1. Solve the differential equation $y' = \cos(2x)e^y$.
2. Solve the differential equation $y' - 4y = x^2$.
3. Solve the differential equation $(x^3 + y^3)dx + 3xy^2dy = 0$.
4. Use Euler's method to solve the initial value problem $y' = 1 + y^2$, $y(0) = 1$, with a step size of $h = 0.1$. Find approximations of $y(0.1)$ and $y(0.2)$.
5. In the previous problem, compare your answers with the exact solution, $y = \tan(x + \pi/4)$. What are the errors at $x = 0.1$ and at $x = 0.2$?
6. A 10 gram sample of a radioactive substance is found to decay by 5% in 8 hours. Identify all variables. Assuming the rate of decay is proportional to the amount present, write down the differential equation problem for the amount of the substance as a function of time, and solve it. What is the half-life of the substance?
7. A baked cake is taken from a $375°F$ oven and placed on a table at $70°F$. Thirty minutes later it has cooled to $100°F$. Identify all variables in the problem, write down the differential equation problem for the temperature, and solve it for the temperature as a function of time. Assume that Newton's law of cooling applies. What is the temperature one hour after it was taken out of the oven?
8. Solve the differential equation $y'' - 6y' + 8y = 0$.
9. Solve the differential equation $y'' + 4y' + 4y = x^2 + 1$.
10. Solve the differential equation $y'' - 6y' + 13y = \cos(3x)$.
11. Solve the differential equation $y'' - 3y' + 2y = x + e^x$.
12. Solve the differential equation $y'' + 4y = \tan(2x)$.
13. Solve the differential equation $x^2y'' - 6xy + 10y = 0$.
14. Solve the differential equation $x^2y'' + 5xy + 5y = 0$.
15. A 1-kilogram mass is attached to a spring with spring constant 12-Newtons per meter. The system is immersed in a liquid that imparts a damping force numerically equal to 8 times the instantaneous velocity. The mass is initially released from rest at a point 0.5 meter below the equilibrium point. Identify all variables and find the equation of motion of the mass.
16. Find the charge on the capacitor in an LRC circuit if $L = 1h$, $R = 100\Omega$, $C = 0.0004f$, $E(t) = 50V$, $q(0) = 0C$, $i(0) = 4A$
17. Solve the eigenvalue problem $y'' + \lambda y = 0$, $y(0) = 0$, $y(\pi/2) = 0$.
18. Consider the series $\sum_{n=1}^{\infty}(x + 2)^n/(n2^n)$. Use the ratio test to determine the radius of convergence.
19. Identify the singular points of $x^2y'' + 2xy' + y = 0$ and tell whether they are regular or irregular.

20. Identify the singular points of $(x-1)^3x(x+1)^2y'' - 3xy' + 2y = 0$ and tell whether they are regular or irregular.

21. Use power series methods to find the solution of $y'' + y = 0$ about $x = 0$.

22. Use power series methods to find the solution of $y'' + (1 - x^2)y = 0$ about $x = 0$.

23. Use power series methods to find the solution of $2xy'' - y' + 2y = 0$ about $x = 0$.

24. Use power series methods to find the solution of $xy'' + 2y' - xy = 0$ about $x = 0$.

25. Use power series methods to find the solution of $xy'' - xy' + y = 0$ about $x = 0$.

26. Use power series methods to find one solution of $x^2y'' + xy' + (x^2 - 4)y = 0$ about $x = 0$.

27. In the previous problem, identify the equation and write down the solution using the special function notation.

28. Use power series methods to find the solution of $(1 - x^2)y'' - 2xy' + 6y = 0$ about $x = 0$.

29. In the previous problem, identify the equation and write down the solution using the special function notation.

30. Define the Laplace transform of a function $f(t)$.

31. Find the Laplace transform of te^t.

32. Find the Laplace transform of $\cos(3t)$.

33. Find the inverse Laplace transform of $4/(s^2 + 9)$.

34. Find the inverse Laplace transform of $5/(s - 4)^3$.

35. Find the inverse Laplace transform of $e^{-4s}/(s(s - 1))$.

36. Find the inverse Laplace transform of $1/(s^2 + 2s + 5)$.

37. Find the inverse Laplace transform of $1/((s-1)(s^2+4))$ using the convolution theorem.

38. Solve the initial value problem $y' - y = \sin(2t)$, $y(0) = 0$ using Laplace transform methods. (Hint: the previous problem might be useful.)

39. Solve the initial value problem $y' - y = te^{2t}$, $y(0) = 1$ using Laplace transform methods.

40. Solve the initial value problem $y'' + 2y' - 3y = e^{-3t}$, $y(0) = 0$, $y'(0) = 2$ using Laplace transform methods.

41. Solve the initial value problem $y'' - y = t\mathcal{U}(t - 4)$, $y(0) = 1$, $y'(0) = 0$ using Laplace transform methods.

42. Solve the system $\mathbf{X}' = A\mathbf{X}$ where $A = \begin{pmatrix} 1 & 2 \\ -1 & 4 \end{pmatrix}$.

43. Find a particular solution of the system $\mathbf{X}' = A\mathbf{X} + F(t)$ where $A = \begin{pmatrix} 1 & 2 \\ -1 & 4 \end{pmatrix}$ and $F(t) = \begin{pmatrix} t \\ 1 \end{pmatrix}$.

44. Solve the system $\mathbf{X}' = A\mathbf{X}$ where $A = \begin{pmatrix} 1 & 1 \\ -4 & 1 \end{pmatrix}$.

45. Solve the system $\mathbf{X}' = A\mathbf{X}$ where $A = \begin{pmatrix} 5 & -4 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 5 \end{pmatrix}$.

46. Let $A = \begin{pmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 3 & 2 & 0 \end{pmatrix}$. Calculate e^{At}.

47. Consider the initial value problem $y' = 3xy$, $y(0) = 1$. Solve for $y(0.1)$ using the improved Euler method with a step size of $h = 0.1$.

48. In the previous problem, solve for $y(0.1)$ using the most popular second order Runge–Kutta method with a step size of $h = 0.1$.

49. In the previous two problems, solve for $y(0.1)$ using the classical fourth order Runge–Kutta method with a step size of $h = 0.1$.

50. In the previous three problems, comment on the expected local and global errors of the three methods.

ANSWER KEY
Zill Differential Equations 9e Final Exam 1 Form A

1. $y = -\ln(c - \sin(2x)/2)$

2. $y = -x^2/4 - x/8 - 1/32 + ce^{4x}$

3. $x^4/4 + xy^3 = c$

4. $y(0.1) \approx 1.2$, $y(0.2) \approx 1.444$

5. $e(0.1) = \tan(0.1 + \pi/4) - 1.2 = 0.023$, $e(0.2) = \tan(0.2 + \pi/4) - 1.444 = 0.064$

6. $x(t)$ = amount of the substance at time t, $\frac{dx}{dt} = kx$, $x = 10e^{kt}$, $k = \ln(0.95)/8 = -0.00641$, half-life $= -\ln 2/k = 108$ hours

7. $T(t)$ = temperature at time t, $\frac{dT}{dt} = k(T - 70)$, $T = 70 + 305e^{kt}$, $k = \ln(30/305)/30 = -0.0773$, $T = 70 + 305e^{-0.0773t}$, $T(60) = 73.0°F$

8. $y = c_1e^{2x} + c_2e^{4x}$

9. $y = c_1e^{-2x} + c_2xe^{-2x} + x^2/4 - x/2 + 5/8$

10. $y = c_1e^{3x}\cos(2x) + c_2e^{3x}\sin(2x) + (2\cos(3x) - 9\sin(3x))/170$

11. $y = c_1e^{2x} + c_2e^{x} + x/2 + 3/4 - xe^{x}$

12. $y = c_1\cos(2x) + c_2\sin(2x) - \ln|\sec(2x) + \tan(2x)|\cos(2x)/4$

13. $y = c_1x^2 + c_2x^5$

14. $y = x^{-2}[c_1\cos(\ln x) + c_2\sin(\ln x)]$

15. $x(t)$ = position at time t, $x'' + 8x' + 12x = 0$, $x(0) = 0.5$, $x'(0) = 0$, $x = (3e^{-2t} - e^{-6t})/4$

16. $q(t) = (1 - e^{-50t} + 150te^{-50t})/50$

17. $\lambda = 4n^2$, $y = \sin(2nx)$, $n = 1, 2, 3, \ldots$

18. $R = 2$

19. $x = 0$ is a regular singular point

20. $x = 0, -1$ are regular singular points, $x = 1$ is an irregular singular point

21. $y = c_0\sum_{n=0}^{\infty}(-1)^n x^{2n}/(2n)! + c_1\sum_{n=0}^{\infty}(-1)^n x^{2n+1}/(2n+1)!$

22. $y = c_0(1 - x^2/2 + x^4/8 + \ldots) + c_1(x - x^3/6 + 7x^5/120 + \ldots)$

23. $y = c_0[1 + \sum_{n=1}^{\infty}(-2)^n x^n/(n!(-1)\cdot 1\cdots(2n-3))] + c_1x^{3/2}[1 + \sum_{n=1}^{\infty}(-2)^n x^n/(n!5\cdot 7\cdots(2n+3))]$

24. $y = c_0\sum_{n=0}^{\infty} x^{2n-1}/(2n)! + c_1\sum_{n=0}^{\infty} x^{2n}/(2n+1)!$

25. $y = c_0x + c_1[x\ln x - 1 + x^2/2 + x^3/12 + \cdots]$

26. $y = c_0[x^2 + \sum_{n=1}^{\infty}(-1)^n x^{2n+2}/(2^{2n}n!(n+2)!)]$

27. Bessel's equation of order 2, $y = c_0J_2(x) + c_1Y_2(x)$

28. $y = c_0(1 - 3x^2) + c_1[x + \sum_{n=1}^{\infty} 2^n(n+1)!(-1) \cdot 1 \cdot 3 \cdots (2n-3)x^{2n+1}/(2n+1)!]$

29. Legendre's equation of order 2, $y = c_0 P_2(x) + c_1[x + \sum_{n=1}^{\infty} 2^n(n+1)!(-1) \cdot 1 \cdot 3 \cdots (2n-3)x^{2n+1}/(2n+1)!]$

30. $\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$

31. $1/(s-1)^2$

32. $s/(s^2+9)$

33. $4\sin(3t)/3$

34. $5t^2e^{4t}/2$

35. $\mathcal{U}(t-4)(e^{t-4}-1)$

36. $e^{-t}\sin(2t)/2$

37. $(2e^t - 2\cos(2t) - \sin(2t))/10$

38. $y = (2e^t - 2\cos(2t) - \sin(2t))/5$

39. $y = 2e^t - e^{2t} + te^{2t}$

40. $y = (9e^t - 9e^{-3t} - 4te^{-3t})/16$

41. $y = (e^t + e^{-t})/2 + (-t + (5e^{t-4} + 3e^{-(t-4)})/2)\mathcal{U}(t-4)$

42. $X = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{3t}$

43. $X_p = \begin{pmatrix} -2t/3 - 1/18 \\ -t/6 - 11/36 \end{pmatrix}$

44. $X = c_1 e^t \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos(2t) - \begin{pmatrix} 0 \\ 2 \end{pmatrix} \sin(2t) \right] + c_2 e^t \left[\begin{pmatrix} 0 \\ 2 \end{pmatrix} \cos(2t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin(2t) \right]$

45. $X = c_1 \begin{pmatrix} 4 \\ 5 \\ -2 \end{pmatrix} + c_2 \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} e^{5t} + c_3 \left[\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} te^{5t} + \begin{pmatrix} -5/2 \\ -1/2 \\ 0 \end{pmatrix} e^{5t} \right]$

46. $e^{At} = \begin{pmatrix} 1 & 0 & 0 \\ 5t & 1 & 0 \\ 3t + 5t^2 & 2t & 1 \end{pmatrix}$

47. $y_1 = 1.015$

48. $y_1 = 1.015$

49. $y_1 = 1.015113$

50. The local errors are $O(h^3)$, $O(h^3)$ and $O(h^5)$, respectively. The global errors are $O(h^2)$, $O(h^2)$ and $O(h^4)$, respectively.

1. The differential equation $y' = x^2y^2$ is

   Select all that apply.

   (a) linear
   (b) separable
   (c) exact
   (d) non-linear
   (e) Bernoulli

2. The differential equation $y' + y = x^2$ is

   Select all that apply.

   (a) linear
   (b) separable
   (c) exact
   (d) non-linear
   (e) Bernoulli

3. The differential equation $xdy - ydx = 0$ is

   Select all that apply.

   (a) linear
   (b) separable
   (c) exact
   (d) non-linear
   (e) Bernoulli

4. The solution of $y' - y = x$ is

   Select the correct answer.

   (a) $y = x - 1 + ce^x$
   (b) $y = -x + 1 + ce^x$
   (c) $y = -x - 1 + ce^x$
   (d) $y = -x - 1 + ce^{-x}$
   (e) $y = x + 1 + ce^{-x}$

5. The solution of $x^2ydx + (x^3/3 + y)dy = 0$ is

   Select the correct answer.

   (a) $x^3y/3 + y^2/2$
   (b) $x^3y/3 - y^2/2$
   (c) $x^3y/3 + y^2/2 - c$
   (d) $x^3y/3 + y^2/2 = c$
   (e) $x^3y/3 - y^2/2 = c$

6. The solution of $xy' = (x + 1)y^2$

   Select the correct answer.

   (a) $y = 1/(x + \ln x + c)$
   (b) $y = 1/(x - \ln x + c)$
   (c) $y = -c/(x + \ln x)$
   (d) $y = -c/(x - \ln x)$
   (e) $y = -1/(x + \ln x + c)$

7. A frozen chicken at $0°C$ is taken out of the freezer and placed on a table at $20°C$. One hour later the temperature of the chicken is $18°C$. The mathematical model for the temperature $T(t)$ as a function of time t is (assuming Newton's law of warming)

   Select the correct answer.

   (a) $\frac{dT}{dt} = kT$, $T(0) = 0$, $T(1) = 18$
   (b) $\frac{dT}{dt} = k(T - 20)$, $T(0) = 0$, $T(1) = 18$
   (c) $\frac{dT}{dt} = (T - 20)$, $T(0) = 0$, $T(1) = 18$
   (d) $\frac{dT}{dt} = T$, $T(0) = 0$, $T(1) = 18$
   (e) $\frac{dT}{dt} = k(T - 18)$, $T(0) = 0$, $T(1) = 18$

8. In the previous problem, the solution of the differential equation is

   Select the correct answer.

   (a) $T = Ce^{kt}$
   (b) $T = Ce^{-kt}$
   (c) $T = 20 + Ce^{kt}$
   (d) $T = 20 + Ce^{-kt}$
   (e) $T = 18 + Ce^{kt}$

9. In the previous two problems, the solution for the temperature is

   Select the correct answer.

   (a) $T(t) = 20 - 20e^{-2.30t}$
   (b) $T(t) = 20 - 20e^{2.30t}$
   (c) $T(t) = 18 - 18e^{-2.30t}$
   (d) $T(t) = 18 - 18e^{2.30t}$
   (e) $T(t) = 18e^{-2.30t}$

10. The solution of $y'' + 4y' + 4y = 0$ is

    Select the correct answer.

    (a) $y = c_1e^{-2x} + c_2xe^{-2x}$
    (b) $y = c_1e^{-2x} + c_2e^{-2x}$
    (c) $y = c_1e^{2x} + c_2e^{2x}$
    (d) $y = c_1e^{2x} + c_2xe^{2x}$
    (e) $y = c_1e^{2x} + c_2e^{4x}$

11. The auxiliary equation of $y'' - 5y' + 6y = 0$ is

    Select the correct answer.

    (a) $m^2 - 5m - 6 = 0$
    (b) $m^2 - 5m + 6 = 0$
    (c) $m^2 - 5m + 6 = 1$
    (d) $m^2 - 5m + 6$
    (e) $m^2 - 5m - 6$

12. The solution of $y'' - 5y' + 6y = 0$ is

    Select the correct answer.

    (a) $y = c_1e^{-2x} + c_2e^{-3x}$
    (b) $y = c_1e^{2x} + c_2xe^{3x}$
    (c) $y = c_1e^{-2x} + c_2xe^{-3x}$
    (d) $y = c_1e^{2x} + c_2e^{3x}$
    (e) $y = c_1e^{2x} + c_2e^{-3x}$

13. The solution of $y'' - 4y' + 13y = 0$ is

    Select the correct answer.

    (a) $y = c_1 e^{-2x}\cos(3x) + c_2 e^{-2x}\sin(3x)$
    (b) $y = c_1 e^{-2x}\cos(3x) + c_2 e^{2x}\sin(3x)$
    (c) $y = c_1 e^{2x}\cos(3x) + c_2 e^{2x}\sin(3x)$
    (d) $y = c_1 e^{2x} + c_2 e^{3x}$
    (e) $y = c_1\cos(3x) + c_2\sin(3x)$

14. The correct form of the particular solution of $y'' - 2y' + y = e^x$ is

    Select the correct answer.

    (a) $y_p = Ae^x$
    (b) $y_p = Axe^x$
    (c) $y_p = Ax^2e^x$
    (d) $y_p = Ax^3e^x$
    (e) none of the above

15. The correct form of the particular solution of $y'' - 2y' = x + e^x$ is

    Select the correct answer.

    (a) $y_p = Ax + B + Ce^x$
    (b) $y_p = (Ax + B + Ce^x)x$
    (c) $y_p = Ax^2 + B + Ce^x$
    (d) $y_p = Ax^2 + Bx + Ce^x$
    (e) $y_p = Ax + B + Cxe^x$

16. The solution of $y'' - 2y' = x + e^x$ is

    Select the correct answer.

    (a) $y = c_1 + c_2e^{2x} - x^2/4 - x/4 - e^x$
    (b) $y = c_1 + c_2e^{2x} - x^2/4 - x/4 + e^x$
    (c) $y = c_1 + c_2e^{2x} + x^2/4 - x/4 - e^x$
    (d) $y = c_1 + c_2e^{2x} + x^2/4 + x/4 - e^x$
    (e) $y = c_1 + c_2e^{2x} + x^2/4 + x/4 + e^x$

17. The solution of $y'' + 3y' - 4y = \cos x$ is

    Select the correct answer.

    (a) $y = c_1 e^x + c_2 e^{-4x} + (5\sin x + 3\cos x)/34$
    (b) $y = c_1 e^x + c_2 e^{-4x} + (-5\sin x + 3\cos x)/34$
    (c) $y = c_1 e^x + c_2 e^{-4x} + (-5\cos x - 3\sin x)/34$
    (d) $y = c_1 e^x + c_2 e^{-4x} + (5\cos x + 3\sin x)/34$
    (e) $y = c_1 e^x + c_2 e^{-4x} + (-5\cos x + 3\sin x)/34$

18. The solution of $y'' + y = \tan x$ is

    Select the correct answer.

    (a) $y = c_1 \cos x + c_2 \sin x + \cos x \ln |\sec x + \tan x|$
    (b) $y = c_1 \cos x + c_2 \sin x - \cos x \ln |\sec x + \tan x|$
    (c) $y = c_1 \cos x + c_2 \sin x + \cos x \ln |\sec x|$
    (d) $y = c_1 \cos x + c_2 \sin x - \cos x \ln |\tan x|$
    (e) $y = c_1 \cos x + c_2 \sin x - \cos x \ln |\sec x - \tan x|$

19. The solution of $x^2 y'' + xy' = 0$ is

    Select the correct answer.

    (a) $y = c_1 + c_2 x^{-1}$
    (b) $y = c_1 \ln x + c_2 x^{-1}$
    (c) $y = c_1 + c_2 \ln x$
    (d) $y = c_1 + c_2 x$
    (e) $y = c_1 + c_2 x^{-2}$

20. The solution of $x^2 y'' + 3xy' - 3y = 0$ is

    Select the correct answer.

    (a) $y = c_1 x + c_2 x^{-3}$
    (b) $y = c_1 x^{-1} + c_2 x^3$
    (c) $y = c_1 e^x + c_2 e^{-3x}$
    (d) $y = c_1 e^{-x} + c_2 e^{3x}$
    (e) $y = c_1 x + c_2 x^3$

21. The solution of $yy'' = (y')^2$ is

    Select the correct answer.

    (a) $y = c_1 e^{c_2 x}$
    (b) $y \ln(c_1) + y \ln y - y = -x + c_2$
    (c) $y = c_1 x + c_2$
    (d) $y \ln y - y = c_1 x + c_2$
    (e) $y = c_1 \ln(c_2 x)$

22. A 2-pound weight is hung on a spring and stretches it 1/2 foot. The mass spring system is then put into motion in a medium offering a damping force numerically equal to the velocity. If the mass is pulled down 4 inches from equilibrium and released, the initial value problem describing the position, $x(t)$, of the mass at time t is

    Select the correct answer.

    (a) $x'' - 16x' + 64x = 0,\ x(0) = 4,\ x'(0) = 0$
    (b) $x'' + 16x' + 64x = 0,\ x(0) = 4,\ x'(0) = 0$
    (c) $x'' - 16x' + 64x = 0,\ x(0) = 1/3,\ x'(0) = 0$
    (d) $x'' + 16x' + 64x = 0,\ x(0) = 1/3,\ x'(0) = 0$
    (e) $x'' + 64x = 16,\ x(0) = 1/3,\ x'(0) = 0$

23. In the previous problem, the solution for the position, $x(t)$, is

    Select the correct answer.

    (a) $x = (e^{8t} + 8te^{8t})/3$
    (b) $x = (e^{-8t} + 8te^{-8t})/3$
    (c) $x = \cos(8t)/12 + 1/4$
    (d) $x = (4e^{-8t} + 32te^{-8t})$
    (e) $x = (4e^{8t} - 32te^{8t})$

24. The solution of the eigenvalue problem $y'' + \lambda y = 0,\ y'(0) = 0,\ y(1) = 0$ is

    Select the correct answer.

    (a) $\lambda = n^2\pi^2/4,\ y = \cos(n\pi x/2),\ n = 1, 2, 3, \ldots$
    (b) $\lambda = n\pi/2,\ y = \cos(n\pi x/2),\ n = 1, 2, 3, \ldots$
    (c) $\lambda = n^2\pi^2/4,\ y = \sin(n\pi x/2),\ n = 1, 2, 3, \ldots$
    (d) $\lambda = (2n-1)\pi/2,\ y = \cos((2n-1)\pi x/2),\ n = 1, 2, 3, \ldots$
    (e) $\lambda = (2n-1)^2\pi^2/4,\ y = \cos((2n-1)\pi x/2),\ n = 1, 2, 3, \ldots$

25. Using power series methods, the solution of $2xy'' + y' + 2y = 0$ is

    Select the correct answer.

    (a) $y = c_0 \sum_{n=1}^{\infty}(-2)^n x^n/(n!(1\cdot 3\cdots(2n-1))) + c_1 x^{1/2}\sum_{n=1}^{\infty}(-2)^n x^n/(n!(3\cdot 5\cdots(2n+1)))$
    (b) $y = c_0 \sum_{n=1}^{\infty}(-2)^n x^n/(n!(1\cdot 3\cdots(2n-1))) + c_1 x^{1/2}[1+\sum_{n=1}^{\infty}(-2)^n x^n/(n!(3\cdot 5\cdots(2n+1)))]$
    (c) $y = c_0[1+\sum_{n=1}^{\infty}(-2)^n x^n/(n!(1\cdot 3\cdots(2n-1)))] + c_1[1+\sum_{n=1}^{\infty}(-2)^n x^n/(n!(3\cdot 5\cdots(2n+1)))]$
    (d) $y = c_0[1+\sum_{n=1}^{\infty}(-2)^n x^n/(n!(1\cdot 3\cdots(2n-1)))] + c_1 x^{1/2}[1+\sum_{n=1}^{\infty}(-2)^n x^n/(n!(3\cdot 5\cdots(2n+1)))]$
    (e) $y = [1+\sum_{n=1}^{\infty}(-2)^n x^n/(n!(1\cdot 3\cdots(2n-1)))] + x^{1/2}[1+\sum_{n=1}^{\infty}(-2)^n x^n/(n!(3\cdot 5\cdots(2n+1)))]$

26. Using power series methods, the solution of $xy'' - xy' + y = 0$ is

    Select the correct answer.

    (a) $y = c_0 x + c_1[xlnx - 1 + \sum_{n=2}^{\infty} x^n/n!]$
    (b) $y = c_0 x + c_1[xlnx - 1 + \sum_{n=1}^{\infty} x^n/(n!(n+1))]$
    (c) $y = c_0 x + c_1[xlnx + \sum_{n=2}^{\infty} x^n/(n!(n-1))]$
    (d) $y = c_0 x + c_1[xlnx + \sum_{n=1}^{\infty} x^n/(n!(n-1))]$
    (e) $y = c_0 x + c_1[xlnx - 1 + \sum_{n=2}^{\infty} x^n/(n!(n-1))]$

27. $\mathcal{L}\{t\cos t\} =$

    Select the correct answer.

    (a) $(s^2+1)/(s^2-1)$
    (b) $(s^2+1)/(s^2-1)^2$
    (c) $(s^2-1)/(s^2+1)$
    (d) $s^2/(s^2+1)$
    (e) $(s^2-1)/(s^2+1)^2$

28. $\mathcal{L}^{-1}\{1/(s^2+4s+29)\} =$

    Select the correct answer.

    (a) $\cos(5t)e^{-2t}/5$
    (b) $\cos(5t)e^{2t}/5$
    (c) $\sin(5t)e^{-2t}/5$
    (d) $\sin(5t)e^{2t}$
    (e) $\sin(5t)e^{-2t}$

29. Using the convolution theorem, we find that $\mathcal{L}^{-1}\{1/((s+1)(s^2+1))\} =$

    Select the correct answer.

    (a) $(e^{-t} + \sin t - \cos t)/2$
    (b) $(e^{t} + \sin t - \cos t)/2$
    (c) $(e^{-t} + \sin t + \cos t)/2$
    (d) $(e^{t} - \sin t - \cos t)/2$
    (e) $(e^{-t} - \sin t - \cos t)/2$

30. Using Laplace transform methods, the solution of $y' + y = 2\sin t$, $y(0) = 1$ is (Hint: the previous problem might be useful.)

    Select the correct answer.

    (a) $y = 2e^{-t} + \sin t + \cos t$
    (b) $y = e^{t} + e^{-t} - \sin t - \cos t$
    (c) $y = 2e^{-t} - \sin t - \cos t$
    (d) $y = 2e^{-t} + \sin t - \cos t$
    (e) $y = e^{t} + e^{-t} + \sin t - \cos t$

31. Using Laplace transform methods, the solution of $y'' + 2y' + y = e^{-t}$, $y(0) = 1$, $y'(0) = 0$ is

    Select the correct answer.

    (a) $y = e^{t} + te^{t} + t^2e^{t}/2$
    (b) $y = e^{-t} + te^{-t} + t^2e^{-t}/2$
    (c) $y = e^{t} - te^{t} + t^2e^{t}/2$
    (d) $y = e^{-t} - te^{-t} - t^2e^{-t}/2$
    (e) $y = e^{-t} + te^{-t} - t^2e^{-t}/2$

32. Using Laplace transform methods, the solution of $y'' + y = \delta(t-\pi)$, $y(0) = 1$, $y'(0) = 0$ is

    Select the correct answer.

    (a) $y = \sin t + \sin(t-\pi)\mathcal{U}(t-\pi)$
    (b) $y = \sin t - \cos(t-\pi)\mathcal{U}(t-\pi)$
    (c) $y = \cos t + \sin(t-\pi)\mathcal{U}(t-\pi)$
    (d) $y = \cos t + \cos(t-\pi)\mathcal{U}(t-\pi)$
    (e) $y = \cos t - \sin(t-\pi)\mathcal{U}(t-\pi)$

33. A uniform beam of length 10 has a concentrated load w_0 at $x = 5$. It is embedded at both ends. The boundary value problem for the deflections, $y(x)$, for this system is

    Select the correct answer.

    (a) $y'''' = EIw_0\delta(x-5)$, $y(0) = 0$, $y'(0) = 0$, $y(10) = 0$, $y'(10) = 0$
    (b) $y'' = EIw_0\delta(x-10)$, $y(0) = 0$, $y'(0) = 0$, $y(10) = 0$, $y'(10) = 0$
    (c) $EIy'' = w_0\delta(x-5)$, $y(0) = 0$, $y'(0) = 0$, $y(10) = 0$, $y'(10) = 0$
    (d) $EIy'''' = w_0\delta(x-5)$, $y(0) = 0$, $y'(0) = 0$, $y(10) = 0$, $y'(10) = 0$
    (e) $EIy'''' = w_0\delta(x-10)$, $y(0) = 0$, $y'(0) = 0$, $y(10) = 0$, $y'(10) = 0$

34. The solution of the previous problem is

    Select the correct answer.

    (a) $y = w_0[4(t-5)^3\mathcal{U}(t-5) + 15t^2 - 2t^3]/(24EI)$
    (b) $y = w_0[4(t-5)^3\mathcal{U}(t-5) + 15t^2 - 2t^3]/(12EI)$
    (c) $y = w_0EI[4(t-5)^3\mathcal{U}(t-5) + 15t^2 - 2t^3]/24$
    (d) $y = w_0EI[4(t-5)^3\mathcal{U}(t-5) - 15t^2 - 2t^3]/12$
    (e) $y = w_0[4(t-5)^3\mathcal{U}(t-5) - 15t^2 - 2t^3]/(24EI)$

35. The eigenvalues of the matrix $A = \begin{pmatrix} 1 & -2 \\ 1 & 4 \end{pmatrix}$ are

    Select the correct answer.

    (a) $(5 \pm \sqrt{17})/2$
    (b) $(-5 \pm \sqrt{17})/2$
    (c) 1, 2
    (d) 2, 3
    (e) 2, 2

36. The solution of $\mathbf{X}' = \begin{pmatrix} 1 & -2 \\ 1 & 4 \end{pmatrix}\mathbf{X}$ is

    Select the correct answer.

    (a) $\mathbf{X} = c_1\begin{pmatrix} 2 \\ 1 \end{pmatrix}e^{-2t} + c_2\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{-3t}$

    (b) $\mathbf{X} = c_1\begin{pmatrix} 2 \\ 1 \end{pmatrix}e^{-2t} + c_2\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{-3t}$

    (c) $\mathbf{X} = c_1\begin{pmatrix} -2 \\ -1 \end{pmatrix}e^{2t} + c_2\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{3t}$

    (d) $\mathbf{X} = c_1\begin{pmatrix} 1 \\ -2 \end{pmatrix}e^{2t} + c_2\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{3t}$

    (e) $\mathbf{X} = c_1\begin{pmatrix} 2 \\ -1 \end{pmatrix}e^{2t} + c_2\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{3t}$

37. The eigenvalues of the matrix $A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & -1 & 1 \end{pmatrix}$ are

Select the correct answer.

(a) 1, 2, 3
(b) 2, 2, 3
(c) 1, 2, 2
(d) -2, -2, 3
(e) -1, -2, 3

38. The eigenvectors of the matrix $A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & -1 & 1 \end{pmatrix}$ are

Select all that apply.

(a) $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

(b) $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

(d) $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

(e) $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

39. The solution of $\mathbf{X}' = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & -1 & 1 \end{pmatrix} \mathbf{X}$ is

Select the correct answer.

(a) $X = c_1 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{2t} + c_3 \left[\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} te^{2t} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{2t} \right]$

(b) $X = c_1 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{2t} + c_3 \left[\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} te^{2t} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{2t} \right]$

(c) $X = c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{2t} + c_3 \left[\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} te^{2t} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{2t} \right]$

(d) $X = c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} e^{2t} + c_3 \left[\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} te^{2t} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{2t} \right]$

(e) $X = c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} e^{2t} + c_3 \left[\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} te^{2t} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{2t} \right]$

40. The eigenvalues of the matrix $A = \begin{pmatrix} 1 & -2 \\ 1 & -1 \end{pmatrix}$ are

Select the correct answer.

(a) $\pm\sqrt{3}$

(b) $\pm\sqrt{3}i$

(c) ± 1

(d) $\pm 2i$

(e) $\pm i$

41. The solution of $\mathbf{X}' = \begin{pmatrix} 1 & -2 \\ 1 & -1 \end{pmatrix} \mathbf{X}$ is

Select the correct answer.

(a) $\mathbf{X} = c_1 \left[\begin{pmatrix} 2 \\ 1 \end{pmatrix} \cos t - \begin{pmatrix} 0 \\ -1 \end{pmatrix} \sin t \right] + c_2 \left[\begin{pmatrix} 2 \\ 1 \end{pmatrix} \sin t + \begin{pmatrix} 0 \\ -1 \end{pmatrix} \cos t \right]$

(b) $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{\sqrt{3}t} + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-\sqrt{3}t}$

(c) $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-t}$

(d) $\mathbf{X} = c_1 \left[\begin{pmatrix} 2 \\ 1 \end{pmatrix} \cos(\sqrt{3}t) - \begin{pmatrix} 0 \\ -1 \end{pmatrix} \sin(\sqrt{3}t) \right] +$
$c_2 \left[\begin{pmatrix} 2 \\ 1 \end{pmatrix} \sin(\sqrt{3}t) + \begin{pmatrix} 0 \\ -1 \end{pmatrix} \cos(\sqrt{3}t) \right]$

(e) $\mathbf{X} = c_1 \left[\begin{pmatrix} 2 \\ 1 \end{pmatrix} \cos(2t) - \begin{pmatrix} 0 \\ -1 \end{pmatrix} \sin(2t) \right] + c_2 \left[\begin{pmatrix} 2 \\ 1 \end{pmatrix} \sin(2t) + \begin{pmatrix} 0 \\ -1 \end{pmatrix} \cos(2t) \right]$

42. A particular solution of $\mathbf{X}' = \begin{pmatrix} 1 & -2 \\ 1 & -1 \end{pmatrix} \mathbf{X} + \begin{pmatrix} 2 \\ t \end{pmatrix}$ is

Select the correct answer.

(a) $\mathbf{X}_p = \begin{pmatrix} 2t+2 \\ -t+3 \end{pmatrix}$

(b) $\mathbf{X}_p = \begin{pmatrix} -2t+2 \\ -t+3 \end{pmatrix}$

(c) $\mathbf{X}_p = \begin{pmatrix} -2t+2 \\ -t-3 \end{pmatrix}$

(d) $\mathbf{X}_p = \begin{pmatrix} -2t+2 \\ t+3 \end{pmatrix}$

(e) $\mathbf{X}_p = \begin{pmatrix} -2t-2 \\ -t+3 \end{pmatrix}$

43. Let $A = \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$. Then $e^{At} =$

    Select the correct answer.

    (a) $I + At$
    (b) $I + At + A^2t^2/2$
    (c) $At + A^2t^2/2$
    (d) $\begin{pmatrix} e^{2t} & 0 \\ 0 & e^{-3t} \end{pmatrix}$
    (e) $\begin{pmatrix} e^{2t} - 1 & 0 \\ 0 & e^{-3t} - 1 \end{pmatrix}$

44. Using the improved Euler method with a step size of $h = 0.1$, the solution of $y' = 1 - y^2$, $y(0) = 0$ at $x = 0.1$ is

    Select the correct answer.

    (a) $y_1 = 0.095$
    (b) $y_1 = 0.995$
    (c) $y_1 = 0.95$
    (d) $y_1 = 0.00995$
    (e) $y_1 = 0.0995$

45. In the previous problem, the exact solution of the initial value problem is

    Select the correct answer.

    (a) $y = (e^{2x} - 1)/(e^{2x} + 1)$
    (b) $y = (e^{2x} + 1)/(e^{2x} - 1)$
    (c) $y = (e^{-2x} - 1)/(e^{-2x} + 1)$
    (d) $y = -(e^{-2x} + 1)/(e^{-2x} - 1)$
    (e) $y = -(e^{2x} - 1)/(e^{2x} + 1)$

46. In the previous two problems, the error in the improved Euler method at $x = 0.1$ is

    Select the correct answer.

    (a) 0.00467
    (b) 0.000168
    (c) 0.870
    (d) 0.895
    (e) 0.0897

47. Using the classical Runge–Kutta method of order 4 with a step size of $h = 0.1$, the solution of $y' = 1 - y^2$, $y(0) = 0$ at $x = 0.1$ is

    Select the correct answer.

    (a) 0.099588
    (b) 0.099668
    (c) 0.099688
    (d) 0.099768
    (e) 0.099788

48. In the previous problem, the error in the classical Runge–Kutta method at $x = 0.1$ is (Hint: see the previous five problems.)

    Select the correct answer.

    (a) 0.0008
    (b) 0.00008
    (c) 0.00000008
    (d) 0.000008
    (e) 0.0000008

49. Consider the boundary-value problem $y'' - 4y' + 3y = x$, $y(0) = 1$, $y(1) = 2$. Replace the derivatives with central differences with a step size of $h = 1/4$. The resulting equations are

    Select the correct answer.

    (a) $12y_{i+1} - 29y_i - 24y_{i-1} = x_i$
    (b) $12y_{i+1} + 17y_i + 12y_{i-1} = x_i$
    (c) $8y_{i+1} + 17y_i + 12y_{i-1} = x_i$
    (d) $8y_{i+1} - 29y_i + 24y_{i-1} = x_i$
    (e) $8y_{i+1} + 29y_i - 24y_{i-1} = x_i$

50. The solution of the system in the previous problem is

    Select the correct answer.

    (a) $y_1 = 1.228$, $y_2 = 1.482$, $y_3 = 1.753$
    (b) $y_1 = 1.228$, $y_2 = 1.646$, $y_3 = 1.753$
    (c) $y_1 = 1.126$, $y_2 = 1.646$, $y_3 = 2.903$
    (d) $y_1 = 1.126$, $y_2 = 1.786$, $y_3 = 2.903$
    (e) $y_1 = 1.016$, $y_2 = 1.786$, $y_3 = 2.903$

ANSWER KEY
Zill Differential Equations 9e Final Exam 1 Form B

1. b, d, e
2. a, e
3. a, b, e
4. c
5. d
6. e
7. b
8. c
9. a
10. a
11. b
12. d
13. c
14. c
15. d
16. a
17. e
18. b
19. c
20. a
21. a
22. d
23. b
24. e
25. d
26. e
27. e
28. c
29. a

30. d
31. b
32. c
33. d
34. a
35. d
36. e
37. b
38. a, b
39. c
40. e
41. a
42. b
43. d
44. e
45. a
46. b
47. b
48. c
49. d
50. a

1. The differential equation $y' = x^2 y + \cos x$ is

   Select all that apply.

   (a) linear
   (b) separable
   (c) exact
   (d) non-linear
   (e) Bernoulli

2. The differential equation $y' = x^2 y \ln y$ is

   Select all that apply.

   (a) linear
   (b) separable
   (c) exact
   (d) non-linear
   (e) Bernoulli

3. The differential equation $xdy - y^2 dx = 0$ is

   Select all that apply.

   (a) linear
   (b) separable
   (c) exact
   (d) non-linear
   (e) Bernoulli

4. The solution of $y' + y = x$ is

   Select the correct answer.

   (a) $y = x - 1 + ce^x$
   (b) $y = -x + 1 + ce^x$
   (c) $y = -x - 1 + ce^x$
   (d) $y = -x - 1 + ce^{-x}$
   (e) $y = x - 1 + ce^{-x}$

5. The solution of $y \cos x dx + \sin x dy = 0$ is

   Select the correct answer.

   (a) $y \sin x$
   (b) $y \cos x$
   (c) $y \sin x = c$
   (d) $y \cos x = c$
   (e) $y \cos x - c$

6. Use Euler's method to solve the initial value problem $y' = 1 - y^2$, $y(0) = 0$, with a step size of $h = 0.1$. Find approximations of $y(0.1)$ and $y(0.2)$.

7. In the previous problem, compare your answers with the exact solution, $y = (e^{2x} - 1)/(e^{2x} + 1)$. What are the errors at $x = 0.1$ and at $x = 0.2$?

8. The half life of a C-14 is 5600 years. A charcoal sample is discovered with 20% of its original C-14 remaining. How old is the sample?

9. Consider the logistic equation for a population of a city that is at 5,000, and that has a limiting size of 20,000. The intrinsic growth rate of the city is 2%. Write down the mathematical model for the population, $P(t)$, as a function of t.

10. In the previous problem, what is the population as a function of t?

11. Solve the differential equation $y'' - 6y' + 8y = 0$.

12. The solution of $y'' + 4y' - 5y = 0$ is

    Select the correct answer.

    (a) $y = c_1 e^{-x} + c_2 e^{5x}$
    (b) $y = c_1 e^{x} + c_2 e^{5x}$
    (c) $y = c_1 e^{x} + c_2 e^{-5x}$
    (d) $y = c_1 e^{-x} + c_2 x e^{-5x}$
    (e) $y = c_1 e^{2x} + c_2 e^{3x}$

13. The auxiliary equation of $y'' - 6y' + 9y = 0$ is

    Select the correct answer.

    (a) $m^2 - 6m - 9 = 0$
    (b) $m^2 - 6m + 9 = 0$
    (c) $m^2 - 7m + 9 = 0$
    (d) $m^2 - 6m + 9$
    (e) $m^2 - 7m + 9$

14. The solution of $y'' - 6y' + 9y = 0$ is

    Select the correct answer.

    (a) $y = c_1e^{3x} + c_2xe^{3x}$
    (b) $y = c_1e^{-6x} + c_2e^{-3x}$
    (c) $y = c_1e^{(7+\sqrt{13})x/2} + c_2e^{(7-\sqrt{13})x/2}$
    (d) $y = c_1e^{7x/2}\cos(\sqrt{13}x/2) + c_2e^{7x/2}\sin(\sqrt{13}x/2)$
    (e) $y = c_1e^{-3x} + c_2xe^{-3x}$

15. The solution of $y'' - 6y' + 13y = 0$ is

    Select the correct answer.

    (a) $y = c_1e^{-3x}\cos(2x) + c_2e^{3x}\sin(2x)$
    (b) $y = c_1e^{-3x}\cos(2x) + c_2e^{-3x}\sin(2x)$
    (c) $y = c_1e^{3x}\cos(2x) + c_2e^{3x}\sin(2x)$
    (d) $y = c_1e^{3x} + c_2e^{3x}$
    (e) $y = c_1\cos(2x) + c_2\sin(2x)$

16. The correct form of the particular solution of $y'' - 3y' + 2y = e^x$ is

    Select the correct answer.

    (a) $y_p = Ax^2e^x$
    (b) $y_p = Ax^3e^x$
    (c) $y_p = Ae^x$
    (d) $y_p = Axe^x$
    (e) none of the above

17. Solve the differential equation $y'' - 6y' + 13y = \cos(3x)$.

18. Solve the differential equation $y'' - 4y' + 3y = x + e^x$.

19. Solve the differential equation $y'' + y = \sec x$.

20. Solve the differential equation $x^2y'' - 4xy + 4y = 0$.

21. Solve the differential equation $x^2y'' - 3xy + 5y = 0$.

22. A 2-kilogram mass is hung on a spring. The spring constant is $k = 4N/m$ The mass spring system is then put into motion in a medium offering a damping force numerically equal to six times the velocity. If the mass is pulled down 40 centimeters from equilibrium and released, and a forcing function equal to $2e^{-3t}$ is applied to the system, the initial value problem describing the position, $x(t)$, of the mass at time t is

    Select the correct answer.

    (a) $x'' - 3x' + 2x = e^{-3t}$, $x(0) = .4$, $x'(0) = 0$
    (b) $x'' + 3x' + 2x = e^{-3t}$, $x(0) = .4$, $x'(0) = 0$
    (c) $x'' - 3x' + x/2 = e^{-3t}$, $x(0) = .4$, $x'(0) = 0$
    (d) $x'' + 3x' + x/2 = e^{-3t}$, $x(0) = 40$, $x'(0) = 0$
    (e) $x'' + 2x = 3 + e^{-3t}$, $x(0) = 40$, $x'(0) = 0$

23. In the previous problem, the solution for the position, $x(t)$, is

    Select the correct answer.

    (a) $x = 1.4e^{-t} - 1.3e^{-2t} + e^{-3t}/2$
    (b) $x = 1.4e^{-t} + 1.3e^{-2t} + e^{-3t}/2$
    (c) $x = 1.4e^{-t} - 1.3e^{-2t} - e^{-3t}/2$
    (d) $x = 1.3e^{-t} + 1.4e^{-2t} + e^{-3t}/2$
    (e) $x = 1.3e^{-t} - 1.4e^{-2t} + e^{-3t}/2$

24. The solution of the eigenvalue problem $y'' + \lambda y = 0$, $y'(0) = 0$, $y'(1) = 0$ is

    Select the correct answer.

    (a) $\lambda = n^2\pi^2$, $y = \cos(n\pi x)$, $n = 0, 1, 2, \ldots$
    (b) $\lambda = n\pi$, $y = \cos(n\pi x)$, $n = 0, 1, 2, \ldots$
    (c) $\lambda = n^2\pi^2$, $y = \sin(n\pi x)$, $n = 1, 2, 3, \ldots$
    (d) $\lambda = (2n-1)\pi/2$, $y = \cos((2n-1)\pi x/2)$, $n = 1, 2, 3, \ldots$
    (e) $\lambda = (2n-1)^2\pi^2/4$, $y = \cos((2n-1)\pi x/2)$, $n = 1, 2, 3, \ldots$

25. Use power series methods to find the solution of $xy'' - xy' + y = 0$ about $x = 0$.

26. Use power series methods to find the solution of $xy'' + 2y' - xy = 0$ about $x = 0$.

27. Use power series methods to find the solution of $x^2y'' + xy' + (x^2 - 4/9)y = 0$ about $x = 0$.

28. In the previous problem, identify the equation and write down the solution using the special function notation.

29. $\mathcal{L}\{t^2 e^{2t}\} =$

    Select the correct answer.

    (a) $1/(s-2)^2$
    (b) $2/(s-2)^2$
    (c) $2/(s-2)^3$
    (d) $1/(s-2)^3$
    (e) none of the above

30. $\mathcal{L}\{t^2 \sin t\} =$

    Select the correct answer.

    (a) $(8s^2-4)/(s^2+1)^3$
    (b) $(6s^2-2)/(s^2+1)^2$
    (c) $(6s^2+2)/(s^2+1)^2$
    (d) $(6s^2-2)/(s^2+1)^3$
    (e) $(6s^2+2)/(s^2+1)^3$

31. $\mathcal{L}^{-1}\{(s+1)/(s^2+16)\} =$

    Select the correct answer.

    (a) $\cos(4t)+\sin(4t)$
    (b) $\cos(4t)-\sin(4t)$
    (c) $\cos(4t)+\sin(4t)/16$
    (d) $\cos(4t)-\sin(4t)/4$
    (e) $\cos(4t)+\sin(4t)/4$

32. $\mathcal{L}^{-1}\{(s+1)/(s^2+6s+34)\} =$

    Select the correct answer.

    (a) $\cos(5t)e^{-3t}+\sin(5t)e^{-3t}/5$
    (b) $\cos(5t)e^{-3t}-\sin(5t)e^{-3t}$
    (c) $\cos(5t)e^{-3t}-2\sin(5t)e^{-3t}$
    (d) $\cos(5t)e^{-3t}-2\sin(5t)e^{-3t}/5$
    (e) $\cos(5t)e^{-3t}+2\sin(5t)e^{-3t}/5$

33. Using the convolution theorem, we find that $\mathcal{L}^{-1}\{1/((s-1)(s^2+4))\} =$

    Select the correct answer.

    (a) $(e^t - \sin(2t) - \cos(2t))/10$
    (b) $(2e^t - \sin(2t) - \cos(2t))/10$
    (c) $(e^t + \sin(2t) - 2\cos(2t))/10$
    (d) $(2e^t + \sin(2t) + 2\cos(2t))/10$
    (e) $(2e^t - \sin(2t) - 2\cos(2t))/10$

34. Using Laplace transform methods, the solution of $y' - y = 2\sin(2t)$, $y(0) = 0$ is (Hint: the previous problem might be useful.)

    Select the correct answer.

    (a) $y = 2(2e^t + \sin(2t) + 2\cos(2t))/5$
    (b) $y = 2(2e^t - \sin(2t) - 2\cos(2t))/5$
    (c) $y = 2(e^t - \sin(2t) - \cos(2t))/5$
    (d) $y = 2(2e^t - \sin(2t) - \cos(2t))/5$
    (e) $y = 2(e^t + \sin(2t) - 2\cos(2t))/5$

35. Using Laplace transform methods, the solution of $y'' + 3y' + 2y = e^{-t}$, $y(0) = 0$, $y'(0) = 0$ is

    Select the correct answer.

    (a) $y = -e^{-t} - te^{-t} + e^{-2t}$
    (b) $y = -e^{-t} + te^{-t} - e^{-2t}$
    (c) $y = -e^{-t} + te^{-t} + e^{-2t}$
    (d) $y = e^{-t} + te^{-t} - e^{-2t}$
    (e) $y = e^{-t} + te^{-t} + e^{-2t}$

36. Solve the initial value problem $y'' + y = t\,\mathcal{U}(t-2)$, $y(0) = 1$, $y'(0) = 0$ using Laplace transform methods.

37. Solve the system $\mathbf{X}' = A\mathbf{X}$ where $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$.

38. Find a particular solution of the system $\mathbf{X}' = A\mathbf{X} + F(t)$ where $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$ and $F(t) = \begin{pmatrix} e^t \\ 0 \end{pmatrix}$, using a fundamental matrix.

39. Solve the system $\mathbf{X}' = A\mathbf{X}$ where $A = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$.

40. Solve the system $\mathbf{X}' = A\mathbf{X}$ where $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$.

41. Let $A = \begin{pmatrix} 0 & 0 & 0 \\ 6 & 0 & 0 \\ 4 & -1 & 0 \end{pmatrix}$. Calculate e^{At}.

42. Using the improved Euler method with a step size of $h = 0.1$, the solution of $y' = 1+y^2$, $y(0) = 1$ at $x = 0.1$ is

    Select the correct answer.

    (a) $y_1 = 1.211$
    (b) $y_1 = 1.2$
    (c) $y_1 = 1.221$
    (d) $y_1 = 1.222$
    (e) $y_1 = 1.121$

43. In the previous problem, the exact solution of the initial value problem is

    Select the correct answer.

    (a) $y = \tan x$
    (b) $y = \tan(x + \pi/4)$
    (c) $y = \tan(x + \pi/2)$
    (d) $y = \sec(x + \pi/4)$
    (e) $y = \sec x$

44. In the previous two problems, the error in the improved Euler method at $x = 0.1$ is

    Select the correct answer.

    (a) 0.00105
    (b) 0.0106
    (c) 0.0011
    (d) 0.0105
    (e) 0.0165

45. Using the classical Runge–Kutta method of order 4 with a step size of $h = 0.1$, the solution of $y' = 1 + y^2$, $y(0) = 1$ at $x = 0.1$ is

    Select the correct answer.

    (a) 1.228
    (b) 1.231
    (c) 1.023
    (d) 1.218
    (e) 1.223

46. In the previous problem, the error in the classical Runge–Kutta method at $x = 0.1$ is (Hint: see the previous four problems.)

    Select the correct answer.

    (a) 0.0000001
    (b) 0.00000002
    (c) 0.00000003
    (d) 0.0000004
    (e) 0.0000005

47. What is the expected order of the local truncation error in the improved Euler method?

48. What is the expected order of the local truncation error in the classical Runge–Kutta method of order 4?

49. What is the expected order of the global truncation error in the improved Euler method?

50. What is the expected order of the global truncation error in the classical Runge–Kutta method of order 4?

ANSWER KEY
Zill Differential Equations 9e Final Exam 1 Form C

1. a, e
2. b, d
3. b, d, e (a, b, e if x is the dependent variable)
4. e
5. c
6. $y_1 = 0.1$, $y_2 = 0.199$
7. $e_1 = 0.000332$, $e_2 = 0.00162$
8. $t = \ln(0.2)/(-0.000124) = 13000$ years
9. $\frac{dP}{dt} = 0.000001P(20000 - P)$, $P(0) = 5000$
10. $P = 20000/(1 + 3e^{-0.02t})$
11. $y = c_1e^{2x} + c_2e^{4x}$
12. c
13. b
14. a
15. c
16. d
17. $y = c_1e^{3x}\cos(2x) + c_2e^{3x}\sin(2x) + (2\cos(3x) - 9\sin(3x))/170$
18. $y = c_1e^x + c_2e^{3x} + x/3 + 4/9 - xe^x/2$
19. $y = c_1\cos x + c_2\sin x + x\sin x + \cos x \ln|\cos x|$
20. $y = c_1x + c_2x^4$
21. $y = c_1x^2\cos(\ln x) + c_2x^2\sin(\ln x)$
22. b
23. e
24. a
25. $y = c_1x + c_2(x\ln x - 1 + \sum_{n=2}^{\infty} x^n/(n!(n-1)))$
26. $y = c_0x^{-1}\sum_{n=0}^{\infty} x^{2n}/(2n)! + c_1x^{-1}\sum_{n=0}^{\infty} x^{2n+1}/(2n+1)!$
27. $y = c_0\sum_{n=0}^{\infty}(-1)^n(x/2)^{2n+2/3}/(n!(\Gamma(n+5/3))) +$
    $c_1\sum_{n=0}^{\infty}(-1)^n(x/2)^{2n-2/3}/(n!(\Gamma(n+1/3)))$
28. Bessel's equation of order 2/3, $y = c_1J_{-2/3}(x) + c_2J_{2/3}(x)$
29. c

30. d

31. e

32. d

33. e

34. b

35. c

36. $y = (t - \sin(t-2) - 2\cos(t-2))\,\mathcal{U}(t-2) + \cos t$

37. $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{2t}$

38. $\mathbf{X}_p = \begin{pmatrix} te^t \\ 0 \end{pmatrix}$

39. $\mathbf{X} = c_1 e^t \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos t - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sin t \right] + c_2 e^t \left[\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cos t + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin t \right]$

40. $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{2t} + c_3 \left[\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} te^{2t} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{2t} \right]$

41. $e^{At} = \begin{pmatrix} 1 & 0 & 0 \\ 6t & 1 & 0 \\ 4t - 3t^2 & -t & 1 \end{pmatrix}$

42. d

43. b

44. a

45. e

46. c

47. $O(h^3)$

48. $O(h^5)$

49. $O(h^2)$

50. $O(h^4)$