- 1. Is the set of functions  $\{f_1(x) = 1, f_2(x) = \sin^2 x, f_3(x) = \cos^2 x\}$  linearly dependent or linearly independent on  $(-\infty, \infty)$ ?
- 2. One solution of the differential equation y'' y' = 0 is  $y = e^x$ . Use the method of reduction of order to find a second linearly independent solution.
- 3. Solve the differential equation y'' + 3y' + 2y = 0.
- 4. Solve the differential equation y'' + 4y' + 4y = 0.
- 5. Solve the differential equation y'' + 4y' + 5y = 0.
- 6. Solve the differential equation y'' + 3y' + 2y = 3x + 1.
- 7. Solve the differential equation  $y'' + 4y' + 4y = \cos(2x)$ .
- 8. Solve the differential equation  $y'' + 4y' + 5y = 2xe^x$ .
- 9. Solve the differential equation y'' + 3y' = 4x 3.
- 10. Solve the differential equation  $y'' + 3y' + 2y = 4e^{-x}$ .
- 11. Solve the differential equation  $y'' + 4y' + 4y = e^{-2x}$ .
- 12. Without solving for the undetermined coefficients, what is the correct form of the particular solution of the differential equation  $y'' + 4y' + 5y = e^{-2x} \cos x$ ?
- 13. Solve the differential equation  $x^2y'' + xy' y = 0$ .
- 14. Solve the differential equation  $x^2y'' + 3xy' + y = 0$ .
- 15. Solve the differential equation  $x^2y'' + 5xy' + 5y = 0$ .
- 16. Solve the system of differential equations

$$\frac{dx}{dt} = 4x + 7y$$
$$\frac{dy}{dt} = x - 2y.$$

17. Solve the system of differential equations

$$\frac{dx}{dt} = 4y + 3$$
$$\frac{dy}{dt} = -x + 2$$

18. Solve the initial value problem

$$\frac{dx}{dt} = 4y + 3$$
$$\frac{dy}{dt} = -x + 2$$
$$x(0) = 0, y(0) = 0$$

19. Solve the differential equation  $x^2y'' + (y')^2 = 0$ .

1.

20. Solve the differential equation  $xy'' = y' + (y')^3$ .

1. linearly dependent 2. y = c3.  $y = c_1 e^{-2x} + c_2 e^{-x}$ 4.  $y = c_1 e^{-2x} + c_2 x e^{-2x}$ 5.  $y = c_1 e^{-2x} \cos x + c_2 e^{-2x} \sin x$ 6.  $y = c_1 e^{-2x} + c_2 e^{-x} + 3x/2 - 7/4$ 7.  $y = c_1 e^{-2x} + c_2 x e^{-2x} + \sin(2x)/8$ 8.  $y = c_1 e^{-2x} \cos x + c_2 e^{-2x} \sin x + x e^x / 5 - 3 e^x / 25$ 9.  $y = c_1 + c_2 e^{-3x} + 2x^2/3 - 13x/9$ 10.  $y = c_1 e^{-2x} + c_2 e^{-x} + 4x e^{-x}$ 11.  $y = c_1 e^{-2x} + c_2 x e^{-2x} + x^2 e^{-2x}/2$ 12.  $y_p = Axe^{-2x}\cos x + Bxe^{-x}\sin x$ 13.  $y = c_1 x + c_2 x^{-1}$ 14.  $y = c_1 x^{-1} + c_2 x^{-1} \ln x$ 15.  $y = c_1 x^{-2} \cos(\ln x) + c_2 x^{-2} \sin(\ln x)$ 16.  $x = 7c_1e^{5t} - c_2e^{-3t}, y = c_1e^{5t} + c_2e^{-3t}$ 17.  $x = 2c_1 \sin(2t) - 2c_2 \cos(2t) + 2$ ,  $y = c_1 \cos(2t) + c_2 \sin(2t) - 3/4$ 18.  $x = 2 - 2\cos(2t) + 7\sin(2t)/2, y = 7\cos(2t)/4 + \sin(2t) - 3/4$ 19.  $y = c_1 x + c_1^2 \ln(x - c_1) + c_2$ 20.  $y = -\sqrt{c_1 - x^2} + c_2$ 

- 1. Is the set of functions  $\{f_1(x) = 1, f_2(x) = x + 1, f_3(x) = x^2\}$  linearly dependent or linearly independent on  $(-\infty, \infty)$ ?
- 2. One solution of the differential equation y'' y = 0 is  $y = e^x$ . Use the method of reduction of order to find a second linearly independent solution.
- 3. Solve the differential equation y'' 4y' 5y = 0.
- 4. Solve the differential equation y'' 6y' + 9y = 0.
- 5. Solve the differential equation y'' + 6y' + 13y = 0.
- 6. Solve the differential equation y'' + 4y' + 3y = 5x 2.
- 7. Solve the differential equation  $y'' + 6y' + 9y = \cos(3x)$ .
- 8. Solve the differential equation  $y'' + 2y' + 5y = 2xe^{-x}$ .
- 9. Solve the differential equation y'' 4y' = 5x 2.
- 10. Solve the differential equation  $y'' + 5y' + 4y = 2e^{-x}$ .
- 11. Solve the differential equation  $y'' 4y' + 4y = e^{2x}$ .
- 12. Without solving for the undetermined coefficients, what is the correct form of the particular solution of the differential equation  $y'' + 4y' + 13y = e^{-2x} \cos(3x)$ ?
- 13. Solve the differential equation  $x^2y'' xy' + y = 0$ .
- 14. Solve the differential equation  $x^2y'' + 3xy' + 10y = 0$ .
- 15. Solve the differential equation  $x^2y'' + 5xy' + 3y = 0$ .
- 16. Solve the system of differential equations

$$\frac{dx}{dt} = 2x + 2y$$
$$\frac{dy}{dt} = x + 3y.$$

17. Solve the system of differential equations

$$\frac{dx}{dt} = 2x + 3y + 1$$
$$\frac{dy}{dt} = -x - 2y + 4.$$

- 18. Solve the initial value problem
  - $\begin{array}{l} \frac{dx}{dt} = 3x 3y \\ \frac{dy}{dt} = 2x 2y, \\ x(0) = 0, \ y(0) = 1 \end{array}$
- 19. Solve the differential equation  $y^2y'' = y'$ .
- 20. Obtain the first four nonzero terms of a Taylor series solution, centered at x = 0, of the differential equation  $y'' + y^2 = 1$ , y(0) = 1, y'(0) = 2.

1. linearly independent 2.  $y = e^{-x}$ 3.  $y = c_1 e^{5x} + c_2 e^{-x}$ 4.  $y = c_1 e^{3x} + c_2 x e^{3x}$ 5.  $y = c_1 e^{-3x} \cos(2x) + c_2 e^{-3x} \sin(2x)$ 6.  $y = c_1 e^{-3x} + c_2 e^{-x} + 5x/3 - 26/9$ 7.  $y = c_1 e^{-3x} + c_2 x e^{-3x} + \sin(3x)/18$ 8.  $y = c_1 e^{-x} \cos(2x) + c_2 e^{-x} \sin(2x) + x e^{-x}/2$ 9.  $y = c_1 + c_2 e^{4x} - 5x^2/8 + 3x/16$ 10.  $y = c_1 e^{-4x} + c_2 e^{-x} + 2x e^{-x}/3$ 11.  $y = c_1 e^{2x} + c_2 x e^{2x} + x^2 e^{2x}/2$ 12.  $y_p = Axe^{-2x}\cos(3x) + Bxe^{-2x}\sin(3x)$ 13.  $y = c_1 x + c_2 x \ln x$ 14.  $y = c_1 x^{-1} \cos(3 \ln x) + c_2 x^{-1} \sin(3 \ln x)$ 15.  $y = c_1 x^{-1} + c_2 x^{-3}$ 16.  $x = -2c_1e^t + c_2e^{4t}, y = c_1e^t + c_2e^{4t}$ 17.  $x = -3c_1e^t - c_2e^{-t} - 14, y = c_1e^t + c_2e^{-t} + 9$ 18.  $x = 3 - 3e^t$ ,  $y = 3 - 2e^t$ 19.  $c_1y - \ln(1 + c_1y) = -c_1^2(x + c_2)$ 20.  $y = 1 + 2x - 2x^3/3 - x^4/3$ 

- 1. Which of the following sets of functions are linearly independent on  $(0, \infty)$ ? Select all that apply.
  - (a)  $\{1, \sin^2 x, \cos^2 x\}$
  - (b)  $\{1, x+3, 2x\}$
  - (c)  $\{\sqrt{x}, x, x^2\}$
  - (d)  $\{1, \tan^2 x, \sec^2 x\}$
  - (e)  $\{1/x, x, \ln x\}$
- 2. One solution of the differential equation y'' + y' = 0 is  $y = e^{-x}$ . A second linearly independent solution is

- (a) y = c
- (b)  $y = e^x$
- (c)  $y = xe^x$
- (d)  $y = xe^{-x}$
- (e)  $y = e^{-x}$
- 3. Two linearly independent solutions of the differential equation y'' 4y' + 4y = 0 are Select the correct answer.
  - (a)  $y_1 = e^{2x}, y_2 = e^{2x}$
  - (b)  $y_1 = e^{2x}, y_2 = xe^{2x}$ (c)  $y_1 = e^{2x}, y_2 = e^{-2x}$
  - (c)  $y_1 = e^{-2x}, y_2 = e^{-2x}$ (d)  $y_1 = e^{-2x}, y_2 = xe^{-2x}$
  - (e)  $y_1 = e^{-2x}, y_2 = xe^{2x}$
- 4. Two linearly independent solutions of the differential equation y'' 5y' 6y = 0 are Select the correct answer.
  - (a)  $y_1 = e^{6x}, y_2 = e^x$
  - (b)  $y_1 = e^{6x}, y_2 = xe^{-x}$
  - (c)  $y_1 = e^{6x}, y_2 = e^{-x}$
  - (d)  $y_1 = e^{-6x}, y_2 = xe^{-x}$
  - (e)  $y_1 = e^{-6x}, y_2 = e^x$

- 5. Two linearly independent solutions of the differential equation y'' 6y' + 25y = 0 are Select the correct answer.
  - (a)  $y_1 = e^{3x}, y_2 = e^{4x}$ (b)  $y_1 = e^{-3x}, y_2 = e^{-4x}$ (c)  $y_1 = e^{-3x} \cos(4x), y_2 = e^{-3x} \sin(4x)$ (d)  $y_1 = e^{3x} \cos(4x), y_2 = e^{3x} \sin(4x)$ (e)  $y_1 = e^{4x} \cos(3x), y_2 = e^{4x} \sin(3x)$
- 6. A particular solution of the differential equation y'' + 3y' + 2y = 4x + 3 is Select the correct answer.
  - (a)  $y_p = 4x + 3$ (b)  $y_p = 2x + 3/2$ (c)  $y_p = 2x - 3/2$ (d)  $y_p = 4x^2 + 3x$ (e)  $y_p = 2x - 3$
- 7. A particular solution of the differential equation  $y'' + 2y' + y = e^x$  is Select the correct answer.
  - (a)  $y_p = 4xe^x$
  - (b)  $y_p = x^2 e^x / 2$
  - (c)  $y_p = 2x^2 e^x$
  - (d)  $y_p = e^x/4$
  - (e)  $y_p = e^x$
- 8. A particular solution of the differential equation  $y'' 2y' + y = \cos x$  is Select the correct answer.
  - (a)  $y_p = \cos x$
  - (b)  $y_p = \sin x$
  - (c)  $y_p = \sin x/2$
  - (d)  $y_p = \cos x/2$
  - (e)  $y_p = -\sin x/2$

- 9. A particular solution of the differential equation  $y'' 2y' + y = e^x$  is Select the correct answer.
  - (a)  $y_p = x^2 e^x / 2$
  - (b)  $y_p = xe^x/2$
  - (c)  $y_p = xe^x$
  - (d)  $y_p = e^x$
  - (e)  $y_p = e^{-x}/4$
- 10. A particular solution of the differential equation  $y'' + 3y' 4y = e^x$  is Select the correct answer.
  - (a)  $y_p = x^2 e^x$
  - (b)  $y_p = xe^x/5$
  - (c)  $y_p = xe^x$
  - (d)  $y_p = e^x$
  - (e)  $y_p = e^{-x}/5$
- 11. Without solving for the undetermined coefficients, the correct form of a particular solution of the differential equation  $y'' + 4y = \cos(2x)$  is

- (a)  $y_p = A\cos(2x)$
- (b)  $y_p = A\cos(2x) + B\sin(2x)$
- (c)  $y_p = Ax\cos(2x)$
- (d)  $y_p = Ax\cos(2x) + B\sin(2x)$
- (e)  $y_p = Ax\cos(2x) + Bx\sin(2x)$
- 12. Without solving for the undetermined coefficients, the correct form of a particular solution of the differential equation  $y'' + 4y' + 5y = e^{-2x} \cos x$  is

- (a)  $y_p = Ae^{-2x} \cos x$
- (b)  $y_p = Ae^{-2x}\cos x + Be^{-2x}\sin x$
- (c)  $y_p = Ae^{-2x} \sin x$
- (d)  $y_p = Axe^{-2x}\cos x + Bxe^{-2x}\sin x$
- (e)  $y_p = Axe^{-2x}\cos x + Be^{-2x}\sin x$

- 13. The auxiliary equation for the differential equation  $x^2y'' + 3xy' + y = 6$  is Select the correct answer.
  - (a)  $m^2 + 3m + 1$
  - (b)  $m^2 + 3m + 1 = 0$
  - (c)  $m^2 + 2m + 1 = 0$
  - (d)  $m^2 + 3m + 1 = 6$
  - (e)  $m^2 + 2m + 1 = 6$
- 14. The solution of the differential equation  $x^2y'' + 3xy' + y = 0$  is Select the correct answer.
  - (a)  $y = c_1 x^{-1} + c_2 x^{-1} \ln x$ (b)  $y = c_1 x^{-1} + c_2 x^{-2}$ (c)  $y = c_1 x^{(-3+\sqrt{5})/2} + c_2 x^{(-3-\sqrt{5})/2}$ (d)  $y = c_1 x + c_2 x \ln x$ (e)  $y = c_1 x + c_2 x^2$
- 15. The solution of the differential equation  $x^2y'' 2xy' + 2y = 0$  is Select the correct answer.
  - (a)  $y = c_1 x \cos(\ln x) + c_2 x \sin(\ln x)$ (b)  $y = c_1 x^{1/2} \cos(\sqrt{3} \ln x/2) + c_2 x^{1/2} \sin(\sqrt{3} \ln x/2)$ (c)  $y = c_1 x^{(1+\sqrt{3})/2} + c_2 x^{(1-\sqrt{3})/2}$ (d)  $y = c_1 x + c_2 x \ln x$ (e)  $y = c_1 x + c_2 x^2$
- 16. A solution of the differential equation  $y'' = 2x(y')^2$  is Select the correct answer.
  - (a)  $y = \ln(c_1 x^2) + c_2$ (b)  $y = \ln((c_1 - x)/(c_1 + x)) + c_2$ (c)  $y = \ln((c_1 + x)/(c_1 - x)) + c_2$ (d)  $y = \ln(((c_1 + x)/(c_1 - x))^2) + c_2$ (e)  $y = \ln(((c_1 - x)/(c_1 + x))^2) + c_2$

17. The solution of the system of differential equations

$$\frac{dx}{dt} = x + 2y$$
$$\frac{dy}{dt} = 4x + 3y$$
is

Select the correct answer.

(a) 
$$x = c_1 e^{5t}/2 + c_2 e^{-t}, y = c_1 e^{5t} + c_2 e^{-t}$$
  
(b)  $x = c_1 e^{5t}/2 - c_2 e^{-t}, y = c_1 e^{5t} - c_2 e^{-t}$   
(c)  $x = c_1 e^{5t} - c_2 e^{-t}, y = c_1 e^{5t} + c_2 e^{-t}$   
(d)  $x = c_1 e^{5t} + c_2 e^{-t}, y = c_1 e^{5t} + c_2 e^{-t}$   
(e)  $x = c_1 e^{5t}/2 - c_2 e^{-t}, y = c_1 e^{5t} + c_2 e^{-t}$ 

18. The solution of the system of differential equations

$$\frac{dx}{dt} = 3x - y$$
$$\frac{dy}{dt} = 9x - 3y$$
is

Select the correct answer.

- (a)  $x = (c_1 + 3c_1t + 3c_2)/9, y = c_1t + c_2$ (b)  $x = (3c_1t + 3c_2)/9, y = c_1t + c_2$ (c)  $x = (c_1 + 3c_1t)/9, y = c_1t + c_2$ (d)  $x = (c_1 + 3c_1t + 3c_2), y = c_1t + c_2$ (e)  $x = (c_1 - 3c_1t + 3c_2)/9, y = c_1t + c_2$
- 19. The solution of the initial value problem

1

$$\frac{dx}{dt} = 10x - 5y$$
$$\frac{dy}{dt} = 8x - 12y,$$
$$x(0) = 2, y(0) =$$
is

(a) 
$$x = (35e^{8t} - e^{-10t})/18, y = (7e^{8t} + 2e^{-10t})/9$$
  
(b)  $x = (35e^{8t} + e^{-10t})/18, y = (7e^{8t} + 2e^{-10t})/9$   
(c)  $x = (35e^{8t} + e^{-10t})/9, y = (7e^{8t} + 2e^{-10t})/9$   
(d)  $x = (35e^{8t} - e^{-10t})/9, y = (7e^{8t} - 2e^{-10t})/9$   
(e)  $x = (35e^{8t} + e^{-10t})/18, y = (7e^{8t} - 2e^{-10t})/9$ 

- 20. The solution of the differential equation y'y'' = 4 is Select the correct answer.
  - (a)  $y = (8x + c_1)^{3/2}/24 + c_2$
  - (b)  $y = (4x + c_1)^{3/2}/12 + c_2$
  - (c)  $y = (4x + c_1)^{3/2}/24 + c_2$
  - (d)  $y = (8x + c_1)^{1/2}/12 + c_2$
  - (e)  $y = (8x + c_1)^{3/2}/12 + c_2$

1. c, e 2. a 3. b 4. c 5. d 6. c 7. d 8. e 9. a 10. b 11. e 12. d 13. c 14. a 15. e 16. e 17. a 18. b 19. b

20. e

- 1. The Wronskian of the functions  $e^x$  and  $e^{3x}$  is Select the correct answer.
  - (a)  $2e^{4x}$
  - (b)  $-2e^{4x}$
  - (c)  $2e^{2x}$
  - (d)  $4e^{4x}$
  - (e)  $4e^{2x}$
- 2. One solution of the differential equation y'' + y = 0 is  $y = \cos x$ . A second linearly independent solution is

- (a)  $y = \cos x$
- (b)  $y = \sin x$
- (c)  $y = e^x$
- (d)  $y = e^{-x}$
- (e)  $y = x \cos x$
- 3. Two linearly independent solutions of the differential equation y'' 4y' + 5y = 0 are Select the correct answer.
  - (a)  $y_1 = e^x, y_2 = e^{5x}$
  - (b)  $y_1 = e^{-x}, y_2 = e^{-5x}$
  - (c)  $y_1 = e^{2x} \cos x, y_2 = e^{2x} \sin x$
  - (d)  $y_1 = e^x \cos(2x), y_2 = e^x \sin(2x)$
  - (e)  $y_1 = e^{-x}\cos(2x), y_2 = e^{-2x}\sin(2x)$
- 4. Two linearly independent solutions of the differential equation y'' 5y' + 6y = 0 are Select the correct answer.
  - (a)  $y_1 = e^{2x}, y_2 = e^{3x}$ (b)  $y_1 = e^{-2x}, y_2 = e^{-3x}$ (c)  $y_1 = e^{6x}, y_2 = e^{-x}$ (d)  $y_1 = e^{-6x}, y_2 = e^x$ (e)  $y_1 = e^{-6x}, y_2 = e^{-x}$

- 5. Two linearly independent solutions of the differential equation y'' + 6y' + 9y = 0 are Select the correct answer.
  - (a)  $y_1 = e^{3x}, y_2 = xe^{3x}$ (b)  $y_1 = e^{-3x}, y_2 = xe^{-3x}$ (c)  $y_1 = e^{3x}, y_2 = e^{-3x}$ (d)  $y_1 = e^{3x} \cos(x), y_2 = e^{3x} \sin(x)$ (e)  $y_1 = e^{-3x} \cos(3x), y_2 = e^{-3x} \sin(x)$
- 6. A particular solution of the differential equation y'' + 3y' + 4y = 8x + 2 is Select the correct answer.
  - (a)  $y_p = 2x + 1$
  - (b)  $y_p = 8x + 2$
  - (c)  $y_p = 2x 1$
  - (d)  $y_p = x^2 + 3x$
  - (e)  $y_p = 2x 3$
- 7. A particular solution of the differential equation  $y'' + 4y' + 4y = e^{2x}$  is Select the correct answer.
  - (a)  $y_p = e^{2x}$ (b)  $y_p = x^2 e^{2x}/2$
  - (c)  $y_p = 2xe^{2x}$
  - (d)  $y_p = e^{2x}/4$
  - (e)  $y_p = e^{2x}/16$
- 8. A particular solution of the differential equation  $y'' 2y' + y = \sin x$  is Select the correct answer.
  - (a)  $y_p = \cos x$
  - (b)  $y_p = \sin x$
  - (c)  $y_p = \sin x/2$
  - (d)  $y_p = \cos x/2$
  - (e)  $y_p = -\cos x/2$

- 9. A particular solution of the differential equation  $y'' + 2y' + y = e^{-x}$  is Select the correct answer.
  - (a)  $y_p = x^2 e^{-x}/2$ (b)  $y_p = x e^{-x}/2$
  - (c)  $y_p = xe^{-x}$
  - (d)  $y_p = e^{-x}$
  - (e)  $y_p = e^{-x}/4$
- 10. A particular solution of the differential equation  $y'' 3y' 4y = e^{4x}$  is Select the correct answer.
  - (a)  $y_p = x^2 e^{4x}$ (b)  $y_p = x e^{4x} / 7$ (c)  $y_p = x e^{4x} / 5$ (d)  $y_p = e^{4x}$ (e)  $y_p = e^{4x} / 9$
- 11. Without solving for the undetermined coefficients, the correct form of a particular solution of the differential equation  $y'' + 9y = \sin(3x)$  is

- (a)  $y_p = A\cos(3x)$
- (b)  $y_p = Ax\cos(3x) + Bx\sin(3x)$
- (c)  $y_p = A\sin(3x)$
- (d)  $y_p = Ax\cos(3x) + B\sin(3x)$
- (e)  $y_p = A\cos(3x) + B\sin(3x)$
- 12. Without solving for the undetermined coefficients, the correct form of a particular solution of the differential equation  $y'' + 6y' + 13y = e^{-3x}\cos(2x)$  is Select the correct answer.
  - (a)  $y_p = Ae^{-3x}\cos(2x)$ (b)  $y_p = Ae^{-3x}\cos(2x) + Be^{-3x}\sin(2x)$ (c)  $y_p = Ae^{-3x}\cos(2x)$ (d)  $y_p = Axe^{-3x}\cos(2x) + Bxe^{-3x}\sin(2x)$
  - (e)  $y_p = Axe^{-3x}\cos(2x) + Be^{-3x}\sin(2x)$

- 13. The auxiliary equation for the differential equation  $x^2y'' + 5y' + 4y = 6$  is Select the correct answer.
  - (a)  $m^2 + 5m + 4$
  - (b)  $m^2 + 4m + 4 = 0$
  - (c)  $m^2 + 5m + 4 = 0$
  - (d)  $m^2 + 5m + 4 = 6$
  - (e)  $m^2 + 4m + 4 = 6$

14. The solution of the differential equation  $x^2y'' - 3xy' + 4y = 0$  is Select the correct answer.

(a) 
$$y = c_1 x + c_2 x^2$$
  
(b)  $y = c_1 x + c_2 x^3$   
(c)  $y = c_1 x^{(3+\sqrt{7})/2} + c_2 x^{(3-\sqrt{7})/2}$   
(d)  $y = c_1 x^2 + c_2 x^2 \ln x$   
(e)  $y = c_1 x + c_2 x \ln x$ 

- 15. The solution of the differential equation  $x^2y'' 5xy' + 5y = 0$  is Select the correct answer.
  - (a)  $y = c_1 x + c_2 x^5$
  - (b)  $y = c_1 x^2 \cos(\ln x) + c_2 x^2 \sin(\ln x)$
  - (c)  $y = c_1 x \cos(2 \ln x) + c_2 x \sin(2 \ln x)$
  - (d)  $y = c_1 x^{(5+\sqrt{5})/2} + c_2 x^{(5-\sqrt{5})/2}$
  - (e)  $y = c_1 e^{2x} \cos x + c_2 x e^{2x} \sin x$

16. The solution of the differential equation  $y'' = 2x(y')^2$  is Select the correct answer.

- (a)  $y = -\tan^{-1}(x/c_1)/c_1 + c_2$ (b)  $y = c_1 \int e^{x^2} dx + c_2$
- (c)  $y = c_1 e^x + c_2$
- (d)  $y = c_1 x^3 + c_2$
- (e)  $y = c_1 x^3 + c_2 x$

17. The solution of the system of differential equations

$$\frac{dx}{dt} = -6x + 5y$$
$$\frac{dy}{dt} = -5x + 4y$$
is

Select the correct answer.

(a) 
$$x = (c_1 - c_2/5)e^t + c_2te^t, y = c_1e^t + c_2te^t$$
  
(b)  $x = (c_1 - c_2)e^{-t} + c_2te^{-t}, y = c_1e^{-t} + c_2te^{-t}$   
(c)  $x = (c_1 + c_2)e^{-t} + c_2te^{-t}, y = c_1e^{-t} + c_2te^{-t}$   
(d)  $x = (c_1 + c_2/5)e^{-t} + c_2te^{-t}, y = c_1e^{-t} + c_2te^{-t}$   
(e)  $x = (c_1 - c_2/5)e^{-t} + c_2te^{-t}, y = c_1e^{-t} + c_2te^{-t}$ 

18. The solution of the initial value problem

$$\frac{dx}{dt} = -6x + 5y$$
$$\frac{dy}{dt} = -5x + 4y,$$
$$x(0) = 1/3, y(0) = 0$$
is

Select the correct answer.

(a) 
$$x = (e^t - 5te^t)/3, y = -5te^t/3$$
  
(b)  $x = (e^{-t} - te^{-t})/3, y = -te^{-t}/3$   
(c)  $x = (e^{-t} + te^{-t})/3, y = te^{-t}/3$   
(d)  $x = (e^{-t} + 5te^{-t})/3, y = 5te^{-t}/3$   
(e)  $x = (e^{-t} - 5te^{-t})/3, y = -5te^{-t}/3$ 

19. The solution of the system of differential equations

$$\frac{dx}{dt} = -6x + 5y + t$$
$$\frac{dy}{dt} = -5x + 4y + 1$$
is

(a) 
$$x = (c_1 + c_2/5)e^{-t} + c_2te^{-t} - 4t + 69/5, y = c_1e^{-t} + c_2te^{-t} - 5t - 16$$
  
(b)  $x = (c_1 + c_2/5)e^{-t} + c_2te^{-t} + 4t + 69/5, y = c_1e^{-t} + c_2te^{-t} - 5t + 16$   
(c)  $x = (c_1 + c_2/5)e^t + c_2te^t - 4t + 69/5, y = c_1e^t + c_2te^t - 5t + 16$   
(d)  $x = (c_1 - c_2/5)e^{-t} + c_2te^{-t} - 4t + 69/5, y = c_1e^{-t} + c_2te^{-t} - 5t + 16$   
(e)  $x = (c_1 + c_2/5)e^{-t} + c_2te^{-t} - 4t + 69/5, y = c_1e^{-t} + c_2te^{-t} - 5t + 16$ 

- 20. The solution of the initial value problem  $2y'' = 3y^2$ , y(0) = 1, y'(0) = 1 is Select the correct answer.
  - (a)  $y = 2/(x+2)^2$
  - (b)  $y = 4/(x+2)^3$
  - (c)  $y = 1/(x+1)^3$
  - (d)  $y = 1/(x+1)^2$
  - (e)  $y = 4/(x+2)^2$

- 1. a
- 2. b
- 3. c
- 4. a
- 5. b
- 6. c
- 7. e
- 8. d
- 9. a
- 10. c
- 11. b
- 12. d
- 13. b
- 14. d
- 15. a
- 16. a
- 17. a
- 18. e
- 19. a
- 20. e

- 1. The Wronskian of the functions x,  $x^2$  and  $e^x$  is Select the correct answer.
  - (a)  $x^2 e^x$
  - (b)  $-x^2 e^x$
  - (c)  $(x^2 2x + 2)e^x$
  - (d)  $-(x^2 2x + 2)e^x$
  - (e)  $2x^2e^x 2xe^x$
- 2. One solution of the differential equation y'' 4y = 0 is  $y = e^{2x}$ . A second linearly independent solution is

- (a)  $e^{2x}$
- (b)  $xe^{-2x}$
- (c)  $xe^{2x}$
- (d)  $e^{-2x}$
- (e)  $\cos(2x)$
- 3. Solve the differential equation y'' 2y' + y = 0.
- 4. Solve the differential equation y'' 3y' + 2y = 0.
- 5. Solve the differential equation y'' 2y' + 5y = 0.
- 6. A particular solution of the differential equation y'' + y' 2y = 8x + 2 is Select the correct answer.
  - (a)  $y_p = 4x 6$
  - (b)  $y_p = 8x + 2$
  - (c)  $y_p = 4x 1$
  - (d)  $y_p = x^2 + 3x$
  - (e)  $y_p = -4x 3$
- 7. Solve the differential equation  $y'' + 2y' + 5y = xe^{-x}$ .
- 8. Solve the differential equation y'' + 2y' = 2x 3.

- 9. A particular solution of the differential equation  $y'' + 2y' 8y = -\sin(2x)$  is Select the correct answer.
  - (a)  $y_p = \sin(2x)$
  - (b)  $y_p = \cos(2x) + \sin(2x)$
  - (c)  $y_p = (\cos(2x) 3\sin(2x))/4$
  - (d)  $y_p = (\cos(2x) + 3\sin(2x))/40$
  - (e)  $y_p = (3\cos(2x) + \sin(2x))/40$
- 10. A particular solution of the differential equation  $y'' + 2y' 8y = e^{2x}$  is Select the correct answer.
  - (a)  $y_p = xe^{2x}/6$ (b)  $y_p = xe^{2x}/2$ (c)  $y_p = x^2e^{2x}/6$ (d)  $y_p = xe^{2x}$ (e)  $y_p = e^{2x}$
- 11. A particular solution of the differential equation  $y'' + 4y = \sin(2x)$  is Select the correct answer.
  - (a)  $y_p = \sin(2x)$
  - (b)  $y_p = x \cos(2x) + \sin(2x)$
  - (c)  $y_p = -x\cos(2x)/4$
  - (d)  $y_p = \sin(2x)/4$
  - (e)  $y_p = \cos(2x)$
- 12. Without solving for the undetermined coefficients, what is the correct form of a particular solution of the differential equation  $y'' + 2y' + 5y = e^{-x} \cos(2x)$ ?
- 13. The auxiliary equation for the differential equation  $x^2y'' + 4y' + 2y = 6$  is Select the correct answer.
  - (a)  $m^2 + 3m + 2$
  - (b)  $m^2 + 4m + 2 = 6$
  - (c)  $m^2 + 3m + 2 = 6$
  - (d)  $m^2 + 4m + 2 = 0$
  - (e)  $m^2 + 3m + 2 = 0$

14. Solve the differential equation  $x^2y'' - 2xy' + 2y = x$ .

15. Solve the differential equation  $x^3y''' - 3x^2y'' + 6xy' - 6y = 0$ .

16. Consider the system of differential equations

$$\frac{\frac{dx}{dt}}{\frac{dy}{dt}} = 2x - y$$

After eliminating one variable, the auxiliary equation for the other is Select the correct answer.

(a)  $m^2 + 2m + 1 = 0$ 

(b) 
$$m^2 - 2m + 1 = 0$$

- (c)  $m^2 2m 1 = 0$
- (d)  $m^2 + 2m 1 = 0$
- (e)  $m^2 2m = 0$
- 17. The solution of the system of differential equations in the previous problem is Select the correct answer.
  - (a)  $x = c_1 e^t + c_2 e^t + c_2 t e^t$ ,  $y = c_1 e^t + c_2 t e^t$ (b)  $x = -c_1 e^{-t} + c_2 e^{-t} - c_2 t e^{-t}$ ,  $y = c_1 e^{-t} + c_2 t e^{-t}$ (c)  $x = c_1 e^t - c_2 e^t + c_2 t e^t$ ,  $y = c_1 e^t + c_2 t e^t$ (d)  $x = -c_1 e^{-t} - c_2 e^{-t} - c_2 t e^{-t}$ ,  $y = c_1 e^{-t} + c_2 t e^{-t}$ (e)  $x = c_1 e^t + 2c_2 e^{2t}$ ,  $y = c_1 e^t + c_2 e^{2t}$
- 18. Solve the system of differential equations

$$\frac{\frac{dx}{dt}}{\frac{dy}{dt}} = 6x - y$$
$$\frac{\frac{dy}{dt}}{\frac{dy}{dt}} = 5x + 2y$$

19. The solution of the differential equation  $y'' + (y')^2 + 1 = 0$  is Select the correct answer.

- (a)  $y = -\ln(\cos(c_1 x)) + c_2$ (b)  $y = -\ln(\sin(c_1 - x)) + c_2$ (c)  $y = \ln(\cos(c_1 - x)) + c_2$ (d)  $y = \ln(\sin(c_1 - x)) + c_2$ (e)  $y = \tan^{-1}(c_1 - x) + c_2$
- 20. Write down the first four nonzero terms in the Taylor expansion about x = 0 of the solution of the initial value problem  $y'' + y^2 = 1$ , y(0) = 2, y'(0) = 1.

```
1. \ c
 2. d
 3. y = c_1 e^x + c_2 x e^x
 4. y = c_1 e^x + c_2 e^{2x}
 5. y = c_1 e^x \cos(2x) + c_2 e^x \sin(2x)
 6. e
 7. y = c_1 e^{-x} \cos(2x) + c_2 e^{-x} \sin(2x) + x e^{-x}/4
 8. y = c_1 + c_2 e^{-2x} + x^2/2 - 2x
 9. d
10. a
11. c
12. y_p = Axe^{-x}\cos(2x) + Bxe^{-x}\sin(2x)
13. e
14. y = c_1 x + c_2 x^2 - x \ln x - x
15. y = c_1 x + c_2 x^2 + c_3 x^3
16. b
17. a
18. x = e^{4t}(c_1 \cos t + c_2 \sin t), y = e^{4t}(c_1(2\cos t + \sin t) + c_2(2\sin t - \cos t))
19. c
20. y = 2 + x - 3x^2/2 - 2x^3/3
```

- 1. Evaluate the Wronskian of 1, x,  $e^x$ ,  $e^{2x}$ .
- 2. One solution of the differential equation xy'' 3y' = 0 is  $y = x^4$ . A second linearly independent solution is

- (a)  $y = x^3$
- (b)  $y = x^{-3}$
- (c)  $y = x^4 \ln x$
- (d)  $y = x^3 \ln x$
- (e) y = c
- 3. Solve the differential equation y'' 4y' + 4y = 0.
- 4. Solve the differential equation y'' 5y' 6y = 0.
- 5. Solve the differential equation y'' 4y' + 5y = 0.
- 6. A particular solution of the differential equation y'' y' 2y = 4x 1 is Select the correct answer.
  - (a)  $y_p = -2x + 3/2$
  - (b)  $y_p = 2x 3$
  - (c)  $y_p = 4x 1$
  - (d)  $y_p = x^2 + 2x$
  - (e)  $y_p = -4x 3$
- 7. A particular solution of the differential equation  $y'' y' 6y = \sin(2x)$  is Select the correct answer.
  - (a)  $y_p = \sin(2x)$
  - (b)  $y_p = (\cos(2x) + \sin(2x))/52$
  - (c)  $y_p = (\cos(2x) 3\sin(2x))/52$
  - (d)  $y_p = (\cos(2x) 5\sin(2x))/52$
  - (e)  $y_p = (3\cos(2x) + \sin(2x))/52$
- 8. Solve the differential equation  $y'' 4y' + 5y = xe^x$ .
- 9. Solve the differential equation y'' 3y' = 6x 2.

- 10. A particular solution of the differential equation  $y'' + y' 2y = e^x$  is Select the correct answer.
  - (a)  $y_p = xe^x/2$ (b)  $y_p = xe^x/3$
  - (c)  $y_p = x^2 e^x/2$
  - (d)  $y_p = xe^x$
  - (e)  $y_p = e^x$
- 11. A particular solution of the differential equation  $y'' + 9y = \sin(3x)$  is Select the correct answer.
  - (a)  $y_p = \sin(3x)$ (b)  $y_p = x \cos(3x) + \sin(3x)$ (c)  $y_p = -x \cos(3x)/6$ (d)  $y_p = x \sin(3x)/4$

(e) 
$$y_p = \cos(3x)$$

- 12. Without solving for the undetermined coefficients, what is the correct form of a particular solution of the differential equation  $y'' - 4y' + 5y = e^{2x} \sin x$ ?
- 13. The auxiliary equation for the differential equation  $x^2y'' 4y' + 4y = 6$  is Select the correct answer.
  - (a)  $m^2 4m + 4$
  - (b)  $m^2 4m + 4 = 0$
  - (c)  $m^2 5m + 4 = 6$
  - (d)  $m^2 3m + 4 = 6$
  - (e)  $m^2 5m + 4 = 0$
- 14. Solve the differential equation  $x^2y'' 4xy' + 4y = 0$ .
- 15. Solve the differential equation  $x^3y''' 3x^2y'' + 6xy' 6y = 0$ .

16. Consider the system of differential equations

$$\frac{dx}{dt} = 3x - 2y$$
$$\frac{dy}{dt} = x$$

After eliminating one variable, the auxiliary equation for the other is Select the correct answer.

- (a)  $m^2 + 3m 2 = 0$
- (b)  $m^2 3m 2 = 0$
- (c)  $m^2 + 3m + 2 = 0$
- (d)  $m^2 3m + 2 = 0$
- (e)  $m^2 3m = 0$
- 17. The solution of the system of differential equations in the previous problem is Select the correct answer.
  - (a)  $x = c_1 e^{-t} c_2 e^{-2t}, y = c_1 e^{-t} + c_2 e^{-2t}$ (b)  $x = c_1 e^t - 2c_2 e^{2t}, y = c_1 e^t + c_2 e^{2t}$ (c)  $x = c_1 e^t + c_2 e^{2t}, y = c_1 e^t + c_2 e^{2t}$ (d)  $x = c_1 e^{-t} - 2c_2 e^{-2t}, y = c_1 e^{-t} + c_2 e^{-2t}$ (e)  $x = c_1 e^t + 2c_2 e^{2t}, y = c_1 e^t + c_2 e^{2t}$
- 18. The solution of the system of differential equations
  - $\frac{\frac{dx}{dt}}{\frac{dy}{dt}} = 5x 4y$ is

- (a)  $x = c_1 e^{-t} + 4c_2 e^{-4t}, y = c_1 e^{-t} + c_2 e^{-4t}$ (b)  $x = c_1 e^t - 4c_2 e^{4t}, y = c_1 e^t + c_2 e^{4t}$ (c)  $x = c_1 e^t + 4c_2 e^{4t}, y = c_1 e^t + c_2 e^{4t}$ (d)  $x = c_1 e^{-t} - 4c_2 e^{-4t}, y = c_1 e^{-t} + c_2 e^{-4t}$ (e)  $x = c_1 e^t + c_2 e^{4t}, y = c_1 e^t + c_2 e^{4t}$
- 19. Find the first four nonzero terms in the Taylor expansion about x = 0 of the solution of the initial value problem  $y'' = e^y$ , y(0) = 1, y'(0) = -1.
- 20. The solution of the differential equation  $y^2y'' = y'$  is Select the correct answer.
  - (a)  $c_1 y + \ln(c_1 y 1) = c_1^2 x + c_2$
  - (b)  $c_1 y + \ln(c_1 y + 1) = c_1^2 x + c_2$
  - (c)  $c_1 y + \ln(c_1 y 1) = c_1^2 x^2 + c_2$
  - (d)  $c_1 y + \ln(c_1 y + 1) = c_1^2 x^2 + c_2$
  - (e)  $c_1 y + \ln(c_1 y 1) = c_1^2 x^2 + c_2 x$

```
1. 4e^{3x}
 2. e
 3. y = c_1 e^{2x} + c_2 x e^{2x}
 4. y = c_1 e^{6x} + c_2 e^{-x}
 5. y = c_1 e^{2x} \cos x + c_2 e^{2x} \sin x
 6. a
 7. d
 8. y = c_1 e^{2x} \cos x + c_2 e^{2x} \sin x + (x/2 + 1/2)e^x
 9. y = c_1 + c_2 e^{3x} - x^2
10. b
11. c
12. y_p = Axe^{2x}\cos x + Be^{2x}\sin x
13. e
14. y = c_1 x + c_2 x^4
15. y = c_1 x + c_2 x^2 + c_3 x^3
16. d
17. e
18. c
19. y = 1 - x + ex^2/2 - ex^3/6
20. \ a
```

- 1. The functions  $y_1 = e^x$ ,  $y_2 = e^{2x}$ ,  $y_3 = e^{3x}$  are solutions of the differential equation y''' 6y'' + 11y' 6y = 0. Write the general solution.
- 2. One solution of the differential equation xy'' y' = 0 is  $y = x^2$ . A second linearly independent solution is

(a)  $y = x^4$ (b)  $y = x^3$ (c) y = c(d)  $y = x^{-2}$ (e)  $y = x^2 \ln x x$ 

3. The solution of the differential equation y'' - 6y' + 8y = 0 is Select the correct answer.

- (a)  $y = c_1 e^{-6x} + c_2 e^{2x}$
- (b)  $y = c_1 e^{-4x} + c_2 e^{2x}$
- (c)  $y = c_1 e^{6x} + c_2 e^{2x}$
- (d)  $y = c_1 e^{-4x} + c_2 e^{-2x}$
- (e)  $y = c_1 e^{4x} + c_2 e^{2x}$
- 4. Solve the differential equation y'' 6y' + 25y = 0.
- 5. The solution of the differential equation y'' 6y' + 9y = 0 is Select the correct answer.
  - (a)  $y = c_1 e^{-6x} + c_2 e^{3x}$ (b)  $y = c_1 e^{6x} + c_2 e^{3x}$ (c)  $y = c_1 e^{3x} + c_2 e^{-3x}$ (d)  $y = c_1 e^{3x} + c_2 x e^{3x}$ (e)  $y = c_1 e^{-3x} + c_2 x e^{-3x}$
- 6. Solve the differential equation  $y'' 4y' = xe^x$ .
- 7. Solve the differential equation y'' + 4y' = 2x 3.

- 8. A particular solution of the differential equation  $y'' 2y' + y = x^2 1$  is Select the correct answer.
  - (a)  $y_p = x^2 4x + 1$
  - (b)  $y_p = x^2 + 4x + 5$
  - (c)  $y_p = x^2 + 4x 5$
  - (d)  $y_p = x^2 1$
  - (e)  $y_p = x^2 4x + 5$
- 9. A particular solution of the differential equation  $y'' 4y = \sin(2x)$  is Select the correct answer.
  - (a)  $y_p = -\sin(2x)/8$ (b)  $y_p = -\cos(2x)/8$ (c)  $y_p = -x\sin(2x)/4$ (d)  $y_p = -x\cos(2x)/4$ (e)  $y_p = \sin(2x)$
- 10. A particular solution of the differential equation  $y'' + 2y' 3y = e^x$  is Select the correct answer.
  - (a)  $y_p = xe^x$
  - (b)  $y_p = xe^x/3$
  - (c)  $y_p = x^2 e^x / 2$
  - (d)  $y_p = xe^x/4$
  - (e)  $y_p = e^x$
- 11. A particular solution of the differential equation  $y'' + y = \sin x$  is Select the correct answer.
  - (a)  $y_p = \sin x$
  - (b)  $y_p = x \cos x + x \sin x$
  - (c)  $y_p = -x \cos x/2$
  - (d)  $y_p = x \sin x/4$
  - (e)  $y_p = \cos x$
- 12. Without solving for the undetermined coefficients, what is the correct form of a particular solution of the differential equation  $y'' - 4y' + 8y = e^{2x} \sin(2x)$ ?
- 13. Solve the differential equation  $x^2y'' 3xy' + 3y = 0$ .
- 14. Solve the differential equation  $x^2y'' 3xy' + 13y = 0$ .

- 15. The solution of  $x^2y'' + 4xy' + 2y = 0$  is  $y = c_1x^{-1} + c_2x^{-2}$ . Find a particular solution of  $x^2y'' + 4xy' + 2y = x$ .
- 16. Consider the system of differential equations

$$\frac{dx}{dt} = 3x + 2y$$
$$\frac{dy}{dt} = -x$$

After eliminating one variable, the auxiliary equation for the other is Select the correct answer.

(a) 
$$m^2 + 3m - 2 = 0$$
  
(b)  $m^2 - 3m + 2 = 0$   
(c)  $m^2 + 3m + 2 = 0$   
(d)  $m^2 - 3m - 2 = 0$   
(e)  $m^2 - 3m = 0$ 

- 17. The solution of the system of differential equations in the previous problem is Select the correct answer.
  - (a)  $x = c_1 e^t 2c_2 e^{2t}, y = c_1 e^t + c_2 e^{2t}$ (b)  $x = -c_1 e^{-t} + 2c_2 e^{-2t}, y = c_1 e^{-t} + c_2 e^{-2t}$ (c)  $x = c_1 e^{-t} + 2c_2 e^{-2t}, y = c_1 e^{-t} + c_2 e^{-2t}$ (d)  $x = -c_1 e^t - 2c_2 e^{2t}, y = c_1 e^t + c_2 e^{2t}$ (e)  $x = c_1 e^t + 2c_2 e^{2t}, y = c_1 e^t + c_2 e^{2t}$
- 18. The solution of the initial value problem y'' + yy' = 0, y(0) = 2, y'(0) = -2 is Select the correct answer.
  - (a)  $\int e^{y^2/2} dy = c_1 x + c_2$ (b)  $\int e^{-y^2/2} dy = c_1 x + c_2$ (c)  $\int e^{y^2/2} dy = c_1 x^2 + c_2 x$ (d) y = -2/(x-1)(e) y = 2/(x+1)
- 19. Solve the system of differential equations

$$\frac{dx}{dt} = 2x + y$$
$$\frac{dy}{dt} = 3x + 4y$$

20. Solve the differential equation  $x^2y'' + (y')^2 = 0$ 

1.  $y = c_1 e^x + c_2 e^{2x} + c_3 e^{3x}$ 2. c 3. e 4.  $y = c_1 e^{3x} cox(4x) + c_2 e^{3x} sin(4x)$ 5. d 6.  $y = c_1 e^{4x} + c_2 + (-x/3 - 2/9)e^x$ 7.  $y = c_1 + c_2 e^{-4x} + x^2/4 - 7x/8$ 8. b 9. a 10. d 11. c 12.  $y_p = Axe^{2x}\cos(2x) + Bxe^{2x}\sin(2x)$ 13.  $y = c_1 x + c_2 x^3$ 14.  $y = c_1 x^2 \cos(3 \ln x) + c_2 x^2 \sin(3 \ln x)$ 15.  $y_p = x/6$ 16. b 17. d 18. e 19.  $x = c_1 e^t + c_2 e^{5t}, y = -c_1 e^t + 3c_2 e^{5t}$ 20.  $y = c_1 x + c_1^2 \ln(x - c_1) + c_2$ 

- 1. The functions  $y_1 = e^x$ ,  $y_2 = e^{2x}$ ,  $y_3 = e^{3x}$  are solutions of the differential equation y''' 6y'' + 11y' 6y = 0. The function  $y_p = -2x 4$  is a solution of y''' 6y'' + 11y' 6y = 12x + 2. Write the general solution of y''' 6y'' + 11y' 6y = 12x + 2.
- 2. One solution of the differential equation y'' + 4y = 0 is  $y = \cos(2x)$ . A second linearly independent solution is

- (a)  $y = x \cos(2x)$
- (b)  $y = \sin(2x)$
- (c)  $y = -x\cos(2x)$
- (d)  $y = e^{2x}$
- (e)  $y = e^{-2x}$
- 3. The solution of the differential equation y'' 6y' + 10y = 0 is Select the correct answer.
  - (a)  $y = c_1 e^{3x} + c_2 e^x$ (b)  $y = c_1 e^{3x} \cos x + c_2 e^{3x} \sin x$ (c)  $y = c_1 e^{-3x} \cos x + c_2 e^{-3x} \sin x$ (d)  $y = c_1 e^{-3x} + c_2 e^x$ (e)  $y = c_1 e^{4x} + c_2 e^{2x}$
- 4. Solve the differential equation y'' + 4y' 5y = 0.
- 5. The solution of the differential equation y'' + 6y' + 9y = 0 is Select the correct answer.
  - (a)  $y = c_1 e^{-6x} + c_2 e^{3x}$
  - (b)  $y = c_1 e^{6x} + c_2 e^{3x}$
  - (c)  $y = c_1 e^{3x} + c_2 e^{-3x}$
  - (d)  $y = c_1 e^{3x} + c_2 x e^{3x}$
  - (e)  $y = c_1 e^{-3x} + c_2 x e^{-3x}$
- 6. Solve the differential equation  $y'' 4y' + 5y = xe^{2x}$ .
- 7. Solve the differential equation y'' 5y' = 6x 2.

- 8. A particular solution of the differential equation y'' y' 2y = 4x 1 is Select the correct answer.
  - (a)  $y_p = -4x 3$
  - (b)  $y_p = 2x 3$
  - (c)  $y_p = 4x 1$
  - (d)  $y_p = x^2 + 2x$
  - (e)  $y_p = -2x + 3/2$
- 9. A particular solution of the differential equation  $y'' y' 6y = \cos(2x)$  is Select the correct answer.
  - (a)  $y_p = \cos(2x)$
  - (b)  $y_p = (-5\cos(2x) \sin(2x))/52$
  - (c)  $y_p = (5\cos(2x) \sin(2x))/52$
  - (d)  $y_p = (5\cos(2x) + \sin(2x))/52$
  - (e)  $y_p = (-5\cos(2x) + \sin(2x))/52$
- 10. A particular solution of the differential equation  $y'' + y' 2y = e^{-2x}$  is Select the correct answer.
  - (a)  $y_p = -xe^{-2x}/2$ (b)  $y_p = x^2e^{-2x}/2$ (c)  $y_p = -xe^{-2x}/3$ (d)  $y_p = xe^{-2x}$ (e)  $y_p = e^{-2x}$
- 11. A particular solution of the differential equation  $y'' + 4y = \cos(2x)$  is Select the correct answer.
  - (a)  $y_p = \sin(2x)$
  - (b)  $y_p = x\cos(2x) + \sin(2x)$
  - (c)  $y_p = -x\cos(2x)/6$
  - (d)  $y_p = x \sin(2x)/4$
  - (e)  $y_p = \cos(2x)$
- 12. Without solving for the undetermined coefficients, what is the correct form of a particular solution of the differential equation  $y'' - 2y' + 10y = e^x \sin(3x)$ ?
- 13. Solve the differential equation  $x^2y'' 4xy' + 6y = 0$ .
- 14. Solve the differential equation  $x^2y'' 3xy' + 4y = 0$ .

- 15. The solution of  $x^2y'' 2xy' + 2y = 0$ . is  $y = c_1x + c_2x^2$ . Find a particular solution of  $x^2y'' 2xy' + 2y = x$ .
- 16. Consider the system of differential equations

$$\frac{dx}{dt} = -3x + 2y$$
$$\frac{dy}{dt} = -x$$

After eliminating the variable x, the differential equation for the variable y is Select the correct answer.

(a) 
$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 0$$
  
(b)  $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - 2y = 0$   
(c)  $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 0$   
(d)  $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 3y = 0$   
(e)  $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 3y = 0$ 

17. The solution of the system of differential equations in the previous problem is Select the correct answer.

(a) 
$$x = c_1 e^{-t} + 2c_2 e^{-2t}, y = c_1 e^{-t} + c_2 e^{-2t}$$
  
(b)  $x = c_1 e^{-t} - 2c_2 e^{-2t}, y = c_1 e^{-t} + c_2 e^{-2t}$   
(c)  $x = -c_1 e^{-t} - 2c_2 e^{-2t}, y = c_1 e^{-t} + c_2 e^{-2t}$   
(d)  $x = -c_1 e^t - 2c_2 e^{2t}, y = c_1 e^t + c_2 e^{2t}$   
(e)  $x = c_1 e^t + 2c_2 e^{2t}, y = c_1 e^t + c_2 e^{2t}$ 

18. Solve the system of differential equations

$$\frac{\frac{dx}{dt}}{\frac{dy}{dt}} = -6x + 5y$$
$$\frac{\frac{dy}{dt}}{\frac{dy}{dt}} = -x$$

19. Solve the initial value problem  $yy'' = (y')^2/2$ , y(1) = 1, y'(1) = -1

- 20. The solution of the differential equation  $yy'' = (y')^2/2$ Select the correct answer.
  - (a)  $y = c_1 x + c_2$

(b) 
$$y = c_1 x^2 + c_1 x^2 + c_2 x^2 + c_1 x^2 + c_2 x^2 + c_2 x^2 + c_2 x^2 + c_1 x^2 + c_2 x^2 + c_2 x^2 + c_1 x^2 + c_2 x^$$

- (c)  $y = c_1/x^2 + c_2$
- (d)  $y = c_1/x + c_2$
- (e)  $y = (c_1 x + c_2)^2$

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1. y = c_1 e^x + c_2 e^{2x} + c_3 e^{3x} - 2x - 4
 2. b
 3. b
 4. y = c_1 e^{-5x} + c_2 e^x
 5. e
 6. y = c_1 e^{2x} \cos x + c_2 e^{2x} \sin x + x e^{2x}
 7. y = c_1 + c_2 e^{5x} - 3x^2/5 + 4x/25
 8. e
 9. b
10. c
11. d
12. y_p = Axe^x \cos(3x) + Bxe^x \sin(3x)
13. y = c_1 x^2 + c_2 x^3
14. y = c_1 x^2 + c_2 x^2 \ln x
15. y_p = -x \ln x - x
16. c
17. a
18. x = c_1 e^{-t} + c_2 e^{-5t}, y = c_1 e^{-t} + c_2 e^{-5t}/5
19. e
20. y = (3/2 - x^2/2)^2
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