1. Which of the following sets of functions are linearly independent on $(0, \infty)$?

   Select all that apply.

   (a) $\{1, \sin^2 x, \cos^2 x\}$
   (b) $\{1, x + 3, 2x\}$
   (c) $\{\sqrt{x}, x, x^2\}$
   (d) $\{1, \tan^2 x, \sec^2 x\}$
   (e) $\{1/x, x, \ln x\}$

2. One solution of the differential equation $y'' + y' = 0$ is $y = e^{-x}$. A second linearly independent solution is

   Select the correct answer.

   (a) $y = c$
   (b) $y = e^x$
   (c) $y = xe^x$
   (d) $y = xe^{-x}$
   (e) $y = e^{-x}$

3. Two linearly independent solutions of the differential equation $y'' - 4y' + 4y = 0$ are

   Select the correct answer.

   (a) $y_1 = e^{2x}, y_2 = e^{2x}$
   (b) $y_1 = e^{2x}, y_2 = xe^{2x}$
   (c) $y_1 = e^{2x}, y_2 = e^{-2x}$
   (d) $y_1 = e^{-2x}, y_2 = xe^{-2x}$
   (e) $y_1 = e^{-2x}, y_2 = xe^{2x}$

4. Two linearly independent solutions of the differential equation $y'' - 5y' - 6y = 0$ are

   Select the correct answer.

   (a) $y_1 = e^{6x}, y_2 = e^x$
   (b) $y_1 = e^{6x}, y_2 = xe^{-x}$
   (c) $y_1 = e^{6x}, y_2 = e^{-x}$
   (d) $y_1 = e^{-6x}, y_2 = xe^{-x}$
   (e) $y_1 = e^{-6x}, y_2 = e^x$

5. Two linearly independent solutions of the differential equation $y'' - 6y' + 25y = 0$ are
   Select the correct answer.

   (a) $y_1 = e^{3x}$, $y_2 = e^{4x}$
   (b) $y_1 = e^{-3x}$, $y_2 = e^{-4x}$
   (c) $y_1 = e^{-3x}\cos(4x)$, $y_2 = e^{-3x}\sin(4x)$
   (d) $y_1 = e^{3x}\cos(4x)$, $y_2 = e^{3x}\sin(4x)$
   (e) $y_1 = e^{4x}\cos(3x)$, $y_2 = e^{4x}\sin(3x)$

6. A particular solution of the differential equation $y'' + 3y' + 2y = 4x + 3$ is
   Select the correct answer.

   (a) $y_p = 4x + 3$
   (b) $y_p = 2x + 3/2$
   (c) $y_p = 2x - 3/2$
   (d) $y_p = 4x^2 + 3x$
   (e) $y_p = 2x - 3$

7. A particular solution of the differential equation $y'' + 2y' + y = e^x$ is
   Select the correct answer.

   (a) $y_p = 4xe^x$
   (b) $y_p = x^2 e^x / 2$
   (c) $y_p = 2x^2 e^x$
   (d) $y_p = e^x / 4$
   (e) $y_p = e^x$

8. A particular solution of the differential equation $y'' - 2y' + y = \cos x$ is
   Select the correct answer.

   (a) $y_p = \cos x$
   (b) $y_p = \sin x$
   (c) $y_p = \sin x / 2$
   (d) $y_p = \cos x / 2$
   (e) $y_p = -\sin x / 2$

9. A particular solution of the differential equation $y'' - 2y' + y = e^x$ is

Select the correct answer.

(a) $y_p = x^2 e^x / 2$

(b) $y_p = x e^x / 2$

(c) $y_p = x e^x$

(d) $y_p = e^x$

(e) $y_p = e^{-x} / 4$

10. A particular solution of the differential equation $y'' + 3y' - 4y = e^x$ is

Select the correct answer.

(a) $y_p = x^2 e^x$

(b) $y_p = x e^x / 5$

(c) $y_p = x e^x$

(d) $y_p = e^x$

(e) $y_p = e^{-x} / 5$

11. Without solving for the undetermined coefficients, the correct form of a particular solution of the differential equation $y'' + 4y = \cos(2x)$ is

Select the correct answer.

(a) $y_p = A \cos(2x)$

(b) $y_p = A \cos(2x) + B \sin(2x)$

(c) $y_p = Ax \cos(2x)$

(d) $y_p = Ax \cos(2x) + B \sin(2x)$

(e) $y_p = Ax \cos(2x) + Bx \sin(2x)$

12. Without solving for the undetermined coefficients, the correct form of a particular solution of the differential equation $y'' + 4y' + 5y = e^{-2x} \cos x$ is

Select the correct answer.

(a) $y_p = Ae^{-2x} \cos x$

(b) $y_p = Ae^{-2x} \cos x + Be^{-2x} \sin x$

(c) $y_p = Ae^{-2x} \sin x$

(d) $y_p = Axe^{-2x} \cos x + Bxe^{-2x} \sin x$

(e) $y_p = Axe^{-2x} \cos x + Be^{-2x} \sin x$

13. The auxiliary equation for the differential equation $x^2 y'' + 3xy' + y = 6$ is

    Select the correct answer.

    (a) $m^2 + 3m + 1$
    (b) $m^2 + 3m + 1 = 0$
    (c) $m^2 + 2m + 1 = 0$
    (d) $m^2 + 3m + 1 = 6$
    (e) $m^2 + 2m + 1 = 6$

14. The solution of the differential equation $x^2 y'' + 3xy' + y = 0$ is

    Select the correct answer.

    (a) $y = c_1 x^{-1} + c_2 x^{-1} \ln x$
    (b) $y = c_1 x^{-1} + c_2 x^{-2}$
    (c) $y = c_1 x^{(-3+\sqrt{5})/2} + c_2 x^{(-3-\sqrt{5})/2}$
    (d) $y = c_1 x + c_2 x \ln x$
    (e) $y = c_1 x + c_2 x^2$

15. The solution of the differential equation $x^2 y'' - 2xy' + 2y = 0$ is

    Select the correct answer.

    (a) $y = c_1 x \cos(\ln x) + c_2 x \sin(\ln x)$
    (b) $y = c_1 x^{1/2} \cos(\sqrt{3} \ln x / 2) + c_2 x^{1/2} \sin(\sqrt{3} \ln x / 2)$
    (c) $y = c_1 x^{(1+\sqrt{3})/2} + c_2 x^{(1-\sqrt{3})/2}$
    (d) $y = c_1 x + c_2 x \ln x$
    (e) $y = c_1 x + c_2 x^2$

16. A solution of the differential equation $y'' = 2x(y')^2$ is

    Select the correct answer.

    (a) $y = \ln(c_1 - x^2) + c_2$
    (b) $y = \ln((c_1 - x)/(c_1 + x)) + c_2$
    (c) $y = \ln((c_1 + x)/(c_1 - x)) + c_2$
    (d) $y = \ln(((c_1 + x)/(c_1 - x))^2) + c_2$
    (e) $y = \ln(((c_1 - x)/(c_1 + x))^2) + c_2$

17. The solution of the system of differential equations

$\frac{dx}{dt} = x + 2y$

$\frac{dy}{dt} = 4x + 3y$

is

Select the correct answer.

(a) $x = c_1 e^{5t}/2 + c_2 e^{-t}$, $y = c_1 e^{5t} + c_2 e^{-t}$

(b) $x = c_1 e^{5t}/2 - c_2 e^{-t}$, $y = c_1 e^{5t} - c_2 e^{-t}$

(c) $x = c_1 e^{5t} - c_2 e^{-t}$, $y = c_1 e^{5t} + c_2 e^{-t}$

(d) $x = c_1 e^{5t} + c_2 e^{-t}$, $y = c_1 e^{5t} + c_2 e^{-t}$

(e) $x = c_1 e^{5t}/2 - c_2 e^{-t}$, $y = c_1 e^{5t} + c_2 e^{-t}$

18. The solution of the system of differential equations

$\frac{dx}{dt} = 3x - y$

$\frac{dy}{dt} = 9x - 3y$

is

Select the correct answer.

(a) $x = (c_1 + 3c_1 t + 3c_2)/9$, $y = c_1 t + c_2$

(b) $x = (3c_1 t + 3c_2)/9$, $y = c_1 t + c_2$

(c) $x = (c_1 + 3c_1 t)/9$, $y = c_1 t + c_2$

(d) $x = (c_1 + 3c_1 t + 3c_2)$, $y = c_1 t + c_2$

(e) $x = (c_1 - 3c_1 t + 3c_2)/9$, $y = c_1 t + c_2$

19. The solution of the initial value problem

$\frac{dx}{dt} = 10x - 5y$

$\frac{dy}{dt} = 8x - 12y$,

$x(0) = 2$, $y(0) = 1$

is

Select the correct answer.

(a) $x = (35e^{8t} - e^{-10t})/18$, $y = (7e^{8t} + 2e^{-10t})/9$

(b) $x = (35e^{8t} + e^{-10t})/18$, $y = (7e^{8t} + 2e^{-10t})/9$

(c) $x = (35e^{8t} + e^{-10t})/9$, $y = (7e^{8t} + 2e^{-10t})/9$

(d) $x = (35e^{8t} - e^{-10t})/9$, $y = (7e^{8t} - 2e^{-10t})/9$

(e) $x = (35e^{8t} + e^{-10t})/18$, $y = (7e^{8t} - 2e^{-10t})/9$

20. The solution of the differential equation $y'y'' = 4$ is

Select the correct answer.

(a) $y = (8x + c_1)^{3/2}/24 + c_2$

(b) $y = (4x + c_1)^{3/2}/12 + c_2$

(c) $y = (4x + c_1)^{3/2}/24 + c_2$

(d) $y = (8x + c_1)^{1/2}/12 + c_2$

(e) $y = (8x + c_1)^{3/2}/12 + c_2$