singular point, $x = 1$ is a regular singular point
12. e
13. c
14. a
15. $r^2 - 1/9 = 0$
16. $c_k((k+r)^2 - 1/9) + c_{k-2} = 0$
17. $c_0 x^{1/3}[1+\sum_{k=1}^{\infty}(-3)^k x^{2k}/(2^k k! 8\cdot 14\cdots(6k+2)]+c_1 x^{-1/3}[1+\sum_{k=1}^{\infty}(-3)^k x^{2k}/(2^k k! 4\cdot 10\cdots(6k-2))]$
18. Bessel's equation of order 1/3
19. $c_0 J_{1/3}(x) + c_1 J_{-1/3}(x)$
20. $P_2(x) = (3x^2-1)/2$, $P_3(x) = (5x^3 - 3x)/2$

1. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} x^n$.

2. Find the interval of convergence of the power series in the previous problem.

3. The first four nonzero terms in the power series expansion of the function $f(x) = e^{-x}$ about $x = 0$ are

   Select the correct answer.

   (a) $1 - x + x^2/2 - x^3/3$
   (b) $1 - x + x^2/2 - x^3/6$
   (c) $1 + x + x^2/2 + x^3/6$
   (d) $1 + x + x^2/2 + x^3/3$
   (e) $1 - x^2/2 + x^4/24 - x^6/720$

4. The singular points of the differential equation $(1 - e^x)y'' + y'/(x - 5) + y(x - 4)/(x - 7) = 0$ are

   Select the correct answer.

   (a) none
   (b) 0
   (c) 0, 4, 5
   (d) 0, 4, 7
   (e) 0, 5, 7

5. Find a power series solution about $x = 0$ of the differential equation $y'' + y = 0$.

6. Find a power series solution about $x = 0$ of the differential equation $y'' + 4y' = 0$.

7. Find the recurrence relation for the terms in the power series solution about $x = 0$ of the differential equation $y'' - xy = 0$.

8. Find the first four nonzero terms of a power series solution about $x = 0$ of the differential equation $y'' - xy = 0$.

9. For the equation $(x^2 - 1)^3(x + 2)y'' - 2xy' + y = 0$, the point $x = 1$ is

   Select the correct answer.

   (a) an ordinary point
   (b) a regular singular point
   (c) an irregular singular point
   (d) a special point
   (e) none of the above

10. For the differential equation $(x^2-1)^3(x+2)y''-2xy'+y=0$, the point $x=2$ is

    Select the correct answer.

    (a) an ordinary point
    (b) a regular singular point
    (c) an irregular singular point
    (d) a special point
    (e) none of the above

11. For the differential equation $(x-1)^3(x+2)^2y''-2xy'+y=0$, the point $x=-2$ is

    Select the correct answer.

    (a) an ordinary point
    (b) a regular singular point
    (c) an irregular singular point
    (d) a special point
    (e) none of the above

12. The point $x=0$ is a critical point of the differential equation $x^2y''+xy'+(x^2-1/4)y=0$. What is the indicial equation?

13. For the previous problem, find the recurrence relation for the series solution about $x=0$.

14. Find the series solution of $x^2y''+xy'+(x^2-1/4)y=0$.

15. What is the name of the differential equation in the previous problem?

16. The differential equation $x^2y''+xy'+(x^2-1/36)y=0$ is

    Select the correct answer.

    (a) Bessel's equation of order n
    (b) Bessel's equation of order $1/36$
    (c) Bessel's equation of order $1/6$
    (d) Legendre's equation of order $1/36$
    (e) Legendre's equation of order $1/6$

17. The solution of the previous problem is

    Select the correct answer.

    (a) $y = c_1 P_{1/6}(x) + c_2 P_{-1/6}(x)$
    (b) $y = c_1 P_6(x) + c_2 P_{-6}(x)$
    (c) $y = c_1 J_6(x) + c_2 Y_6(x)$
    (d) $y = c_1 J_{1/6}(x) + c_2 J_{-1/6}(x)$
    (e) $y = c_1 J_{1/36}(x) + c_2 J_{-1/36}(x)$

18. Find $P_4(x)$, using the recurrence relation $(k+1)P_{k+1} - (2k+1)xP_k(x) + kP_{k-1}(x) = 0$ and $P_0(x) = 1$, $P_1(x) = x$.

19. The differential equation $(1 - x^2)y'' - 2xy' + 30y = 0$ is

    Select the correct answer.

    (a) Bessel's equation of order 30
    (b) Bessel's equation of order 5
    (c) Legendre's equation of order 5
    (d) Legendre's equation of order 6
    (e) Legendre's equation of order 30

20. The solution of the previous problem is

    Select the correct answer.

    (a) $y = c_1 J_5(x) + c_2 Y_5(x)$
    (b) $y = c_1 J_{1/5}(x) + c_2 Y_{1/5}(x)$
    (c) $y = c_1 J_{30}(x) + c_2 Y_{30}(x)$
    (d) $y = c_1 P_{30}(x) + c_2 P_{-30}(x)$
    (e) $y = c_1 P_5(x) + c_2 Q_5(x)$, where $Q_5(x)$ is given by an infinite series

ANSWER KEY
Zill Differential Equations 9e Chapter 6 Form H

1. $R = 1$
2. $(-1, 1)$
3. b
4. e
5. $c_0 \sum_{k=0}^{\infty} (-1)^k x^{2k}/(2k)! + c_1 \sum_{k=0}^{\infty} (-1)^k x^{2k+1}/(2k+1)!$
6. $c_0 + c_1 \sum_{k=1}^{\infty} (-4x)^k/k!$
7. $c_{k+3}(k+3)(k+2) - c_k = 0$
8. $1 + x^3/6 + x^6/180 + x^9/12960$ 0r $x + x^4/12 + x^7/504 + x^{10}/45360$
9. c
10. a
11. b
12. $r^2 - 1/4 = 0$
13. $c_{k+2}((k+r+2)^2 - 1/4) + c_k = 0$
14. $c_0 x^{-1/2} \sum_{k=0}^{\infty} (-1)^k x^{2k}/(2k)! + c_1 x^{-1/2} \sum_{k=0}^{\infty} (-1)^k x^{2k+1}/(2k+1)!$
15. Bessel's equation of order 1/2
16. c
17. d
18. $P_4(x) = (35x^4 - 30x^2 + 3)/8$
19. c
20. e