1. Explain the term round-off error.

2. What is the round-off error in representing the number 1/3 in a three digit, base ten calculator?

3. Use Euler's method to find an approximation of $y(0.1)$ for the solution of $y' = y^2 + 1$, $y(0) = 0$ with a step size of $h = 0.1$.

4. In the previous problem, what is the exact value of the error?

5. In the previous two problems, explain how the global error would change if you decrease h to 0.05.

6. Write down the improved Euler's method to solve $y' = f(x, y)$, $y(x_0) = y_0$.

7. Use the improved Euler's method to find an approximation of $y(0.1)$ for the solution of $y' = y^2 + 1$, $y(0) = 0$ with a step size of $h = 0.1$.

8. In the previous problem, what is the exact value of the error?

9. In the previous two problems, explain how the global error would change if you decrease h to 0.05.

10. Write down a second order Runge–Kutta method for the solution of $y' = f(x, y)$, $y(x_0) = y_0$.

11. Use the method from the previous problem to solve for $y(0.2)$ in the problem $y' = y^2 + 1$, $y(0) = 0$ with a step size of $h = 0.2$.

12. What is the order of the local error you expect in the previous problem?

13. Write down the most popular fourth order Runge–Kutta method for the solution of $y' = f(x, y)$, $y(x_0) = y_0$.

14. Use the method from the previous problem to solve for $y(0.2)$ in the problem $y' = y^2 + 1$, $y(0) = 0$ with a step size of $h = 0.2$.

15. What is the order of the local error you expect in the previous problem?

16. Write down the Adams–Bashforth formula for y^*_{n+1}, the solution of $y' = f(x, y)$, $y(x_0) = y_0$ at x_{n+1}.

17. Use the value of y^*_{n+1} from the previous problem to write down the Adams–Moulton corrector value for the solution of the same problem.

18. Use the Adams–Bashforth–Moulton method of the previous two problems to find an approximation of the solution of $y' = y + 1$, $y(0) = 1$ at $x = 0.4$, using $h = 0.1$, given $y(0) = 1$, $y(0.1) = 1.21034$, $y(0.2) = 1.44281$, $y(0.3) = 1.69972$.

19. Rewrite the problem $y'' + xy' + y = 0$, $y(0) = 2$, $y'(0) = 1$ as a system of two first order initial value problems.

20. Use Euler's method to solve for $y(0.2)$ in the problem $y'' + xy' + y = 0$, $y(0) = 2$, $y'(0) = 1$, using a step size of $h = 0.1$.

ANSWER KEY

Zill Differential Equations 9e Chapter 9 Form A

1. Round-off error is the error introduced by a calculating machine, due to the finite capacity of the machine, when a number cannot be represented exactly in the machine.

2. 1/3000

3. $y_1 = 0.1$

4. error $= \tan(0.1) - 0.1 \approx 0.000335$

5. The error would decrease by roughly a factor of 1/2.

6. $y_{n+1} = y_n + h(f(x_n, y_n) + f(x_{n+1}, y^*_{n+1}))/2$ where y^*_{n+1} is calculated from Euler's method.

7. $y_1 = 0.1005$

8. error $= \tan(0.1) - 0.1005 \approx -0.000165$

9. The error would decrease by roughly a factor of 1/4.

10. $y_{n+1} = y_n + h(k_1 + k_2)/2$, $y_0 = y(x_0)$, $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h, y_n + hk_1)$

11. $y_1 = .204$

12. error $= O(h^3)$

13. $y_{n+1} = y_n + h(k_1 + 2k_2 + 2k_3 + k_4)/6$, $y_0 = y(x_0)$, $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/2, y_n + hk_1/2)$, $k_3 = f(x_n + h/2, y_n + hk_2/2)$, $k_4 = f(x_n + h, y_n + hk_3)$

14. $y_1 = .2027$

15. error$= O(h^5)$

16. $y^*_{n+1} = y_n + h(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3})/24$, $y'_n = f(x_n, y_n)$, $y'_{n-1} = f(x_{n-1}, y_{n-1})$, $y'_{n-2} = f(x_{n-2}, y_{n-2})$, $y'_{n-3} = f(x_{n-3}, y_{n-3})$

17. $y_{n+1} = y_n + h(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2})/24$, $y'_{n+1} = f(x_{n+1}, y^*_{n+1})$

18. $y^*_4 = 1.98364$, $y_4 = 1.98365$

19. $y' = u$, $u' = -y - xu$, $y(0) = 2$, $u(0) = 1$

20. $y_2 = 2.18$

1. Explain the term truncation error.
2. What is the local truncation error in using Euler's method to solve the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$?
3. Write down Euler's method to solve $y' = f(x, y)$, $y(x_0) = y_0$.
4. Use Euler's method to find an approximation of $y(0.2)$ for the solution of $y' = y + 1$, $y(0) = 1$ with a step size of $h = 0.1$.
5. In the previous problem, what is the exact value of the error?
6. In the previous two problems, explain how the error would change if you decrease h to 0.05.
7. Use the improved Euler's method to find an approximation of $y(0.2)$ for the solution of $y' = y + 1$, $y(0) = 1$ with a step size of $h = 0.1$.
8. In the previous problem, what is the exact value of the error?
9. In the previous two problems, explain how the error would change if you decrease h to 0.05.
10. Write down a second order Runge–Kutta method for the solution of $y' = f(x, y)$, $y(x_0) = y_0$.
11. Use the method from the previous problem to solve for $y(0.2)$ in the problem $y' = y+1$, $y(0) = 1$ with a step size of $h = 0.2$.
12. What is the order of the local error you expect in the previous problem?
13. Write down the most popular fourth order Runge–Kutta method for the solution of $y' = f(x, y)$, $y(x_0) = y_0$.
14. Use the method from the previous problem to solve for $y(0.2)$ in the problem $y' = y+1$, $y(0) = 1$ with a step size of $h = 0.2$.
15. What is the order of the local error you expect in the previous problem?
16. Write down the Adams–Bashforth formula for the solution of $y' = f(x, y)$, $y(x_0) = y_0$ for the solution y^*_{n+1}.
17. Use the value of y^*_{n+1} from the previous problem to write down the Adams–Moulton corrector value for the solution of the same problem.
18. Use the Adams–Bashforth–Moulton method of the previous two problems to find an approximation of the solution of $y' = y - 1$, $y(0) = 2$ at $x = 0.4$, using $h = 0.1$, given $y(0) = 2$, $y(0.1) = 2.1052$, $y(0.2) = 2.2214$, $y(0.3) = 2.3499$.
19. Rewrite the problem $y'' + 2xy' + 3y = 0$, $y(0) = 0$, $y'(0) = 1$ as a system of two first order initial value problems.
20. Use Euler's method to solve for $y(0.2)$ in the problem $y'' + 2xy' + 3y = 0$, $y(0) = 2$, $y'(0) = 1$, using a step size of $h = 0.1$.

ANSWER KEY
Zill Differential Equations 9e Chapter 9 Form B

1. Truncation error is the formula error in using only a finite number of terms of an infinite expansion (for example, of a Taylor's series).
2. $y''(c)h^2/2$ where $x_n < c < x_{n+1}$
3. $y_{n+1} = y_n + hf(x_n, y_n)$, $y_0 = y(x_0)$
4. $y_1 = 1.2$, $y_2 = 1.42$
5. error$= 2e^{0.2} - 1 - 1.42 \approx 0.0228$
6. The local error decreases by roughly a factor of 1/4, the global error decreases by roughly a factor of 1/2.
7. $y_1 = 1.21$, $y_2 = 1.44205$
8. error$= 2e^{0.2} - 1 - 1.44205 \approx 0.00076$
9. the local error decreases by roughly a factor of 1/8, the global error decreases by roughly a factor of 1/4
10. $y_{n+1} = y_n + h(k_1 + k_2)/2$, $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h, y_n + hk_1)$
11. $y_1 = 1.44$
12. error$= O(h^3)$
13. $y_{n+1} = y_n + h(k_1 + 2k_2 + 2k_3 + k_4)/6$, $y_0 = y(x_0)$, $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/2, y_n + hk_1/2)$, $k_3 = f(x_n + h/2, y_n + hk_2/2)$, $k_4 = f(x_n + h, y_n + hk_3)$
14. $y_1 = 1.4428$
15. error$= O(h^5)$
16. $y^*_{n+1} = y_n + h(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3})/24$, $y'_n = f(x_n, y_n)$, $y'_{n-1} = f(x_{n-1}, y_{n-1})$, $y'_{n-2} = f(x_{n-2}, y_{n-2})$, $y'_{n-3} = f(x_{n-3}, y_{n-3})$
17. $y_{n+1} = y_n + h(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2})/24$, $y'_{n+1} = f(x_{n+1}, y^*_{n+1})$
18. $y^*_4 = y_4 = 2.4919$
19. $y' = u$, $u' = -3y - 2xu$, $y(0) = 0$, $u(0) = 1$
20. $y_2 = 0.2$

1. When entering the number 1/3 into a three digit base ten calculator, the actual value entered is

   Select the correct answer.

   (a) 1/3
   (b) .333
   (c) .334
   (d) .300
   (e) .330

2. When entering the number 1/3 into a three digit base ten calculator, the round-off error is

   Select the correct answer.

   (a) 1/30
   (b) 1/300
   (c) 1/3000
   (d) 0.003
   (e) 0.0003

3. Euler's formula for solving $y' = f(x, y)$, $y(\overline{x}) = \overline{y}$ is

   Select the correct answer.

   (a) $y_{n+1} = y_n - f(x_n, y_n)$, $y_0 = \overline{y}$, $n = 0, 1, 2, \ldots$
   (b) $y_{n+1} = y_n - hf(x_n, y_n)$, $y_0 = \overline{y}$, $n = 0, 1, 2, \ldots$
   (c) $y_{n+1} = y_n + hf(x_n, y_n)$, $y_0 = \overline{y}$, $n = 0, 1, 2, \ldots$
   (d) $y_{n+1} = y_n + h(f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})/2$, $y_0 = \overline{y}$, $n = 0, 1, 2, \ldots$ where y^*_{n+1} is predicted from Euler's formula
   (e) $y_{n+1} = y_n + (f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})/2$, $y_0 = \overline{y}$, $n = 0, 1, 2, \ldots$ where y^*_{n+1} is predicted from Euler's formula

4. The solution of $y' = y$, $y(0) = 1$ for $y(0.2)$, using Euler's method with $h = 0.1$, is

   Select the correct answer.

   (a) 1.01
   (b) 1.1
   (c) 1.11
   (d) 1.21
   (e) 1.22

5. In the previous problem, the local truncation error in y_{n+1} is

   Select the correct answer.

   (a) $0.005e^c$, where $x_{n-1} < c < x_n$
   (b) $0.05e^c$, where $x_{n-1} < c < x_n$
   (c) $0.005e^c$, where $x_n < c < x_{n+1}$
   (d) $0.05e^c$, where $x_n < c < x_{n+1}$
   (e) unknown

6. The solution of $y' = y$, $y(0) = 1$ for $y(0.2)$, using the improved Euler's method with $h = 0.1$, is

   Select the correct answer.

   (a) 1.22125
   (b) 1.210625
   (c) 1.226525
   (d) 1.21525
   (e) 1.221025

7. The local truncation error for the improved Euler's method is

   Select the correct answer.

   (a) unknown
   (b) $O(h)$
   (c) $O(h^2)$
   (d) $O(h^3)$
   (e) $O(h^4)$

8. Euler's method is what type of Runge–Kutta method?

   Select the correct answer.

   (a) first order
   (b) second order
   (c) third order
   (d) fourth order
   (e) It is not a Runge–Kutta method

9. A popular second order Runge–Kutta method for the solution of $y' = f(x, y)$, $y(x_0) = y_0$ is

   Select the correct answer.

   (a) $y_{n+1} = y_n + h(k_1 + k_2)/2$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/2, y_n + hk_1/2)$
   (b) $y_{n+1} = y_n + h(k_1 + k_2)/2$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h, y_n + hk_1/2)$
   (c) $y_{n+1} = y_n + h(k_1 + k_2)/2$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/2, y_n + hk_1)$
   (d) $y_{n+1} = y_n + h(k_1 + k_2)/2$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h, y_n + hk_1)$
   (e) none of the above

10. Using the method from the previous problem, the solution of $y' = y$, $y(0) = 1$ for $y(0.2)$ with $h = 0.2$ is

    Select the correct answer.

    (a) 1.2
    (b) 1.21
    (c) 1.214
    (d) 1.22
    (e) 1.24

11. The most popular fourth order Runge–Kutta method for the solution of $y' = f(x, y)$, $y(x_0) = y_0$ is

    Select the correct answer.

    (a) $y_{n+1} = y_n + h(k_1 + 2k_2 + 2k_3 + k_4)/6$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/2, y_n + hk_1/2)$, $k_3 = f(x_n + h/2, y_n + hk_2/2)$, $k_4 = f(x_n + h, y_n + hk_3)$
    (b) $y_{n+1} = y_n + h(2k_1 + k_2 + k_3 + 2k_4)/6$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/2, y_n + hk_1)$, $k_3 = f(x_n + h/2, y_n + hk_2)$, $k_4 = f(x_n + h, y_n + hk_3)$
    (c) $y_{n+1} = y_n + h(k_1 + 2k_2 + 2k_3 + k_4)/6$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/3, y_n + hk_1/2)$, $k_3 = f(x_n + 2h/3, y_n + hk_2/2)$, $k_4 = f(x_n + h, y_n + hk_3)$
    (d) $y_{n+1} = y_n + h(2k_1 + k_2 + k_3 + 2k_4)/6$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/2, y_n + hk_1/2)$, $k_3 = f(x_n + h/2, y_n + hk_2/2)$, $k_4 = f(x_n + h, y_n + hk_3)$
    (e) $y_{n+1} = y_n + h(k_1 + 2k_2 + 2k_3 + k_4)/6$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/2, y_n + hk_1)$, $k_3 = f(x_n + h/2, y_n + hk_2)$, $k_4 = f(x_n + h, y_n + hk_3)$

12. Using the method from the previous problem, the solution of $y' = y$, $y(0) = 1$ for $y(0.2)$ with $h = 0.2$ is

    Select the correct answer.

    (a) 1.24
    (b) 1.241
    (c) 1.214
    (d) 1.2214
    (e) 1.224

13. The Adams–Bashforth formula for finding the solution of $y' = f(x,y)$, $y(x_0) = y_0$ is

Select the correct answer.

(a) $y^*_{n+1} = y_n + h(55y'_n + 59y'_{n-1} - 37y'_{n-2} - 9y'_{n-3})/24$, where $y'_n = f(x_n, y_n)$, $y'_{n-1} = f(x_{n-1}, y_{n-1})$, $y'_{n-2} = f(x_{n-2}, y_{n-2})$, $y'_{n-3} = f(x_{n-3}, y_{n-3})$

(b) $y^*_{n+1} = y_n + h(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3})/24$, where $y'_n = f(x_n, y_n)$, $y'_{n-1} = f(x_{n-1}, y_{n-1})$, $y'_{n-2} = f(x_{n-2}, y_{n-2})$, $y'_{n-3} = f(x_{n-3}, y_{n-3})$

(c) $y^*_{n+1} = y_n + h(55y'_n - 59y'_{n-1} - 37y'_{n-2} + 65y'_{n-3})/24$, where $y'_n = f(x_n, y_n)$, $y'_{n-1} = f(x_{n-1}, y_{n-1})$, $y'_{n-2} = f(x_{n-2}, y_{n-2})$, $y'_{n-3} = f(x_{n-3}, y_{n-3})$

(d) $y^*_{n+1} = y_n + h(59y'_n - 55y'_{n-1} + 37y'_{n-2} - 17y'_{n-3})/24$, where $y'_n = f(x_n, y_n)$, $y'_{n-1} = f(x_{n-1}, y_{n-1})$, $y'_{n-2} = f(x_{n-2}, y_{n-2})$, $y'_{n-3} = f(x_{n-3}, y_{n-3})$

(e) none of the above

14. Using the value of y^*_{n+1} from the previous problem, the Adams–Moulton corrector value for the solution of $y' = f(x,y)$, $y(x_0) = y_0$ is

Select the correct answer.

(a) $y_{n+1} = y_n + h(9y'_{n+1} - 19y'_n + 5y'_{n-1} + y'_{n-2})/24$, where $y'_{n+1} = f(x_{n+1}, y^*_{n+1})$

(b) $y_{n+1} = y_n + h(9y'_{n+1} + 19y'_n + 5y'_{n-1} + y'_{n-2})/34$, where $y'_{n+1} = f(x_{n+1}, y^*_{n+1})$

(c) $y_{n+1} = y_n + h(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2})/24$, where $y'_{n+1} = f(x_{n+1}, y^*_{n+1})$

(d) $y_{n+1} = y_n + h(9y'_{n+1} + 19y'_n - 5y'_{n-1} - y'_{n-2})/24$, where $y'_{n+1} = f(x_{n+1}, y^*_{n+1})$

(e) none of the above

15. Using the method from the previous two problems, using the values $y_0 = 1$, $y_1 = 1.1052$, $y_2 = 1.2214$, $y_3 = 1.3499$, the solution of $y' = y$, $y(0) = 1$ for $y(0.4)$ with $h = 0.1$ is

Select the correct answer.

(a) 1.4919

(b) 1.4967

(c) 1.4978

(d) 1.5003

(e) none of the above

16. The Euler formula for solving the system $y' = u$, $u' = f(x,y,u)$, $y(x_0) = y_0$, $u(x_0) = u_0$ is

Select the correct answer.

(a) $y_{n+1} = y_n + hu_n$, $u_{n+1} = u_n + hf(x_n, y_n, u_n)$

(b) $y_{n+1} = y_n - hu_n$, $u_{n+1} = u_n + hf(x_n, y_n, u_n)$

(c) $y_{n+1} = y_n + hu_n$, $u_{n+1} = u_n - hf(x_n, y_n, u_n)$

(d) $y_{n+1} = y_n + hf(x_n, y_n, u_n)$, $u_{n+1} = u_n + hu_n$

(e) none of the above

17. The problem $y'' + xyy' = 0$, $y(0) = 0$, $y'(0) = 1$ can be written as a system of two equations as follows.

    Select the correct answer.

    (a) $y' = u$, $u' = xyu$, $y(0) = 0$, $u(0) = 0$
    (b) $y' = u$, $u' = -xyu$, $y(0) = 1$, $u(0) = 0$
    (c) $y' = u$, $u' = xyu$, $y(0) = 1$, $u(0) = 0$
    (d) $y' = u$, $u' = xyu$, $y(0) = 0$, $u(0) = 1$
    (e) $y' = u$, $u' = -xyu$, $y(0) = 0$, $u(0) = 1$

18. Using Euler's method on the previous problem and using a value of $h = 0.1$, the solution for $y(0.2)$ is

    Select the correct answer.

    (a) 0.11
    (b) 0.2
    (c) 0.21
    (d) 0.22
    (e) 0.221

19. The fourth order Runge–Kutta method for solving $y'' = f(x, y, y')$, $y(x_0) = y_0$, $y'(x_0) = u_0$ is

    Select the correct answer.

    (a) $y_{n+1} = y_n + h(m_1 - 2m_2 + 2m_3 - m_4)/6$, $u_{n+1} = u_n + h(k_1 - 2k_2 + 2k_3 - k_4)/6$ where $m_1 = u_n$, $k_1 = f(x_n, y_n, u_n)$, $m_2 = u_n + hk_1/2$, $k_2 = f(x_n + h/2, y_n + hm_1/2, u_n + hk_1/2)$,
    $m_3 = u_n + hk_2/2$, $k_3 = f(x_n + h/2, y_n + hm_2/2, u_n + hk_2/2)$, $m_4 = u_n + hk_3$, $k_4 = f(x_n + h, y_n + hm_3, u_n + hk_3)$

    (b) $y_{n+1} = y_n - h(m_1 + 2m_2 + 2m_3 + m_4)/6$, $u_{n+1} = u_n - h(k_1 + 2k_2 + 2k_3 + k_4)/6$ where $m_1 = u_n$, $k_1 = f(x_n, y_n, u_n)$, $m_2 = u_n + hk_1/2, k_2 = f(x_n + h/2, y_n + hm_1/2, u_n + hk_1/2)$,
    $m_3 = u_n + hk_2/2$, $k_3 = f(x_n + h/2, y_n + hm_2/2, u_n + hk_2/2)$, $m_4 = u_n + hk_3$, $k_4 = f(x_n + h, y_n + hm_3, u_n + hk_3)$

    (c) $y_{n+1} = y_n + h(m_1 + 2m_2 + 2m_3 + m_4)/6$, $u_{n+1} = u_n - h(k_1 + 2k_2 + 2k_3 + k_4)/6$ where $m_1 = u_n$, $k_1 = f(x_n, y_n, u_n)$, $m_2 = u_n + hk_1/2$, $k_2 = f(x_n + h/2, y_n + hm_1/2, u_n + hk_1/2)$,
    $m_3 = u_n + hk_2/2$, $k_3 = f(x_n + h/2, y_n + hm_2/2, u_n + hk_2/2)$, $m_4 = u_n + hk_3$, $k_4 = f(x_n + h, y_n + hm_3, u_n + hk_3)$

    (d) $y_{n+1} = y_n - h(m_1 + 2m_2 + 2m_3 + m_4)/6$, $u_{n+1} = u_n + h(k_1 + 2k_2 + 2k_3 + k_4)/6$ where $m_1 = u_n$, $k_1 = f(x_n, y_n, u_n)$, $m_2 = u_n + hk_1/2$, $k_2 = f(x_n + h/2, y_n + hm_1/2, u_n + hk_1/2)$,
    $m_3 = u_n + hk_2/2$, $k_3 = f(x_n + h/2, y_n + hm_2/2, u_n + hk_2/2)$, $m_4 = u_n + hk_3$, $k_4 = f(x_n + h, y_n + hm_3, u_n + hk_3)$

    (e) $y_{n+1} = y_n + h(m_1 + 2m_2 + 2m_3 + m_4)/6$, $u_{n+1} = u_n + h(k_1 + 2k_2 + 2k_3 + k_4)/6$ where $m_1 = u_n$, $k_1 = f(x_n, y_n, u_n)$, $m_2 = u_n + hk_1/2$, $k_2 = f(x_n + h/2, y_n + hm_1/2, u_n + hk_1/2)$,
    $m_3 = u_n + hk_2/2$, $k_3 = f(x_n + h/2, y_n + hm_2/2, u_n + hk_2/2)$, $m_4 = u_n + hk_3$, $k_4 = f(x_n + h, y_n + hm_3, u_n + hk_3)$

20. The solution of $y'' + xyy' = 0$, $y(0) = 0$, $y'(0) = 1$ for $y(0.1)$, using the Runge–Kutta method of order four, and using $h = 0.1$, is

    Select the correct answer.

    (a) 0.0909
    (b) 0.09999
    (c) 0.09099
    (d) 0.09899
    (e) 0.08899

ANSWER KEY

Zill Differential Equations 9e Chapter 9 Form C

1. b
2. c
3. c
4. d
5. c
6. e
7. d
8. a
9. d
10. d
11. a
12. d
13. b
14. c
15. a
16. a
17. e
18. b
19. e
20. b

1. When entering the number 1/7 into a three digit base ten calculator, the actual value entered is

   Select the correct answer.

   (a) 1/7
   (b) .143
   (c) .142
   (d) .140
   (e) .150

2. When entering the number 1/7 into a three digit base ten calculator, the round-off error is

   Select the correct answer.

   (a) 0.00143
   (b) 0.000143
   (c) 1/70
   (d) 1/700
   (e) 1/7000

3. The solution of $y' = x + y$, $y(0) = 1$ for $y(0.2)$, using Euler's method with $h = 0.1$, is

   Select the correct answer.

   (a) 1.01
   (b) 1.11
   (c) 1.21
   (d) 1.22
   (e) 1.23

4. In the previous problem, the local truncation error in y_{n+1} is

   Select the correct answer.

   (a) $0.005y''(c)$, where $x_n < c < x_{n+1}$
   (b) $0.05y''(c)$, where $x_n < c < x_{n+1}$
   (c) $0.005y''(c)$, where $x_{n-1} < c < x_n$
   (d) $0.05y''(c)$, where $x_{n-1} < c < x_n$
   (e) unknown

5. The improved Euler's formula for solving $y' = f(x, y)$, $y(\overline{x}) = \overline{y}$ is

   Select the correct answer.

   (a) $y_{n+1} = y_n - f(x_n, y_n)$, $y_0 = \overline{y}$, $n = 0, 1, 2, \ldots$
   (b) $y_{n+1} = y_n - hf(x_n, y_n)$, $y_0 = \overline{y}$, $n = 0, 1, 2, \ldots$
   (c) $y_{n+1} = y_n + hf(x_n, y_n)$, $y_0 = \overline{y}$, $n = 0, 1, 2, \ldots$
   (d) $y_{n+1} = y_n + (f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})/2$, $y_0 = \overline{y}$, $n = 0, 1, 2, \ldots$ where y^*_{n+1} is predicted from Euler's formula
   (e) $y_{n+1} = y_n + h(f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})/2$, $y_0 = \overline{y}$, $n = 0, 1, 2, \ldots$ where y^*_{n+1} is predicted from Euler's formula

6. The solution of $y' = x + y$, $y(0) = 1$ for $y(0.2)$, using the improved Euler's method with $h = 0.1$, is

   Select the correct answer.

   (a) 1.2055
   (b) 1.21625
   (c) 1.24205
   (d) 1.226525
   (e) 1.235625

7. The local truncation error for the improved Euler's method is

   Select the correct answer.

   (a) $O(h)$
   (b) $O(h^2)$
   (c) $O(h^3)$
   (d) $O(h^4)$
   (e) unknown

8. The improved Euler's method is what type of Runge–Kutta method?

   Select the correct answer.

   (a) first order
   (b) second order
   (c) third order
   (d) fourth order
   (e) It is not a Runge–Kutta method

9. Which of the following are second order Runge–Kutta methods for the solution of $y' = f(x, y)$, $y(x_0) = y_0$?

   Select all that apply.

   (a) $y_{n+1} = y_n + h(k_1 + k_2)/2$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h, y_n + hk_1)$
   (b) $y_{n+1} = y_n + h(k_1/3 + 2k_2/3)$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + 3h/4, y_n + 3hk_1/4)$
   (c) $y_{n+1} = y_n + h(2k_1/3 + k_2/3)$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + 3h/2, y_n + 3hk_1/2)$
   (d) $y_{n+1} = y_n + h(k_1 + k_2)/2$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + 2h, y_n + 2hk_1)$
   (e) $y_{n+1} = y_n + h(3k_1 + k_2)/4$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + 2h, y_n + 2hk_1)$

10. Using the method from part a of the previous problem, the solution of $y' = x + y$, $y(0) = 1$ for $y(0.2)$ with $h = 0.2$ is

    Select the correct answer.

    (a) 1.222
    (b) 1.22
    (c) 1.2213
    (d) 1.24
    (e) 1.21

11. The most popular fourth order Runge–Kutta method for the solution of $y' = f(x, y)$, $y(x_0) = y_0$ is

    Select the correct answer.

    (a) $y_{n+1} = y_n + h(2k_1 + k_2 + k_3 + 2k_4)/6$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/2, y_n + hk_1/2)$, $k_3 = f(x_n + h/2, y_n + hk_2/2)$, $k_4 = f(x_n + h, y_n + hk_3)$
    (b) $y_{n+1} = y_n + h(k_1 + 2k_2 + 2k_3 + k_4)/6$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/2, y_n + hk_1)$, $k_3 = f(x_n + h/2, y_n + hk_2)$, $k_4 = f(x_n + h, y_n + hk_3)$
    (c) $y_{n+1} = y_n + h(k_1 + 2k_2 + 2k_3 + k_4)/6$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/2, y_n + hk_1/2)$, $k_3 = f(x_n + h/2, y_n + hk_2/2)$, $k_4 = f(x_n + h, y_n + hk_3)$
    (d) $y_{n+1} = y_n + h(2k_1 + k_2 + k_3 + 2k_4)/6$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/2, y_n + hk_1)$, $k_3 = f(x_n + h/2, y_n + hk_2)$, $k_4 = f(x_n + h, y_n + hk_3)$
    (e) $y_{n+1} = y_n + h(k_1 + 2k_2 + 2k_3 + k_4)/6$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/3, y_n + hk_1/2)$, $k_3 = f(x_n + 2h/3, y_n + hk_2/2)$, $k_4 = f(x_n + h, y_n + hk_3)$

12. Using the method from the previous problem, the solution of $y' = x + y$, $y(0) = 1$ for $y(0.2)$ with $h = 0.2$ is

    Select the correct answer.

    (a) 1.241
    (b) 1.242
    (c) 1.2422
    (d) 1.2426
    (e) 1.2428

13. The Adams–Bashforth formula for finding the solution of $y' = f(x, y)$, $y(x_0) = y_0$ is

    Select the correct answer.

    (a) $y^*_{n+1} = y_n + h(55y'_n - 59y'_{n-1} - 37y'_{n-2} + 65y'_{n-3})/24$, where $y'_n = f(x_n, y_n)$, $y'_{n-1} = f(x_{n-1}, y_{n-1})$, $y'_{n-2} = f(x_{n-2}, y_{n-2})$, $y'_{n-3} = f(x_{n-3}, y_{n-3})$
    (b) $y^*_{n+1} = y_n + h(59y'_n - 55y'_{n-1} + 37y'_{n-2} - 17y'_{n-3})/24$, where $y'_n = f(x_n, y_n)$, $y'_{n-1} = f(x_{n-1}, y_{n-1})$, $y'_{n-2} = f(x_{n-2}, y_{n-2})$, $y'_{n-3} = f(x_{n-3}, y_{n-3})$
    (c) $y^*_{n+1} = y_n + h(55y'_n + 59y'_{n-1} - 37y'_{n-2} - 9y'_{n-3})/24$, where $y'_n = f(x_n, y_n)$, $y'_{n-1} = f(x_{n-1}, y_{n-1})$, $y'_{n-2} = f(x_{n-2}, y_{n-2})$, $y'_{n-3} = f(x_{n-3}, y_{n-3})$
    (d) $y^*_{n+1} = y_n + h(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3})/24$, where $y'_n = f(x_n, y_n)$, $y'_{n-1} = f(x_{n-1}, y_{n-1})$, $y'_{n-2} = f(x_{n-2}, y_{n-2})$, $y'_{n-3} = f(x_{n-3}, y_{n-3})$
    (e) none of the above

14. Using the Adams–Bashforth method from the previous problem, and using the values $y_0 = 1$, $y_1 = 1.1052$, $y_2 = 1.2214$, $y_3 = 1.3499$, the solution of $y' = y$, $y(0) = 1$ for $y^*_{n+1} = y(0.4)$ with $h = 0.1$ is

    Select the correct answer.

    (a) 1.4978
    (b) 1.5003
    (c) 1.4919
    (d) 1.4967
    (e) none of the above

15. Using the value of y^*_{n+1} from the previous two problems, the Adams–Moulton corrector formula for the solution of $y' = f(x, y)$, $y(x_0) = y_0$ is

    Select the correct answer.

    (a) $y_{n+1} = y_n + h(9y'_{n+1} - 19y'_n + 5y'_{n-1} + y'_{n-2})/24$, where $y'_{n+1} = f(x_{n+1}, y^*_{n+1})$
    (b) $y_{n+1} = y_n + h(9y'_{n+1} + 19y'_n + 5y'_{n-1} + y'_{n-2})/34$, where $y'_{n+1} = f(x_{n+1}, y^*_{n+1})$
    (c) $y_{n+1} = y_n + h(9y'_{n+1} + 19y'_n - 5y'_{n-1} - y'_{n-2})/24$, where $y'_{n+1} = f(x_{n+1}, y^*_{n+1})$
    (d) $y_{n+1} = y_n + h(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2})/24$, where $y'_{n+1} = f(x_{n+1}, y^*_{n+1})$
    (e) none of the above

16. Using the Adams–Bashforth–Moulton method from the previous three problems, the solution of $y' = y$, $y(0) = 1$ for $y(0.4)$ with $h = 0.1$ is

    Select the correct answer.

    (a) 1.5003
    (b) 1.4978
    (c) 1.4919
    (d) 1.4967
    (e) none of the above

17. The Euler's method solution for $y(0.2)$ of $y'' + y = 0$, $y(0) = 0$, $y'(0) = 1$, using $h = 0.1$, is

    Select the correct answer.

    (a) 0.14
    (b) 0.2
    (c) 0.21
    (d) 0.11
    (e) 0.12

18. The standard backward difference approximation of $y'(x)$ is

    Select the correct answer.

    (a) $(y(x+h) - y(x))/h$
    (b) $(y(x) - y(x-h))/h$
    (c) $(y(x+h) - y(x))/h^2$
    (d) $y(x+h) - y(x)$
    (e) $y(x) - y(x-h)$

19. The standard central difference approximation of $y''(x)$ is

    Select the correct answer.

    (a) $(y(x+h) - 2y(x) + y(x-h))/h^2$
    (b) $(y(x+h) + 2y(x) + y(x-h))/h^2$
    (c) $(y(x+h) - 2y(x) + y(x-h))/h$
    (d) $(y(x+h) + 2y(x) + y(x-h))/h$
    (e) $(y(x+h) - y(x-h))/h$

20. Using the notation from the text, the finite difference equation for solving the boundary value problem $y'' + P(x)y' + Q(x)y = f(x)$, $y(a) = \alpha$, $y(b) = \beta$ is

    Select the correct answer.

    (a) $(1 - hP_i/2)y_{i+1} + (-2 + h^2Q_i)y_i + (1 + hP_i/2)y_{i-1} = h^2 f_i$
    (b) $(1 - hP_i/2)y_{i+1} + (-2 + h^2Q_i)y_i + (1 - hP_i/2)y_{i-1} = h^2 f_i$
    (c) $(1 + hP_i/2)y_{i+1} + (-2 + h^2Q_i)y_i + (1 + hP_i/2)y_{i-1} = h^2 f_i$
    (d) $(1 + hP_i/2)y_{i+1} + (2 - h^2Q_i)y_i + (1 - hP_i/2)y_{i-1} = h^2 f_i$
    (e) $(1 + hP_i/2)y_{i+1} + (-2 + h^2Q_i)y_i + (1 - hP_i/2)y_{i-1} = h^2 f_i$

ANSWER KEY

Zill Differential Equations 9e Chapter 9 Form D

1. b
2. e
3. d
4. a
5. e
6. c
7. c
8. b
9. a, b, c, e
10. d
11. c
12. e
13. d
14. c
15. d
16. c
17. b
18. b
19. a
20. e

1. Explain the term round-off error.
2. What is the round-off error in representing the number 1/9 in a three digit, base ten calculator?
3. Use Euler's method to find an approximation of $y(0.2)$ for the solution of $y' = y^2 - 1$, $y(0) = 0$ with a step size of $h = 0.1$.
4. In the previous problem, what is the exact value of the error?
5. In the previous two problems, explain how the error would change if you decrease h to 0.05.
6. The solution of $y' = y - x$, $y(0) = 2$ for $y(0.2)$, using the improved Euler's method with $h = 0.2$, is

   Select the correct answer.

   (a) 2.42
   (b) 2.21
   (c) 2.22
   (d) 2.44
   (e) 2.11

7. The local truncation error for the improved Euler's method is

   Select the correct answer.

   (a) unknown
   (b) $O(h)$
   (c) $O(h^2)$
   (d) $O(h^3)$
   (e) $O(h^4)$

8. The improved Euler's method is what type of Runge–Kutta method?

   Select the correct answer.

   (a) It is not a Runge–Kutta method
   (b) first order
   (c) second order
   (d) third order
   (e) fourth order

9. Which of the following are second order Runge–Kutta methods for the solution of $y' = f(x, y)$, $y(x_0) = y_0$?

   Select all that apply.

   (a) $y_{n+1} = y_n + h(k_1 + k_2)/2$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h, y_n + hk_1)$
   (b) $y_{n+1} = y_n + h(k_1/3 + 2k_2/3)$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + 3h/4, y_n + 3hk_1/4)$
   (c) $y_{n+1} = y_n + h(2k_1/3 + k_2/3)$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + 3h/2, y_n + 3hk_1/2)$
   (d) $y_{n+1} = y_n + h(k_1 + k_2)/2$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + 2h, y_n + 2hk_1)$
   (e) $y_{n+1} = y_n + h(3k_1 + k_2)/4$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + 2h, y_n + 2hk_1)$

10. Using the method from response b of the previous problem, the solution of $y' = y - x$, $y(0) = 2$ for $y(0.2)$ with $h = 0.2$ is

    Select the correct answer.

    (a) 2.11
    (b) 2.12
    (c) 2.11
    (d) 2.42
    (e) 2.44

11. Write down the most popular fourth order Runge–Kutta method for the solution of $y' = f(x, y)$, $y(x_0) = y_0$.

12. Use the method from the previous problem to solve for $y(0.2)$ in the problem $y' = y^2 + y$, $y(0) = 1$ with a step size of $h = 0.2$.

13. In the previous problem, what is the expected order of the local error?

14. The Adams–Bashforth formula for finding the solution of $y' = f(x, y)$, $y(x_0) = y_0$ is

    Select the correct answer.

    (a) $y^*_{n+1} = y_n + h(55y'_n - 59y'_{n-1} - 37y'_{n-2} + 65y'_{n-3})/24$, where $y'_n = f(x_n, y_n)$, $y'_{n-1} = f(x_{n-1}, y_{n-1})$, $y'_{n-2} = f(x_{n-2}, y_{n-2})$, $y'_{n-3} = f(x_{n-3}, y_{n-3})$
    (b) $y^*_{n+1} = y_n + h(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3})/24$, where $y'_n = f(x_n, y_n)$, $y'_{n-1} = f(x_{n-1}, y_{n-1})$, $y'_{n-2} = f(x_{n-2}, y_{n-2})$, $y'_{n-3} = f(x_{n-3}, y_{n-3})$
    (c) $y^*_{n+1} = y_n + h(59y'_n - 55y'_{n-1} + 37y'_{n-2} - 17y'_{n-3})/24$, where $y'_n = f(x_n, y_n)$, $y'_{n-1} = f(x_{n-1}, y_{n-1})$, $y'_{n-2} = f(x_{n-2}, y_{n-2})$, $y'_{n-3} = f(x_{n-3}, y_{n-3})$
    (d) $y^*_{n+1} = y_n + h(55y'_n + 59y'_{n-1} - 37y'_{n-2} - 9y'_{n-3})/24$, where $y'_n = f(x_n, y_n)$, $y'_{n-1} = f(x_{n-1}, y_{n-1})$, $y'_{n-2} = f(x_{n-2}, y_{n-2})$, $y'_{n-3} = f(x_{n-3}, y_{n-3})$
    (e) none of the above

15. Using the Adams–Bashforth method from the previous problem with $h = 0.1$, and using the values $y_0 = 1$, $y_1 = 1.1052$, $y_2 = 1.2214$, $y_3 = 1.3499$, the solution of $y' = y$, $y(0) = 1$ for $y^*_{n+1} = y(0.4)$ is

    Select the correct answer.

    (a) 1.4978
    (b) 1.5003
    (c) 1.4919
    (d) 1.4967
    (e) none of the above

16. Using the value of y^*_{n+1} from the previous two problems, the Adams–Moulton corrector formula for the solution of $y' = f(x, y)$, $y(x_0) = y_0$ is

    Select the correct answer.

    (a) $y_{n+1} = y_n + h(9y'_{n+1} - 19y'_n + 5y'_{n-1} + y'_{n-2})/24$, where $y'_{n+1} = f(x_{n+1}, y^*_{n+1})$
    (b) $y_{n+1} = y_n + h(9y'_{n+1} + 19y'_n + 5y'_{n-1} + y'_{n-2})/34$, where $y'_{n+1} = f(x_{n+1}, y^*_{n+1})$
    (c) $y_{n+1} = y_n + h(9y'_{n+1} + 19y'_n - 5y'_{n-1} - y'_{n-2})/24$, where $y'_{n+1} = f(x_{n+1}, y^*_{n+1})$
    (d) $y_{n+1} = y_n + h(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2})/24$, where $y'_{n+1} = f(x_{n+1}, y^*_{n+1})$
    (e) none of the above

17. Using the Adams–Bashforth–Moulton method from the previous three problems, the solution of $y' = y$, $y(0) = 1$ for $y(0.4)$ with $h = 0.1$ is

    Select the correct answer.

    (a) 1.5003
    (b) 1.4978
    (c) 1.4919
    (d) 1.4967
    (e) none of the above

18. Rewrite the problem $y'' + 2xy' + 3y = 0$, $y(0) = 2$, $y'(0) = 1$ as a system of two first order initial value problems.

19. Use Euler's method to solve for $y(0.2)$ in the problem $y'' + 2xy' + 3y = 0$, $y(0) = 2$, $y'(0) = 1$, using a step size of $h = 0.1$.

20. Use a fourth order Runge–Kutta method to solve for $y(0.2)$ in the problem $y'' + 2xy' + 3y = 0$, $y(0) = 2$, $y'(0) = 1$, using a step size of $h = 0.2$.

ANSWER KEY
Zill Differential Equations 9e Chapter 9 Form E

1. Round-off error is the error introduced into a calculating machine, due to the finite capacity of the machine, when a number cannot be represented exactly in the machine.
2. error = 1/9000
3. $y_1 = -0.1$, $y_2 = -0.199$
4. error $= (1 - e^{0.4})/(1 + e^{0.4}) + 0.199 \approx 0.0016$
5. The local error decreases by roughly a factor of 1/4, the global error decreases by roughly a factor of 1/2.
6. a
7. d
8. c
9. a, b, c, e
10. d
11. $y_{n+1} = y_n + h(k_1 + 2k_2 + 2k_3 + k_4)/6$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/2, y_n + hk_1/2)$, $k_3 = f(x_n + h/2, y_n + hk_2/2)$, $k_4 = f(x_n + h, y_n + hk_3)$
12. $y_1 = 1.5683$
13. error$= O(h^5)$
14. b
15. c
16. d
17. c
18. $y' = u$, $u' = -3y - 2xu$, $y(0) = 2$, $u(0) = 1$
19. $y_1 = 2.1$, $y_2 = 2.14$
20. $y_1 = 2.0762$

1. When entering the number 2/3 into a three digit base ten calculator, the actual value entered is

   Select the correct answer.

   (a) 2/3
   (b) .666
   (c) .667
   (d) .67
   (e) .66

2. When entering the number 2/3 into a three digit base ten calculator, the round-off error is

   Select the correct answer.

   (a) 0.003
   (b) 0.0003
   (c) 1/30
   (d) 1/300
   (e) 1/3000

3. Euler's formula for solving $y' = f(x, y)$, $y(\overline{x}) = \overline{y}$ is

   Select the correct answer.

   (a) $y_{n+1} = y_n + h(f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})/2$, $y_0 = \overline{y}$, $n = 0, 1, 2, \ldots$ where y^*_{n+1} is predicted from Euler's formula
   (b) $y_{n+1} = y_n + (f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})/2$, $y_0 = \overline{y}$, $n = 0, 1, 2, \ldots$ where y^*_{n+1} is predicted from Euler's formula
   (c) $y_{n+1} = y_n - f(x_n, y_n)$, $y_0 = \overline{y}$, $n = 0, 1, 2, \ldots$
   (d) $y_{n+1} = y_n + hf(x_n, y_n)$, $y_0 = \overline{y}$, $n = 0, 1, 2, \ldots$
   (e) $y_{n+1} = y_n - hf(x_n, y_n)$, $y_0 = \overline{y}$, $n = 0, 1, 2, \ldots$

4. The solution of $y' = y + 1$, $y(0) = 1$ for $y(0.2)$, using Euler's method with $h = 0.1$, is

   Select the correct answer.

   (a) 1.11
   (b) 1.22
   (c) 1.34
   (d) 1.44
   (e) 1.42

5. In the previous problem, the local truncation error in y_{n+1} is

   Select the correct answer.

   (a) $0.01e^c$, where $x_n < c < x_{n+1}$
   (b) $0.1e^c$, where $x_n < c < x_{n+1}$
   (c) $0.01e^c$, where $x_{n-1} < c < x_n$
   (d) $0.1e^c$, where $x_{n-1} < c < x_n$
   (e) unknown

6. Write down the improved Euler's method to solve $y' = f(x, y)$, $y(x_0) = y_0$.

7. Use the improved Euler's method to find an approximation of $y(0.2)$ for the solution of $y' = y^2 - y$, $y(0) = 2$ with a step size of $h = 0.2$.

8. In the previous problem, what is the exact value of the error?

9. In the previous two problems, explain how the error would change if you decrease h to 0.05.

10. The most popular fourth order Runge–Kutta method for the solution of $y' = f(x, y)$, $y(x_0) = y_0$ is

    Select the correct answer.

    (a) $y_{n+1} = y_n + h(2k_1 + k_2 + k_3 + 2k_4)/6$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/2, y_n + hk_1)$, $k_3 = f(x_n + h/2, y_n + hk_2)$, $k_4 = f(x_n + h, y_n + hk_3)$
    (b) $y_{n+1} = y_n + h(k_1 + 2k_2 + 2k_3 + k_4)/6$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/3, y_n + hk_1/2)$, $k_3 = f(x_n + 2h/3, y_n + hk_2/2)$, $k_4 = f(x_n + h, y_n + hk_3)$
    (c) $y_{n+1} = y_n + h(2k_1 + k_2 + k_3 + 2k_4)/6$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/2, y_n + hk_1/2)$, $k_3 = f(x_n + h/2, y_n + hk_2/2)$, $k_4 = f(x_n + h, y_n + hk_3)$
    (d) $y_{n+1} = y_n + h(k_1 + 2k_2 + 2k_3 + k_4)/6$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/2, y_n + hk_1/2)$, $k_3 = f(x_n + h/2, y_n + hk_2/2)$, $k_4 = f(x_n + h, y_n + hk_3)$
    (e) $y_{n+1} = y_n + h(k_1 + 2k_2 + 2k_3 + k_4)/6$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/2, y_n + hk_1)$, $k_3 = f(x_n + h/2, y_n + hk_2)$, $k_4 = f(x_n + h, y_n + hk_3)$

11. Using the method from the previous problem, the solution of $y' = y + 1$, $y(0) = 0$ for $y(0.2)$ with $h = 0.2$ is

    Select the correct answer.

    (a) .2414
    (b) .241
    (c) .214
    (d) .221
    (e) .2214

12. Write down the Adams–Bashforth formula for y^*_{n+1}, the solution of $y' = f(x, y)$, $y(x_0) = y_0$ at x_{n+1}.

13. Use the value of y^*_{n+1} from the previous problem to write down the Adams–Moulton corrector value for the solution of the same problem.

14. Use the Adams–Bashforth–Moulton method of the previous two problems to find an approximation of the solution of $y' = y + 1$, $y(0) = 1$ at $x = 0.4$, using $h = 0.1$, given $y(0) = 1$, $y(0.1) = 1.2103$, $y(0.2) = 1.4428$, $y(0.3) = 1.6997$.

15. The standard forward difference approximation of $y'(x)$ is

    Select the correct answer.

    (a) $(y(x + h) + 2y(x) + y(x - h))/h^2$
    (b) $(y(x + h) - 2y(x) + y(x - h))/h^2$
    (c) $(y(x + h) - 2y(x) + y(x - h))/h$
    (d) $(y(x + h) + 2y(x) + y(x - h))/h$
    (e) $(y(x + h) - y(x))/h$

16. The standard central difference approximation of $y''(x)$ is

    Select the correct answer.

    (a) $(y(x + h) - 2y(x) + y(x - h))/h$
    (b) $(y(x + h) + 2y(x) + y(x - h))/h$
    (c) $(y(x + h) - 2y(x) + y(x - h))/h^2$
    (d) $(y(x + h) + 2y(x) + y(x - h))/h^2$
    (e) $(y(x + h) - y(x - h))/h$

17. Using the notation in the text, write down the finite difference equation for the initial value problem $y'' + P(x)y' + Q(x)y = f(x)$, $y(x_0) = y_0$, $y(x_1) = y_1$.

18. Using the finite difference equation from the previous problem, write down the system of equations generated by the problem $y'' + 3y' + 2y = 0$, $y(0) = 1$, $y(1) = 2$ with $n = 4$.

19. Briefly explain the shooting method for solving $y'' = f(x, y, y')$, $y(x_0) = y_0$, $y(x_1) = y_1$.

20. Use the shooting method once along with Euler's method with $n = 2$ with an initial guess of $y'(0) = 1$ to find a solution of $y'' = y + 1$, $y(0) = 0$, $y(1) = 1$. What is the error at $x = 1$?

ANSWER KEY
Zill Differential Equations 9e Chapter 9 Form F

1. c
2. e
3. d
4. e
5. a
6. $y_{n+1} = y_n + h(f(x_n, y_n) + f(x_{n+1}, y'_{n+1}))/2$, where y'_{n+1} is calculated from Euler's method
7. 2.536
8. error $= 2/(2 - e^{0.2}) - 2.536 \approx 0.0327$
9. The error would decrease by roughly a factor of 1/8.
10. d
11. e
12. $y^*_{n+1} = y_n + h(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3})/24$, $y'_n = f(x_n, y_n)$, $y'_{n-1} = f(x_{n-1}, y_{n-1})$, $y'_{n-2} = f(x_{n-2}, y_{n-2})$, $y'_{n-3} = f(x_{n-3}, y_{n-3})$
13. $y_{n+1} = y_n + h(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2})/24$, $y'_{n+1} = f(x_{n+1}, y^*_{n+1})$
14. $y_4 = 1.9836$
15. e
16. c
17. $(1 + hP_i/2)y_{i+1} + (-2 + h^2 Q_i)y_i + (1 - hP_i/2)y_{i-1} = h^2 f_i$
18. $11y_2 - 15y_1 = -5$, $11y_3 - 15y_2 + 5y_1 = 0$, $-15y_3 + 5y_2 = -22$
19. The shooting method is a numerical method for solving the boundary value problem $y'' = f(x, y, y')$, $y(x_0) = y_0$, $y(x_1) = y_1$ by replacing the right hand boundary condition with a guess of the initial condition $y'(x_0) = y_2$, and solving that problem. If the value of the solution at x_1 is not within a predetermined distance from the actual value, y_1, then iterate.
20. $y_2 = 5/4$, error$= 1/4$

1. Explain the term truncation error.
2. What is the round-off error in representing the number 2/3 in a three digit, base ten calculator?
3. Use Euler's method to find an approximation of $y(0.2)$ for the solution of $y' = y - 1$, $y(0) = 0$ with a step size of $h = 0.1$.
4. In the previous problem, what is the exact value of the error?
5. In the previous two problems, explain how the error would change if you decrease h to 0.05.
6. The solution of $y' = y^2$, $y(0) = 2$ for $y(0.2)$, using the improved Euler's method with $h = 0.2$, is

   Select the correct answer.

   (a) 2.784
   (b) 2.884
   (c) 2.984
   (d) 3.084
   (e) 3.184
7. The local truncation error for the improved Euler's method is

   Select the correct answer.

   (a) $O(h^4)$
   (b) $O(h^3)$
   (c) $O(h^2)$
   (d) $O(h)$
   (e) unknown
8. The improved Euler's method is what type of Runge–Kutta method?

   Select the correct answer.

   (a) fourth order
   (b) third order
   (c) second order
   (d) first order
   (e) It is not a Runge–Kutta method

9. Which of the following are second order Runge–Kutta methods for the solution of $y' = f(x, y)$, $y(x_0) = y_0$?

   Select all that apply.

   (a) $y_{n+1} = y_n + h(2k_1/3 + k_2/3)$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + 3h/2, y_n + 3hk_1/2)$
   (b) $y_{n+1} = y_n + h(k_1 + k_2)/2$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + 2h, y_n + 2hk_1)$
   (c) $y_{n+1} = y_n + h(3k_1 + k_2)/4$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + 2h, y_n + 2hk_1)$
   (d) $y_{n+1} = y_n + h(k_1 + k_2)/2$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h, y_n + hk_1)$
   (e) $y_{n+1} = y_n + h(k_1/3 + 2k_2/3)$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + 3h/4, y_n + 3hk_1/4)$

10. Using the method from response c of the previous problem, the solution of $y' = y^2$, $y(0) = 2$ for $y(0.2)$ with $h = 0.2$ is

    Select the correct answer.

    (a) 3.208
    (b) 3.218
    (c) 3.228
    (d) 3.238
    (e) 3.248

11. Write down the most popular fourth order Runge–Kutta method for the solution of $y' = f(x, y)$, $y(x_0) = y_0$.

12. Use the method from the previous problem to solve for $y(0.2)$ in the problem $y' = y^2 - y$, $y(0) = 2$ with a step size of $h = 0.2$.

13. What is the order of the global error you expect in the previous problem if you were to calculate the solution out to $x = 5.0$?

14. The Adams–Bashforth formula for finding the solution of $y' = f(x, y)$, $y(x_0) = y_0$ is

    Select the correct answer.

    (a) $y^*_{n+1} = y_n + h(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3})/24$, where $y'_n = f(x_n, y_n)$, $y'_{n-1} = f(x_{n-1}, y_{n-1})$, $y'_{n-2} = f(x_{n-2}, y_{n-2})$, $y'_{n-3} = f(x_{n-3}, y_{n-3})$
    (b) $y^*_{n+1} = y_n + h(55y'_n - 59y'_{n-1} - 37y'_{n-2} + 65y'_{n-3})/24$, where $y'_n = f(x_n, y_n)$, $y'_{n-1} = f(x_{n-1}, y_{n-1})$, $y'_{n-2} = f(x_{n-2}, y_{n-2})$, $y'_{n-3} = f(x_{n-3}, y_{n-3})$
    (c) $y^*_{n+1} = y_n + h(59y'_n - 55y'_{n-1} + 37y'_{n-2} - 17y'_{n-3})/24$, where $y'_n = f(x_n, y_n)$, $y'_{n-1} = f(x_{n-1}, y_{n-1})$, $y'_{n-2} = f(x_{n-2}, y_{n-2})$, $y'_{n-3} = f(x_{n-3}, y_{n-3})$
    (d) $y^*_{n+1} = y_n + h(55y'_n + 59y'_{n-1} - 37y'_{n-2} - 9y'_{n-3})/24$, where $y'_n = f(x_n, y_n)$, $y'_{n-1} = f(x_{n-1}, y_{n-1})$, $y'_{n-2} = f(x_{n-2}, y_{n-2})$, $y'_{n-3} = f(x_{n-3}, y_{n-3})$
    (e) none of the above

15. Using the Adams–Bashforth method from the previous problem, and using the values $y_0 = 2$, $y_1 = 2.2351$, $y_2 = 2.5687$, $y_3 = 3.0762$, the solution of $y' = y^2 - y$, $y(0) = 2$ for $y^*_{n+1} = y(0.4)$ with $h = 0.1$ is

    Select the correct answer.

    (a) 3.9356
    (b) 3.9346
    (c) 3.9336
    (d) 3.9326
    (e) 3.9316

16. Using the value of y^*_{n+1} from the previous two problems, the Adams–Moulton corrector formula for the solution of $y' = f(x, y)$, $y(x_0) = y_0$ is

    Select the correct answer.

    (a) $y_{n+1} = y_n + h(9y'_{n+1} - 19y'_n + 5y'_{n-1} + y'_{n-2})/24$, where $y'_{n+1} = f(x_{n+1}, y^*_{n+1})$
    (b) $y_{n+1} = y_n + h(9y'_{n+1} + 19y'_n + 5y'_{n-1} + y'_{n-2})/34$, where $y'_{n+1} = f(x_{n+1}, y^*_{n+1})$
    (c) $y_{n+1} = y_n + h(9y'_{n+1} + 19y'_n - 5y'_{n-1} - y'_{n-2})/24$, where $y'_{n+1} = f(x_{n+1}, y^*_{n+1})$
    (d) $y_{n+1} = y_n + h(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2})/24$, where $y'_{n+1} = f(x_{n+1}, y^*_{n+1})$
    (e) none of the above

17. Using the Adams–Bashforth–Moulton method from the previous three problems, the solution of $y' = y^2 - y$, $y(0) = 2$ for $y(0.4)$ with $h = 0.1$ is

    Select the correct answer.

    (a) 3.9316
    (b) 3.9326
    (c) 3.9336
    (d) 3.9346
    (e) 3.9356

18. Rewrite the problem $y'' - 2xy' + 4y = 0$, $y(0) = 1$, $y'(0) = 2$ as a system of two first order initial value problems.

19. Use Euler's method to solve for $y(0.2)$ in the problem $y'' - 2xy' + 4y = 0$, $y(0) = 1$, $y'(0) = 2$, using a step size of $h = 0.1$.

20. Use a fourth order Runge–Kutta method with a step size of $h = 0.2$ to solve for $y(0.2)$ in the problem $y'' - 2xy' + 4y = 0$, $y(0) = 1$, $y'(0) = 2$.

ANSWER KEY
Zill Differential Equations 9e Chapter 9 Form G

1. Truncation error is the formula error in using only a finite number of terms of an infinite expansion (for example, of a Taylor's series).
2. error = $1/3000$
3. $y_1 = -0.1$, $y_2 = -0.21$
4. error = $1 - e^{0.2} - (-.21) \approx -0.0114$
5. The local error would decrease by about a factor of $1/4$, and the global error would decrease by about a factor of $1/2$.
6. e
7. b
8. c
9. a, c, d, e
10. e
11. $y_{n+1} = y_n + h(k_1 + 2k_2 + 2k_3 + k_4)/6$ where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/2, y_n + hk_1/2)$, $k_3 = f(x_n + h/2, y_n + hk_2/2)$, $k_4 = f(x_n + h, y_n + hk_3)$
12. $y_1 = 2.5682$
13. error= $O(h^4)$
14. a
15. a
16. d
17. e
18. $y' = u$, $u' = 2xu - 4y$, $y(0) = 1$, $u(0) = 2$
19. $y_2 = 1.36$
20. $y_1 = 1.3146$

1. When entering the number 2/7 into a three digit base ten calculator, the actual value entered is

   Select the correct answer.

   (a) 2/7
   (b) .286
   (c) .285
   (d) .28
   (e) .29

2. When entering the number 2/7 into a three digit base ten calculator, the round-off error is

   Select the correct answer.

   (a) 1/350
   (b) 1/3500
   (c) 1/35
   (d) 0.001
   (e) 0.0001

3. Write down Euler's method to solve $y' = f(x, y)$, $y(x_0) = y_0$.

4. Use Euler's method with a step size of $h = 0.1$ to find an approximation of $y(0.2)$, where y is the solution of $y' = y^2 + 1$, $y(0) = 0$.

5. In the previous problem, what is the exact value of the error?

6. In the previous two problems, explain how the error would change if you decrease h to 0.05.

7. The improved Euler's formula for solving $y' = f(x, y)$, $y(\overline{x}) = \overline{y}$ is

   Select the correct answer.

   (a) $y_{n+1} = y_n + h(f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})/2$, $y_0 = \overline{y}$, $n = 0, 1, 2, \ldots$ where y^*_{n+1} is predicted from Euler's formula
   (b) $y_{n+1} = y_n + (f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})/2$, $y_0 = \overline{y}$, $n = 0, 1, 2, \ldots$ where y^*_{n+1} is predicted from Euler's formula
   (c) $y_{n+1} = y_{n-1} - f(x_n, y_n)$, $y_0 = \overline{y}$, $n = 0, 1, 2, \ldots$
   (d) $y_{n+1} = y_n + hf(x_n, y_n)$, $y_0 = \overline{y}$, $n = 0, 1, 2, \ldots$
   (e) $y_{n+1} = y_n - hf(x_n, y_n)$, $y_0 = \overline{y}$, $n = 0, 1, 2, \ldots$

8. The solution of $y' = y + 1$, $y(0) = 1$ for $y(0.2)$, using the improved Euler's method with $h = 0.1$, is

   Select the correct answer.

   (a) 1.43
   (b) 1.431
   (c) 1.44205
   (d) 1.44315
   (e) 1.44505

9. In the previous problem, the local truncation error in y_{n+1} is

   Select the correct answer.

   (a) $0.1e^c$, where $x_n < c < x_{n+1}$
   (b) $0.01e^c$, where $x_n < c < x_{n+1}$
   (c) $O(h^2)$
   (d) $O(h^3)$
   (e) unknown

10. The most popular fourth order Runge–Kutta method for the solution of $y' = f(x, y)$, $y(x_0) = y_0$ is

    Select the correct answer.

    (a) $y_{n+1} = y_n + h(k_1 + 2k_2 + 2k_3 + k_4)/6$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/2, y_n + hk_1/2)$, $k_3 = f(x_n + h/2, y_n + hk_2/2)$, $k_4 = f(x_n + h, y_n + hk_3)$
    (b) $y_{n+1} = y_n + h(2k_1 + k_2 + k_3 + 2k_4)/6$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/2, y_n + hk_1)$, $k_3 = f(x_n + h/2, y_n + hk_2)$, $k_4 = f(x_n + h, y_n + hk_3)$
    (c) $y_{n+1} = y_n + h(k_1 + 2k_2 + 2k_3 + k_4)/6$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/3, y_n + hk_1/2)$, $k_3 = f(x_n + 2h/3, y_n + hk_2/2)$, $k_4 = f(x_n + h, y_n + hk_3)$
    (d) $y_{n+1} = y_n + h(2k_1 + k_2 + k_3 + 2k_4)/6$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/2, y_n + hk_1/2)$, $k_3 = f(x_n + h/2, y_n + hk_2/2)$, $k_4 = f(x_n + h, y_n + hk_3)$
    (e) $y_{n+1} = y_n + h(k_1 + 2k_2 + 2k_3 + k_4)/6$, where $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + h/2, y_n + hk_1)$, $k_3 = f(x_n + h/2, y_n + hk_2)$, $k_4 = f(x_n + h, y_n + hk_3)$

11. Using the method from the previous problem, the solution of $y' = y - 1$, $y(0) = -1$ for $y(0.2)$ with $h = 0.2$ is

    Select the correct answer.

    (a) -1.4418
    (b) -1.4428
    (c) -1.4438
    (d) -1.4448
    (e) -1.4458

12. Write down the Adams–Bashforth formula for y^*_{n+1}, the solution of $y' = f(x, y)$, $y(x_0) = y_0$ at x_{n+1}.

13. Use the value of y^*_{n+1} from the previous problem to write down the Adams–Moulton corrector value for the solution of the same problem.

14. Use the Adams–Bashforth–Moulton method of the previous two problems to find an approximation of the solution of $y' = y - 2$, $y(0) = 1$ at $x = 0.4$, using $h = 0.1$, given $y(0) = 1$, $y(0.1) = 0.8948$, $y(0.2) = 0.7786$, $y(0.3) = 0.6501$.

15. Using the notation from the text, the finite difference equation for solving the boundary value problem $y'' + P(x)y' + Q(x)y = f(x)$, $y(a) = \alpha$, $y(b) = \beta$ is

    Select the correct answer.

    (a) $(1 - hP_i/2)y_{i+1} + (-2 + h^2Q_i)y_i + (1 + hP_i/2)y_{i-1} = h^2 f_i$
    (b) $(1 - hP_i/2)y_{i+1} + (-2 + h^2Q_i)y_i + (1 - hP_i/2)y_{i-1} = h^2 f_i$
    (c) $(1 + hP_i/2)y_{i+1} + (-2 + h^2Q_i)y_i + (1 + hP_i/2)y_{i-1} = h^2 f_i$
    (d) $(1 + hP_i/2)y_{i+1} + (2 - h^2Q_i)y_i + (1 - hP_i/2)y_{i-1} = h^2 f_i$
    (e) $(1 + hP_i/2)y_{i+1} + (-2 + h^2Q_i)y_i + (1 - hP_i/2)y_{i-1} = h^2 f_i$

16. Using the finite difference formula from the previous problem, the system of equations you derive for the problem $y'' + y = 0$, $y(0) = 0$, $y(1) = 1$ with $n = 2$ is

    Select the correct answer.

    (a) $1 - 7y_1/4 = 0$
    (b) $1 + 7y_1/4 = 0$
    (c) $1 - 7y_2/4 + y_1 = 0$, $y_2 - 7y_1/4 = 0$
    (d) $1 - 3y_2/4 + 3y_1/4 = 0$, $y_2 - 3y_1/4 = 0$
    (e) $1 - y_2/4 + y_1/4 = 0$, $y_2 - y_1/4 = 0$

17. Using the finite difference formula from above, the system of equations you derive for the problem $y'' + y = 0$, $y(0) = 0$, $y(1) = 1$ with $n = 4$ is

    Select the correct answer.

    (a) $y_2 + 31y_1/16 = 0$, $y_3 + 31y_2/16 + y_1 = 0$, $1 + 31y_3/16 + y_2 = 0$
    (b) $y_2 - 15y_1/16 = 0$, $y_3 - 15y_2/16 + y_1 = 0$, $1 - 15y_3/16 + y_2 = 0$
    (c) $y_2 - 31y_1/16 = 0$, $y_3 - 31y_2/16 + y_1 = 0$, $1 - 31y_3/16 + y_2 = 0$
    (d) $y_2 + 15y_1/16 = 0$, $y_3 + 15y_2/16 + y_1 = 0$, $1 + 15y_3/16 + y_2 = 0$
    (e) $y_2 - 15y_1/8 = 0$, $y_3 - 15y_2/8 + y_1 = 0$, $1 - 15y_3/8 + y_2 = 0$

18. The numerical solution of the problem in the previous problem is

    Select the correct answer.

    (a) $y_0 = 0,\ y_1 = 1/4,\ y_2 = 1/2,\ y_3 = 3/4,\ y_4 = 1$
    (b) $y_0 = 0,\ y_1 = 1/3,\ y_2 = 1/2,\ y_3 = 2/3,\ y_4 = 1$
    (c) $y_0 = 0,\ y_1 = .2943,\ y_2 = .5702,\ y_3 = .8104,\ y_4 = 1$
    (d) $y_0 = 0,\ y_1 = .2934,\ y_2 = .5072,\ y_3 = .8014,\ y_4 = 1$
    (e) $y_0 = 0,\ y_1 = .2839,\ y_2 = .5772,\ y_3 = .8004,\ y_4 = 1$

19. Briefly explain the finite difference method for solving $y'' = f(x, y, y'),\ y(x_0) = y_0,\ y(x_1) = y_1$.

20. Use the finite difference method with $n = 2$ to find a solution of $y'' + y = 0,\ y(0) = 2,\ y(1) = 1$.

ANSWER KEY
Zill Differential Equations 9e Chapter 9 Form H

1. b
2. b
3. $y_{n+1} = y_n + hf(x_n, y_n)$, $y_0 = y(x_0)$
4. $y_1 = 0.1$, $y_2 = 0.201$
5. error $= \tan(0.2) - 0.201 \approx 0.00171$
6. local error decreases by about a factor of 1/4, global error decreases by about a factor of 1/2
7. a
8. c
9. d
10. a
11. b
12. $y^*_{n+1} = y_n + h(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3})/24$ where $y'_n = f(x_n, y_n)$, $y'_{n-1} = f(x_{n-1}, y_{n-1})$, $y'_{n-2} = f(x_{n-2}, y_{n-2})$, $y'_{n-3} = f(x_{n-3}, y_{n-3})$
13. $y_{n+1} = y_n + h(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2})/24$ where $y'_{n+1} = f(x_{n+1}, y^*_{n+1})$
14. $y_1 = 0.5082$
15. e
16. a
17. c
18. c
19. Divide the interval into n equal subintervals. Use a central difference approximation to approximate $y''(x)$ at each of the $n-1$ interior points. There are $n-1$ linear equations in the $n-1$ unknown values y_1, y_2, …, y_{n-1} and the known values $y_0 = y(x_0)$, $y_n = y(x_n)$. Solve this system of linear equations for the unknown solution values.
20. $y(0) = 2$, $y(0.5) = 12/7$, $y(1) = 1$