1. A spring attached to the ceiling is stretched 6 inches by a two pound weight. What is the value of the Hooke's Law spring constant, k? Include units in your answer.

2. In the previous problem, what is the mass, m? If the system is set in motion, what is the natural frequency, ω? Include units.

3. For the previous two problems, write down the initial value problem for the motion, if the mass is pulled down 1 foot from equilibrium and released.

4. Solve the initial value problem that you derived in the previous problem.

5. A vibrating spring problem has the solution $x = 4\cos 3t - 3\sin 3t$. What is the period? What is the amplitude? Write the solution as $x = A\sin(\omega t + \phi)$.

6. A spring with spring constant 12 pounds per foot is attached to the ceiling. A 4 pound weight is attached. How far does the spring stretch?

7. The mass-spring system of the previous problem is set in motion in a medium that imparts a damping force numerically equal to 2 times the velocity. The mass is pushed up three inches from the equilibrium position and given a downward velocity of 2 feet per second. Write down the initial value problem for the position, $x(t)$, as a function of time, t.

8. Solve the initial value problem from the previous problem.

9. A beam of length L is embedded horizontally at one end and free at the other end. The weight density, $w(x)$, is constant, w_0. Its flexural rigidity, EI, is given. Write down the correct boundary value problem for the deflection, $y(x)$, as a function of distance, x, from the embedded end.

10. Solve the boundary value problem of the previous problem.

11. Solve the eigenvalue problem $y'' + \lambda y = 0$, $y(0) = 0$, $y(\pi) = 0$.

12. Solve the eigenvalue problem $y'' + \lambda y = 0$, $y'(0) = 0$, $y'(\pi/2) = 0$.

13. A model for a nonlinear mass-spring system is $m\frac{d^2x}{dt^2} + kx^3 = 0$, where x is displacement and m is mass. Solve this for dx/dt by multiplying by dx/dt and integrating.

14. Write down the differential equation for the angle θ between a pendulum of length l and a vertical line.

15. Linearize the equation of the previous problem and solve the resulting differential equation.

16. A rocket with mass m is launched vertically upward from the surface of the earth with a velocity, v_0. Let $y(t)$ be the distance of the rocket from the center of the earth at time t. Write down the initial value problem for the position, $y(t)$, assuming that the only force acting on the rocket is gravity, which is inversely proportional to the square of the distance from the center of the earth.

17. Solve the differential equation of the previous problem for the velocity by multiplying by the velocity and integrating.

18. In the previous problem, what is the escape velocity?

19. A 10 foot chain of weight density 1 pound per foot is coiled on the ground. One end is pulled upward by a force of 10 pounds. Write down the differential equation for the height, $x(t)$, of the end of the chain above the ground at time t.

20. Solve the differential equation in the previous problem.

ANSWER KEY
Zill Differential Equations 9e Chapter 5 Form A

1. $k = 4\text{lbs/ft}$
2. $m = 1/16$ slug, $\omega = 8\text{sec}^{-1}$
3. $\frac{d^2x}{dt^2} + 64x = 0$, $x(0) = 1$, $\frac{dx}{dt}(0) = 0$
4. $x = \cos(8t)$
5. $T = 2\pi/3$, $A = 5$, $x = 5\sin(3t + \phi)$, where $\phi = \tan^{-1}(-4/3) + \pi$
6. $s = 1/3$ ft
7. $\frac{d^2x}{dt^2} + 16\frac{dx}{dt} + 96x = 0$, $x(0) = -1/4$, $\frac{dx}{dt}(0) = 2$
8. $x = -e^{-8t}\cos(4\sqrt{2}t)/4$
9. $\frac{d^4y}{dx^4} = w_0/EI$, $y(0) = 0$, $y'(0) = 0$, $y''(L) = 0$, $y'''(L) = 0$
10. $y = w_0(L^2x^2/4 - Lx^3/6 + x^4/24)/EI$
11. $\lambda_n = n^2$, $y_n = c_n\sin(nx)$, $n = 1, 2, 3, \ldots$
12. $\lambda_n = 4n^2$, $y_n = c_n\cos(2nx)$, $n = 1, 2, 3, \ldots$
13. $\frac{dx}{dt} = \sqrt{c - kx^4/(2m)}$
14. $\frac{d^2\theta}{dt^2} + g\sin\theta/l = 0$
15. $\theta = c_1\cos(\sqrt{g/l}t) + c_2\sin(\sqrt{g/l}t)$
16. $\frac{d^2y}{dt^2} = -k/(my^2)$, $y(0) = R$(radius of earth), $\frac{dy}{dt}(0) = v_0$
17. $v = \frac{dy}{dt} = \sqrt{v_0^2 - 2k(1/R - 1/y)/m}$
18. $v_0 = \sqrt{2k/(mR)}$
19. $x\frac{dv}{dt} + v\frac{dx}{dt} = 320 - 32x$
20. $x = 15 - 15(1 - 4\sqrt{5}t/15)^2$, assuming $x(0) = 0$

1. A spring attached to the ceiling is stretched 5 centimeters by a two kilogram mass. What is the value of the Hooke's Law spring constant, k? Include units in your answer.

2. In the previous problem, if the system is set in motion, what is the natural frequency, ω? Include units.

3. For the previous two problems, write down the initial value problem for the motion, if the mass is pulled down 6 centimeters from equilibrium and released.

4. Solve the initial value problem that you derived in the previous problem.

5. A vibrating spring problem has the solution $x = 5\cos 4t + 12\sin 4t$. What is the period? What is the amplitude? Write the solution as $x = A\sin(\omega t + \phi)$.

6. A spring with spring constant 8 Newtons per meter is attached to the ceiling. A 2 kilogram mass is attached. How far does the spring stretch?

7. The mass-spring system of the previous problem is set in motion in a medium that imparts a damping force numerically equal to 8 times the velocity. The mass is started from the equilibrium position with a downward velocity of 4 centimeters per second. An external force $f(t) = 12\sin t$ Newtons is applied to the system. Write down the initial value problem for the position, $x(t)$, as a function of time, t.

8. Solve the initial value problem from the previous problem.

9. A beam of length L is embedded horizontally at one end and simply supported at the other end. The weight density, $w(x)$, is constant, w_0. Its flexural rigidity, EI, is given. Write down the correct boundary value problem for the deflection, $y(x)$ as a function of distance, x, from the embedded end.

10. Solve the boundary value problem of the previous problem.

11. A thin vertical homogeneous column of length L is subjected to a constant load, P. It is hinged at both ends. Its flexural rigidity, EI, is given. Write down the correct boundary value problem for the deflection, $y(x)$, of this column as a function of distance, x, from the top.

12. Solve the boundary value problem in the previous problem.

13. A model for a nonlinear mass-spring system is $\frac{d^2x}{dt^2}+x+x^3 = 0$, where x is displacement and m is mass. Solve this for dx/dt by multiplying by dx/dt and integrating.

14. Write down the differential equation for the angle θ between a pendulum of length 2 feet and a vertical line.

15. Linearize the equation of the previous problem and solve the resulting initial value problem if the pendulum is pulled to an angle of 30° and released.

16. A rocket with mass m is launched vertically upward from the surface of the earth with a velocity of 5 miles per second. Let $y(t)$ be the distance of the rocket from the center of the earth at time t. Write down the initial value problem for the position, $y(t)$, assuming that the only force acting on the rocket is gravity, which is inversely proportional to the square of the distance from the center of the earth.

17. Solve the differential equation of the previous problem for the velocity by multiplying by the velocity and integrating. Assume that the radius of the earth is $R = 4000$ miles.

18. In the previous problem, what value of k implies that the rocket will continue upward forever?

19. A 20 foot chain of weight density 1 pound per foot is coiled on the ground. One end is pulled upward by a force of 10 pounds. Write down the differential equation for the height, $x(t)$, of the end of the chain above the ground at time t.

20. Solve the differential equation in the previous problem.

ANSWER KEY

Zill Differential Equations 9e Chapter 5 Form B

1. $k = 392$ Newtons per meter
2. $\omega = 14\text{sec}^{-1}$
3. $\frac{d^2x}{dt^2} + 196x = 0$, $x(0) = 0.06$, $\frac{dx}{dt}(0) = 0$
4. $x = 0.06\cos(14t)$
5. $T = \pi/2$, $A = 13$, $x = 13\sin(4t + \phi)$, where $\phi = \tan^{-1}(5/12)$
6. $s = 2.45$ meters
7. $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 6\sin t$, $x(0) = 0$, $\frac{dx}{dt}(0) = 0.04$
8. $x = (24e^{-2t} + 31te^{-2t} - 24\cos t + 18\sin t)/25$
9. $y'''' = w_0/(EI)$, $y(0) = 0$, $y'(0) = 0$, $y(L) = 0$, $y''(L) = 0$
10. $y = w_0(L^2x^2/16 - 5Lx^3/48 + x^4/24)/(EI)$
11. $y'' + Py/(EI) = 0$, $y(0) = 0$, $y(L) = 0$
12. $P_n = EI(n\pi/L)^2$, $y_n = c_n\sin(n\pi x/L)$
13. $\frac{dx}{dt} = \sqrt{c - x^2 - x^4/2}$
14. $\frac{d^2\theta}{dt^2} + 16\sin\theta = 0$
15. $\theta = \pi\cos(4t)/6$
16. $m\frac{d^2y}{dx^2} = -k/y^2$
17. $\frac{dy}{dt} = \sqrt{25 + 2k(1/y - 1/4000)/m}$
18. $k \leq 50000m$
19. $x\frac{dv}{dt} + v\frac{dx}{dt} + 32x = 320$, where $v = \frac{dx}{dt}$
20. $x = 15 - 15(1 - 4\sqrt{5}t/15)^2$ assuming $x(0) = 0$

1. A spring attached to the ceiling is stretched one foot by a four pound weight. The value of the Hooke's Law spring constant, k, is

   Select the correct answer.

   (a) 4 pounds per foot
   (b) 1/4 pound per foot
   (c) 1/4 foot-pound
   (d) 4 foot-pounds
   (e) none of the above

2. In the previous problem, if the mass is set in motion, the natural frequency, ω, is

   Select the correct answer.

   (a) $4\sqrt{2}\text{sec}$
   (b) $4\sqrt{2}\text{sec}^{-1}$
   (c) 32 sec
   (d) 32 sec^{-1}
   (e) sec^{-1}

3. In the previous two problems, the correct differential equation for the position, $x(t)$, of the mass at a function of time, t, is

   Select the correct answer.

   (a) $\frac{d^2x}{dt^2} + x/4 = 0$
   (b) $\frac{d^2x}{dt^2} + 2x = 0$
   (c) $\frac{d^2x}{dt^2} + 4x = 0$
   (d) $\frac{d^2x}{dt^2} + 8x = 0$
   (e) $\frac{d^2x}{dt^2} + 32x = 0$

4. If the mass in the previous problem is pulled down two feet and released, the solution for the position is

   Select the correct answer.

   (a) $x = 2\cos(4\sqrt{2}t) + 2\sin(4\sqrt{2}t)$
   (b) $x = 2\sin(4\sqrt{2}t)$
   (c) $x = 2\cos(4\sqrt{2}t)$
   (d) $x = 2\sin(4t)$
   (e) $x = 2\cos(4t)$

5. The solution of a vibrating spring problem is $x = 4\cos t - 3\sin t$. The amplitude is

   Select the correct answer.

   (a) 7
   (b) 1
   (c) 25
   (d) 5
   (e) -1

6. In the previous problem, the function x can be written as

   Select the correct answer.

   (a) $x = 3\sin(t+\phi)$, where $\tan\phi = -4/3$
   (b) $x = 4\sin(t+\phi)$, where $\tan\phi = -4/3$
   (c) $x = 5\sin(t+\phi)$, where $\tan\phi = -4/3$
   (d) $x = 5\sin(t+\phi)$, where $\tan\phi = -3/4$
   (e) $x = 4\sin(t+\phi)$, where $\tan\phi = -3/4$

7. The moment of inertia of a cross section of a beam is I, and the Young's modulus is E. Its flexural rigidity is

   Select the correct answer.

   (a) E/I
   (b) EI
   (c) I/E
   (d) $EI\kappa$, where κ is the curvature
   (e) κ/EI, where κ is the curvature

8. A beam of length L is simply supported at one end and free at the other end. The weight density is constant, $w(x) = w_0$. Let $y(x)$ represent the deflection at point x. The correct form of the boundary value problem for this beam is

   Select the correct answer.

   (a) $\frac{d^2y}{dx^2} = w_0/EI$, $y(0) = 0$, $y'''(L) = 0$
   (b) $\frac{d^4y}{dx^4} = w_0/EI$, $y(0) = 0$, $y''(0) = 0$, $y''(L) = 0$, $y'''(L) = 0$
   (c) $\frac{d^2y}{dx^2} = w_0EI$, $y(0) = 0$, $y'''(L) = 0$
   (d) $\frac{d^4y}{dx^4} = w_0/EI$, $y(0) = 0$, $y''(0) = 0$, $y''(L) = 0y'''(L) = 0$
   (e) none of the above

9. The eigenvalue problem $y'' + \lambda y = 0$, $y(0) = 0$, $y(\pi) = 0$ has the solution

   Select the correct answer.

   (a) $y = \sin(nx)$, $\lambda = n^2$, $n = 1, 2, 3, \ldots$
   (b) $y = \cos(nx)$, $\lambda = n^2$, $n = 1, 2, 3, \ldots$
   (c) $y = \sin(nx)$, $\lambda = n$, $n = 1, 2, 3, \ldots$
   (d) $y = \cos(nx)$, $\lambda = n$, $n = 1, 2, 3, \ldots$
   (e) none of the above

10. The eigenvalue problem $y'' + \lambda y = 0$, $y'(0) = 0$, $y'(\pi/2) = 0$ has the solution

    Select the correct answer.

    (a) $y = \sin(2nx)$, $\lambda = 4n^2$, $n = 1, 2, 3, \ldots$
    (b) $y = \cos(2nx)$, $\lambda = 4n^2$, $n = 1, 2, 3, \ldots$
    (c) $y = \sin(2nx)$, $\lambda = 2n$, $n = 1, 2, 3, \ldots$
    (d) $y = \cos(2nx)$, $\lambda = 2n$, $n = 1, 2, 3, \ldots$
    (e) none of the above

11. The boundary value problem $T\frac{d^2y}{dx^2} + \rho\omega^2 y = 0$, $y(0) = 0$, $y(L) = 0$ is a model of the shape of a rotating string. Suppose T and ρ are constants. The critical angular rotation speeds, $\omega = \omega_n$, for which there exist non-trivial solutions are

    Select the correct answer.

    (a) $\omega_n = (T/\rho)(\frac{n\pi}{L})^2$
    (b) $\omega_n = \sqrt{T/\rho}\frac{n\pi}{L}$
    (c) $\omega_n = \sqrt{T/\rho}\frac{n\pi}{2L}$
    (d) $\omega_n = (T/\rho)(\frac{n\pi}{2L})^2$
    (e) $\omega_n = \sqrt{\rho/T}\frac{n\pi}{L}$

12. In the previous problem the corresponding non-trivial solutions for y are

    Select the correct answer.

    (a) $y = \sin(\omega_n x)$
    (b) $y = \cos(\omega_n x)$
    (c) $y = \sin({\omega_n}^2 x)$
    (d) $y = \cos({\omega_n}^2 x)$
    (e) none of the above

13. A pendulum of length l hangs from the ceiling. Let g represent the gravitational acceleration. The correct linearized differential equation for the angle, θ, that the swinging pendulum makes with the vertical is

    Select the correct answer.

    (a) $\frac{d\theta}{dt} + g\theta/l = 0$
    (b) $\frac{d^2\theta}{dt^2} + g\theta/l = 0$
    (c) $\frac{d\theta}{dt} + l\theta/g = 0$
    (d) $\frac{d^2\theta}{dt^2} + l\theta/g = 0$
    (e) $\frac{d^2\theta}{dt^2} - g\theta/l = 0$

14. The solution of the differential equation of the previous problem is

    Select the correct answer.

    (a) $\theta = ce^{-gt/l}$
    (b) $\theta = ce^{-lt/g}$
    (c) $\theta = ce^{gt/l}$
    (d) $\theta = c_1 \cos(gt/l) + c_2 \sin(gt/l)$
    (e) $\theta = c_1 \cos(\sqrt{g/l}t) + c_2 \sin(\sqrt{g/l}t)$

15. A rocket with mass m is launched vertically upward from the surface of the earth with a velocity v_0. Let $y(t)$ be the distance of the rocket from the center of the earth at time t. Assuming that the only force acting on the rocket is gravity, which is inversely proportional to the square of the distance from the center of the earth, the correct differential equation for the position of the rocket is

    Select the correct answer.

    (a) $\frac{dy}{dt} = -k/y^2$
    (b) $\frac{d^2y}{dt^2} = -k/y^2$
    (c) $m\frac{dy}{dt} = -k/y^2$
    (d) $m\frac{d^2y}{dt^2} = -k/y^2$
    (e) none of the above

16. In the previous problem, the solution for the velocity, v, is

    Select the correct answer.

    (a) $v = -k/y^2$
    (b) $v = k/y^2$
    (c) $v = \sqrt{c + 2k/(my)}$
    (d) $v = c + 2k/(my)$
    (e) none of the above

17. In the previous problem, if $y(0) = R$, what is the escape velocity?

    Select the correct answer.

    (a) $v_0 = \sqrt{2k/(mR)}$
    (b) $v_0 = 2k/(mR)$
    (c) $v_0 = 2m/(kR)$
    (d) $v_0 = \sqrt{2m/(kR)}$
    (e) none of the above

18. A 10 foot chain of weight density 2 pounds per foot is coiled on the ground. One end is pulled upward by a force of 10 pounds. The correct differential equation for the height, $x(t)$, of the end of the chain above the ground at time t is

    Select the correct answer.

    (a) $x\frac{dx}{dt}\frac{d^2x}{dt^2} + \frac{dx}{dt}^2 + 32x = 160$
    (b) $\frac{dx}{dt}\frac{d^2x}{dt^2} + \frac{dx}{dt}^2 + 32x = 160$
    (c) $x\frac{d^2x}{dt^2} + \frac{dx}{dt}^2 + 32x = 160$
    (d) $x\frac{dx}{dt}\frac{d^2x}{dt^2} + \frac{dx}{dt} + 32x = 160$
    (e) $x\frac{dx}{dt}\frac{d^2x}{dt^2} + \frac{dx}{dt}^2 = 160$

19. The solution of the problem given in the previous problem is

    Select the correct answer.

    (a) $x = 15 - 15(1 - 4\sqrt{10}t/15)^2$
    (b) $x = 15/2 - 15(1 - 4\sqrt{10}t/15)/2$
    (c) $x = 15/2 - 15(1 - 2\sqrt{10}t/15)^2/2$
    (d) $x = 15/2 - 15(1 - 4\sqrt{10}t/5)^2/2$
    (e) $x = 15 - 15(1 - 4\sqrt{10}t/15)$

20. The differential equation $\frac{d^2x}{dt^2} + x\cos x = 0$ is a model for an undamped spring-mass system with a nonlinear forcing function. The initial conditions are $x(0) = 0.1$, $x'(0) = -0.1$. The solution of the linearized system is

    Select the correct answer.

    (a) $x = 0.1\cos t$
    (b) $x = 0.1\sin t$
    (c) $x = 0.1\cos t - 0.1\sin t$
    (d) $x = 0.1e^{-t}$
    (e) $x = 0.1e^{t}$

ANSWER KEY
Zill Differential Equations 9e Chapter 5 Form C

1. a
2. b
3. e
4. c
5. d
6. c
7. b
8. e
9. a
10. b
11. b
12. e
13. b
14. e
15. d
16. c
17. a
18. c
19. d
20. c

1. A spring attached to the ceiling is stretched 2.45 meters by a four kilogram mass. The value of the Hooke's Law spring constant, k is

   Select the correct answer.

   (a) 1/4 meter-Newton
   (b) 4 meter-Newtons
   (c) 1/4 Newton per meter
   (d) 16 Newtons per meter
   (e) none of the above

2. In the previous problem, if the mass is set in motion, the natural frequency, ω, is

   Select the correct answer.

   (a) 2sec
   (b) 2sec^{-1}
   (c) 4 sec
   (d) 4 sec^{-1}
   (e) 16 sec^{-1}

3. In the previous two problems, if the mass is set into motion in a medium that imparts a damping force numerically equal to 16 times the velocity, the correct differential equation for the position, $x(t)$, of the mass at a function of time, t, is

   Select the correct answer.

   (a) $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + x/4 = 0$
   (b) $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 2x = 0$
   (c) $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 0$
   (d) $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 8x = 0$
   (e) $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 32x = 0$

4. If the mass in the previous problem is pulled down two centimeters and released, the solution for the position is

   Select the correct answer.

   (a) $x = 0.02e^{-2t} + 0.04te^{-2t}$
   (b) $x = 2e^{-2t} + 4te^{-2t}$
   (c) $x = 0.02e^{2t} - 0.04te^{2t}$
   (d) $x = e^{-2t}\sin t$
   (e) $x = 0.02e^{-2t}\cos t$

5. The solution of a vibrating spring problem is $x = 5\cos t - 12\sin t$. The amplitude is

   Select the correct answer.

   (a) 17
   (b) -7
   (c) 7
   (d) 13
   (e) 60

6. In the previous problem, the function x can be written as

   Select the correct answer.

   (a) $x = 5\sin(t + \phi)$, where $\tan\phi = -5/12$
   (b) $x = 12\sin(t + \phi)$, where $\tan\phi = -5/12$
   (c) $x = 12\sin(t - \phi)$, where $\tan\phi = 5/12$
   (d) $x = 13\sin(t + \phi)$, where $\tan\phi = 5/12$
   (e) $x = 13\sin(t + \phi)$, where $\tan\phi = -5/12$

7. A beam of length L is simply supported at the left end and embedded at the right end. The weight density is constant, $w(x) = w_0$. Let $y(x)$ represent the deflection at point x. The correct form of the boundary value problem for this beam is

   Select the correct answer.

   (a) $\frac{d^2y}{dx^2} = w_0/EI$, $y(0) = 0$, $y'(L) = 0$
   (b) $\frac{d^4y}{dx^4} = w_0/EI$, $y(0) = 0$, $y''(0) = 0$, $y(L) = 0$, $y'(L) = 0$
   (c) $\frac{d^2y}{dx^2} = w_0EI$, $y(0) = 0$, $y''(L) = 0$
   (d) $\frac{d^4y}{dx^4} = w_0/EI$, $y(0) = 0$, $y''(0) = 0$, $y''(L) = 0y'''(L) = 0$
   (e) none of the above

8. The solution of the boundary value problem in the previous problem is

   Select the correct answer.

   (a) $y = w_0/EI(L^3x/48 - Lx^3/16 + x^4/24)$
   (b) $y = w_0/EI(x^2/2 - Lx)$
   (c) $y = w_0EI(L^3x/48 - Lx^3/16 + x^4/24)$
   (d) $y = w_0EI(x^2/2 - Lx)$
   (e) none of the above

9. The boundary value problem $r\frac{d^2u}{dr^2} + 2\frac{du}{dr} = 0$, $u(a) = u_0$, $u(b) = u_1$ is a model for the temperature distribution between two concentric spheres of radii a and b, with $a < b$. The solution of this problem is

   Select the correct answer.

   (a) $u = c_2 + c_1/r$, where $c_1 = ab(u_1 - u_0)/(b-a)$ and $c_2 = (u_1 b - u_0 a)/(b-a)$
   (b) $u = c_2 + c_1/r$, where $c_1 = (u_1 b - u_0 a)/(b-a)$ and $c_2 = ab(u_1 - u_0)/(b-a)$
   (c) $u = c_2 - c_1/r$, where $c_1 = ab(u_1 - u_0)/(b-a)$ and $c_2 = (u_1 b - u_0 a)/(b-a)$
   (d) $u = c_2 - c_1/r$, where $c_1 = (u_1 b - u_0 a)/(b-a)$ and $c_2 = ab(u_1 - u_0)/(b-a)$
   (e) none of the above

10. The eigenvalue problem $y'' + \lambda y = 0$, $y(0) = 0$, $y(\pi/2) = 0$ has the solution

    Select the correct answer.

    (a) $y = \sin(nx)$, $\lambda = 4n^2$, $n = 1, 2, 3, \ldots$
    (b) $y = \cos(nx)$, $\lambda = 4n^2$, $n = 1, 2, 3, \ldots$
    (c) $y = \sin(nx)$, $\lambda = 2n$, $n = 1, 2, 3, \ldots$
    (d) $y = \cos(nx)$, $\lambda = 2n$, $n = 1, 2, 3, \ldots$
    (e) none of the above

11. The eigenvalue problem $y'' + \lambda y = 0$, $y(0) = 0$, $y(1) = 0$ has the solution

    Select the correct answer.

    (a) $y = \sin(n\pi x)$, $\lambda = (n\pi)^2$, $n = 1, 2, 3, \ldots$
    (b) $y = \cos(n\pi x)$, $\lambda = (n\pi)^2$, $n = 1, 2, 3, \ldots$
    (c) $y = \sin(n\pi x)$, $\lambda = n\pi$, $n = 1, 2, 3, \ldots$
    (d) $y = \cos(n\pi x)$, $\lambda = n\pi$, $n = 1, 2, 3, \ldots$
    (e) none of the above

12. A pendulum of length 16 feet hangs from the ceiling. Let $g = 32$ represent the gravitational acceleration. The correct linearized differential equation for the angle, θ, that the swinging pendulum makes with the vertical is

    Select the correct answer.

    (a) $\frac{d\theta}{dt} + 2\theta = 0$
    (b) $\frac{d^2\theta}{dt^2} + 2\theta = 0$
    (c) $\frac{d\theta}{dt} + \theta/2 = 0$
    (d) $\frac{d^2\theta}{dt^2} + \theta/2 = 0$
    (e) $\frac{d^2\theta}{dt^2} - 2\theta = 0$

13. The solution of the differential equation of the previous problem is

    Select the correct answer.

    (a) $\theta = ce^{2t}$
    (b) $\theta = ce^{-2t}$
    (c) $\theta = ce^{-t/2}$
    (d) $\theta = c_1 \cos(2t) + c_2 \sin(2t)$
    (e) $\theta = c_1 \cos(\sqrt{2}t) + c_2 \sin(\sqrt{2}t)$

14. A rocket is launched vertically upward with a speed v_0. Take the upward direction as positive and let the mass be m. Assume that the only force acting on the rocket is gravity, which is inversely proportional to the square of the distance from the center of the earth. Let $y(t)$ be the distance from the center of the earth at time, t. The correct differential equation for the position of the rocket is

    Select the correct answer.

    (a) $m\frac{dy}{dt} = v_0 - k/y^2$
    (b) $m\frac{d^2y}{dt^2} = -k/y^2$
    (c) $\frac{dy}{dt} = v_0 - k/y^2$
    (d) $\frac{d^2y}{dt^2} = -k/y^2$
    (e) none of the above

15. In the previous problem, the solution for the velocity, v, is

    Select the correct answer.

    (a) $v = v_0 - k/y^2$
    (b) $v = v_0 + k/y^2$
    (c) $v = c + 2k/(my)$
    (d) $v = \sqrt{c + 2k/(my)}$
    (e) none of the above

16. In the previous problem, if $y(0) = R$, the escape velocity is

    Select the correct answer.

    (a) $v_0 = 2k/(mR)$
    (b) $v_0 = 2R/(mk)$
    (c) $v_0 = 2m/(kR)$
    (d) $v_0 = \sqrt{2m/(kR)}$
    (e) none of the above

17. The differential equation $\frac{d^2x}{dt^2} + xe^{0.01x} = 0$ is a model for an undamped spring-mass system with a nonlinear restoring force. The initial conditions are $x(0) = 0$, $x'(0) = 1$. The solution of the linearized system is

    Select the correct answer.

    (a) $x = (e^t + e^{-t})/2$
    (b) $x = (e^t - e^{-t})/2$
    (c) $x = \cos t$
    (d) $x = \sin t$
    (e) $x = \cos t - \sin t$

18. The initial value problem $(L - x)\frac{d^2x}{dt^2} - (\frac{dx}{dt})^2 = Lg$, $x(0) = 0$, $x'(0) = 0$ is a model of a chain of length L falling to the ground, where $x(t)$ represents the length of chain on the ground at time t. The solution for $v = \frac{dx}{dt}$ in terms of x is

    Select the correct answer.

    (a) $v = \sqrt{Lg(L^2 - (L - x)^2)}$
    (b) $v = (Lg(L^2 + (L - x)^2))/(L - x)$
    (c) $v = (Lg(L^2 - (L - x)^2))/(L - x)$
    (d) $v = \sqrt{Lg(L^2 + (L - x)^2)}/(L - x)$
    (e) $v = \sqrt{Lg(L^2 - (L - x)^2)}/(L - x)$

19. In the previous problem, the solution for x as a function of t is

    Select the correct answer.

    (a) $x = L - \sqrt{L^2 - Lgt}$
    (b) $x = L - \sqrt{L^2 + Lgt}$
    (c) $x = L - \sqrt{L^2 - Lgt^2}$
    (d) $x = L - \sqrt{L^2 + Lgt^2}$
    (e) $x = L - \sqrt{Lgt^2 - L^2}$

20. In the previous two problems, how long does it take for the chain to fall completely to the ground?

    Select the correct answer.

    (a) $t = (L/g)^2$
    (b) $t = g/L$
    (c) $t = L/g$
    (d) $t = \sqrt{L/g}$
    (e) $t = \sqrt{g/L}$

ANSWER KEY
Zill Differential Equations 9e Chapter 5 Form D

1. d
2. b
3. c
4. a
5. d
6. e
7. b
8. a
9. c
10. e
11. a
12. b
13. e
14. b
15. d
16. e
17. d
18. e
19. c
20. d

1. A spring attached to the ceiling is stretched 6 inches by a two pound weight. What is the value of the Hooke's Law spring constant, k? Include units in your answer.

2. In the previous problem, what is the mass, m? If the system is set in motion, what is the natural frequency, ω? Include units.

3. For the previous two problems, write down the initial value problem for the motion, if the mass is pushed up 6 inches from equilibrium and given an upward velocity of 2 feet per second.

4. Solve the initial value problem that you derived in the previous problem.

5. The solution of a vibrating spring problem is $x = 6\cos t + 8\sin t$. The amplitude is

   Select the correct answer.

   (a) 10
   (b) 14
   (c) 2
   (d) -2
   (e) 48

6. A beam of length L is embedded horizontally at one end and free at the other end. The weight density is given by $w(x) = x$. Its flexural rigidity, EI, is given. Write down the correct boundary value problem for the deflection, $y(x)$.

7. Solve the boundary value problem of the previous problem.

8. A model for a nonlinear mass-spring system is $\frac{d^2x}{dt^2} + x - x^3 = 0$, where x is displacement and m is mass. Solve this for dx/dt by multiplying by dx/dt and integrating.

9. The eigenvalue problem $y'' + \lambda y = 0$, $y(0) = 0$, $y(\pi/2) = 0$ has the solution

   Select the correct answer.

   (a) $y = \sin(nx)$, $\lambda = 4n^2$, $n = 1, 2, 3, \ldots$
   (b) $y = \cos(nx)$, $\lambda = 4n^2$, $n = 1, 2, 3, \ldots$
   (c) $y = \sin(nx)$, $\lambda = 2n$, $n = 1, 2, 3, \ldots$
   (d) $y = \cos(nx)$, $\lambda = 2n$, $n = 1, 2, 3, \ldots$
   (e) none of the above

10. The eigenvalue problem $y'' + \lambda y = 0$, $y(0) = 0$, $y(1) = 0$ has the solution

    Select the correct answer.

    (a) $y = \sin(n\pi x)$, $\lambda = (n\pi)^2$, $n = 1, 2, 3, \ldots$
    (b) $y = \cos(n\pi x)$, $\lambda = (n\pi)^2$, $n = 1, 2, 3, \ldots$
    (c) $y = \sin(n\pi x)$, $\lambda = n\pi$, $n = 1, 2, 3, \ldots$
    (d) $y = \cos(n\pi x)$, $\lambda = n\pi$, $n = 1, 2, 3, \ldots$
    (e) none of the above

11. Write down the differential equation for the angle θ between a pendulum of length 1/2 feet and a vertical line.

12. Linearize the equation of the previous problem and solve the resulting initial value problem if the pendulum starts at an angle of 40° with an angular velocity of 2 rad/sec.

13. A pendulum of length 4 feet hangs from the ceiling. Let $g = 32$ represent the gravitational acceleration. The correct linearized differential equation for the angle, θ, that the swinging pendulum makes with the vertical is

    Select the correct answer.

    (a) $\frac{d\theta}{dt} + 8\theta = 0$
    (b) $\frac{d^2\theta}{dt^2} + 8\theta = 0$
    (c) $\frac{d\theta}{dt} + \theta/8 = 0$
    (d) $\frac{d^2\theta}{dt^2} + \theta/8 = 0$
    (e) $\frac{d^2\theta}{dt^2} - 8\theta = 0$

14. The solution of the differential equation of the previous problem is

    Select the correct answer.

    (a) $\theta = ce^{-t/8}$
    (b) $\theta = c_1 \cos(8t) + c_2 \sin(8t)$
    (c) $\theta = c_1 \cos(\sqrt{8}t) + c_2 \sin(\sqrt{8}t)$
    (d) $\theta = ce^{8t}$
    (e) $\theta = ce^{-8t}$

15. A rocket with mass m is launched vertically upward from the surface of the earth with a velocity v_0. Let $y(t)$ be the distance of the rocket from the center of the earth at time t. Write down the initial value problem for the position, $y(t)$, assuming that the only force acting on the rocket is gravity, which is inversely proportional to the square of the distance from the center of the earth.

16. Solve the differential equation of the previous problem for the velocity by multiplying by the velocity and integrating.

17. In the previous two problems, let R represent the radius of the earth. The escape velocity is

    Select the correct answer.

    (a) $v_0 = 2k/(mR)$
    (b) $v_0 = mR/(2k)$
    (c) $v_0 = \sqrt{mR/(2k)}$
    (d) $v_0 = \sqrt{2k/(mR)}$
    (e) $v_0 = \sqrt{2m/(kR)}$

18. The differential equation $\frac{d^2x}{dt^2} + xe^{-0.1x} = 0$ is a model for an undamped spring-mass system with a nonlinear restoring force. The initial conditions are $x(0) = 0.1$, $x'(0) = -0.1$. The solution of the linearized equation is

    Select the correct answer.

    (a) $x = 0.1e^{-t}$
    (b) $x = 0.1e^{t}$
    (c) $x = 0.1\cos t$
    (d) $x = 0.1\sin t$
    (e) $x = 0.1\cos t - 0.1\sin t$

19. The boundary value problem $EI\frac{d^4y}{dx^4} + P\frac{d^2y}{dx^2} = 0$, $y(0) = 0$, $y''(0) = 0$, $y(L) = 0$, $y''(L) = 0$ is a model for the deflection, $y(x)$, of a vertical column of length L which is hinged at both ends and which is being compressed by a force, P. Assume that EI is constant. The critical buckling loads for this problem are

    Select the correct answer.

    (a) $P_n = EI(L\pi/n)^2$
    (b) $P_n = EI(n\pi/L)^2$
    (c) $P_n = EI(L\pi/n)$
    (d) $P_n = EI(n\pi/L)$
    (e) $P_n = (n\pi/L)^2$

20. In the previous problem, the nontrivial solutions of the boundary value problem are

    Select the correct answer.

    (a) $y = \sin(\sqrt{P_n/(EI)}x)$
    (b) $y = \cos(\sqrt{P_n/(EI)}x)$
    (c) $y = \sin(P_n/(EI)x)$
    (d) $y = \cos(P_n/(EI)x)$
    (e) $y = \sin(\sqrt{P_nEI}x$

ANSWER KEY

Zill Differential Equations 9e Chapter 5 Form E

1. 4 pounds per foot
2. $m = 1/16$ slug, $\omega = 8\text{sec}^{-1}$
3. $\frac{d^2x}{dt^2} + 64x = 0$, $x(0) = -1/2$, $\frac{dx}{dt}(0) = 2$
4. $x = -\cos(8t)/2 - \sin(8t)/4$
5. a
6. $y'''' = x/EI$, $y(0) = 0$, $y'(0) = 0$, $y''(L) = 0$, $y'''(L) = 0$
7. $y = (L^3x^2 - L^2x^3/2 + x^5/20)/(6EI)$
8. $\frac{dx}{dt} = \sqrt{c - x^2 - x^4/2}$
9. e
10. a
11. $\frac{d^2\theta}{dt^2} + 64\sin\theta = 0$
12. $\theta = 2\pi\cos(8t)/9 + \sin(8t)/4$
13. b
14. c
15. $m\frac{d^2y}{dt^2} = -k/y^2$
16. $v = \sqrt{c + 2k/(my)}$
17. d
18. e
19. b
20. a

1. A spring attached to the ceiling is stretched one foot by a four pound weight. The value of the Hooke's Law spring constant, k is

   Select the correct answer.

   (a) 4 pounds per foot
   (b) 1/4 pound per foot
   (c) 1/4 foot pound
   (d) 4 foot pounds
   (e) none of the above

2. In the previous problem, if the mass is set in motion, the natural frequency, ω, is

   Select the correct answer.

   (a) $4\sqrt{2}\text{sec}$
   (b) $4\sqrt{2}\text{sec}^{-1}$
   (c) 32 sec
   (d) 32 sec^{-1}
   (e) sec^{-1}

3. In the previous two problems, if the mass is set into motion in a medium that imparts a damping force numerically equal to the velocity, the correct differential equation for the position, $x(t)$, of the mass at a function of time, t, is

   Select the correct answer.

   (a) $\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + x/4 = 0$
   (b) $\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 2x = 0$
   (c) $\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 4x = 0$
   (d) $\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 8x = 0$
   (e) $\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 32x = 0$

4. If the mass in the previous problem is pulled down two feet and released, the solution for the position is

   Select the correct answer.

   (a) $x = \cos(4\sqrt{2}t) + \sin(4\sqrt{2}t)$
   (b) $x = \sin(4\sqrt{2}t)$
   (c) $x = \cos(4\sqrt{2}t)$
   (d) $x = 2e^{-4t}\cos(4t)$
   (e) $x = 2e^{-4t}(\cos(4t) + \sin(4t))$

5. A vibrating spring problem has the solution $x = 8\cos(4t) + 6\sin(4t)$. What is the period? What is the amplitude? Write the solution as $x = A\sin(\omega t + \phi)$.

6. A mass of 2 kilograms is attached to a spring hanging from the ceiling. It stretches the spring 9.8 meters. If the system is set into motion, what is the period of the motion?

7. In the previous problem, the system is acted upon be a force equal to $30\sin(10t)$ Newtons. What is the initial value problem that describes the motion, if the mass is pulled down 5 centimeters from equilibrium and released.

8. Solve the initial value problem from the previous problem.

9. A beam of length L is simply supported at the left end and embedded at the right end. The weight density is $w(x) = x$. Let $y(x)$ represent the deflection at point x. The correct form of the boundary value problem for this beam is

   Select the correct answer.

   (a) $\frac{d^2y}{dx^2} = x/EI$, $y(0) = 0$, $y'(L) = 0$
   (b) $\frac{d^4y}{dx^4} = x/EI$, $y(0) = 0$, $y''(0) = 0$, $y(L) = 0$, $y'(L) = 0$
   (c) $\frac{d^2y}{dx^2} = xEI$, $y(0) = 0$, $y''(L) = 0$
   (d) $\frac{d^4y}{dx^4} = xEI$, $y(0) = 0$, $y''(0) = 0$, $y''(L) = 0$, $y'''(L) = 0$
   (e) none of the above

10. The solution of the boundary value problem in the previous problem is

    Select the correct answer.

    (a) $y = w_0/EI(L^3x/48 - Lx^3/16 + x^4/24)$
    (b) $y = w_0/EI(x^2/2 - Lx)$
    (c) $y = w_0EI(L^3x/48 - Lx^3/16 + x^4/24)$
    (d) $y = w_0EI(x^2/2 - Lx)$
    (e) none of the above

11. A thin, vertical, homogeneous column of length 6 is subjected to a constant load, P. It is hinged at both ends. Its flexural rigidity is $EI = 4$. Write down the correct boundary value problem for this column.

12. Solve the boundary value problem in the previous problem. What is the first (Euler) buckling load?

13. What are the eigenvalues for the problem $y'' + \lambda y = 0$, $y(0) = 0$, $y(1) = 0$?

14. What are the nontrivial solutions of the boundary value problem of the previous problem?

15. A pendulum of length 16 feet hangs from the ceiling. Let $g = 32$ represent the gravitational acceleration. The correct linearized differential equation for the angle, θ, that the swinging pendulum makes with the vertical is

    Select the correct answer.

    (a) $\frac{d\theta}{dt} + 2\theta = 0$
    (b) $\frac{d^2\theta}{dt^2} + 2\theta = 0$
    (c) $\frac{d\theta}{dt} + \theta/2 = 0$
    (d) $\frac{d^2\theta}{dt^2} + \theta/2 = 0$
    (e) $\frac{d^2\theta}{dt^2} - 2\theta = 0$

16. The solution of the differential equation of the previous problem is

    Select the correct answer.

    (a) $\theta = ce^{2t}$
    (b) $\theta = ce^{-2t}$
    (c) $\theta = ce^{-t/2}$
    (d) $\theta = c_1 \cos(\sqrt{2}t) + c_2 \sin(\sqrt{2}t)$
    (e) $\theta = c_1 \cos(2t) + c_2 \sin(2t)$

17. The differential equation $\frac{d^2x}{dt^2} + x(1 + \ln(1 - x)) = 0$ is a model for an undamped spring-mass system with a nonlinear restoring force. The initial conditions are $x(0) = 0.1$, $x'(0) = 0$. The solution of the linearized system is

    Select the correct answer.

    (a) $x = 0.1 \cos t$
    (b) $x = 0.1 \sin t$
    (c) $x = 0.1 \cos t - 0.1 \sin t$
    (d) $x = 0.1e^{-t}$
    (e) $x = 0.1e^{t}$

18. A rocket with mass m is launched vertically upward from the surface of the earth with a velocity v_0. Let $y(t)$ be the distance of the rocket from the center of the earth at time t. Assuming that the only force acting on the rocket is gravity, which is inversely proportional to the square of the distance from the center of the earth, The correct differential equation for the position of the rocket is

    Select the correct answer.

    (a) $m\frac{dy}{dt} = v_0 - k/y^2$
    (b) $\frac{dy}{dt} = v_0 - k/y^2$
    (c) $m\frac{d^2y}{dt^2} = -k/y^2$
    (d) $\frac{d^2y}{dt^2} = -k/y^2$
    (e) none of the above

19. In the previous problem, the solution for the velocity, v, is

    Select the correct answer.

    (a) $v = \sqrt{c + 2k/(my)}$
    (b) $v = v_0 - k/y^2$
    (c) $v = v_0 + k/y^2$
    (d) $v = c + 2k/(my)$
    (e) none of the above

20. In the previous problem, if $y(0) = R$, the escape velocity is

    Select the correct answer.

    (a) $v_0 = 2k/(mR)$
    (b) $v_0 = \sqrt{2k/(mR)}$
    (c) $v_0 = 2m/(kR)$
    (d) $v_0 = \sqrt{2m/(kR)}$
    (e) none of the above

ANSWER KEY

Zill Differential Equations 9e Chapter 5 Form F

1. a
2. b
3. e
4. e
5. $T = \pi/2$, $A = 10$, $x = 10\sin(4t + \phi)$, where $\tan\phi = 4/3$
6. $T = 2\pi$
7. $\frac{d^2x}{dt^2} + x = 15\sin(10t)$, $x(0) = 0.05$, $\frac{dx}{dt}(0) = 0$
8. $x = 0.05\cos t + (50\sin t - 5\sin(10t))/33$
9. b
10. e
11. $y'' + Py/4 = 0$, $y(0) = 0$, $y(6) = 0$
12. $y = \sin(n\pi x/6)$, $P_n = (n\pi/3)^2$, $P_1 = (\pi/3)^2$
13. $\lambda = (n\pi)^2$
14. $y = c\sin(n\pi x)$
15. b
16. d
17. a
18. c
19. a
20. b

1. A spring attached to the ceiling is stretched 61.25 centimeters by a two kilogram mass. What is the value of the Hooke's Law spring constant, k? Include units in your answer.

2. In the previous problem, if the system is set in motion, what is the natural frequency, ω? Include units.

3. For the previous two problems, write down the initial value problem for the motion, if the mass is pulled down 6 centimeters from equilibrium and released.

4. Solve the problem you derived in the previous problem.

5. A 2 pound weight is attached to a spring and stretches it 6 inches. The system is set into motion by giving it an upward velocity of 4 feet per second in a medium that imparts a damping force numerically equal to the velocity. The initial value problem describing this situation is

   Select the correct answer.

   (a) $\frac{d^2x}{dt^2} + 16\frac{dx}{dt} + 16x/3 = 0$, $x(0) = 0$, $\frac{dx}{dt}(0) = -4$
   (b) $\frac{d^2x}{dt^2} + 16\frac{dx}{dt} + 16x/3 = 0$, $x(0) = 0$, $\frac{dx}{dt}(0) = 4$
   (c) $\frac{d^2x}{dt^2} + 16\frac{dx}{dt} + 64x = 0$, $x(0) = 0$, $\frac{dx}{dt}(0) = -4$
   (d) $\frac{d^2x}{dt^2} + 16\frac{dx}{dt} + 64x = 0$, $x(0) = 0$, $\frac{dx}{dt}(0) = 4$
   (e) $\frac{d^2x}{dt^2} - 16\frac{dx}{dt} - 64x = 0$, $x(0) = 0$, $\frac{dx}{dt}(0) = -4$

6. The solution of the initial value problem of the previous problem is

   Select the correct answer.

   (a) $x = -4te^{-8t}$
   (b) $x = 4te^{-8t}$
   (c) $x = \sqrt{3/44}(e^{(-8t-\sqrt{176/3})t} - e^{(-8t+\sqrt{176/3})t}$
   (d) $x = \sqrt{3/44}(e^{(-8t-\sqrt{176/3})t} + e^{(-8t+\sqrt{176/3})t}$
   (e) none of the above

7. A beam of length L is embedded horizontally at one end and simply supported at the other end. The weight density is given by $w(x) = x$. Its flexural rigidity, EI, is given. Write down the correct boundary value problem for the deflection, $y(x)$ as a function of distance x from the embedded end.

8. Solve the boundary value problem of the previous problem.

9. The solution of a vibrating spring problem is $x = 4\cos t - 3\sin t$. The amplitude is

   Select the correct answer.

   (a) 7
   (b) 1
   (c) 5
   (d) 25
   (e) -1

10. The eigenvalue problem $y'' + \lambda y = 0$, $y(0) = 0$, $y(\pi/2) = 0$ has the solution

    Select the correct answer.

    (a) $y = \sin(nx)$, $\lambda = 4n^2$, $n = 1, 2, 3, \ldots$
    (b) $y = \cos(nx)$, $\lambda = 4n^2$, $n = 1, 2, 3, \ldots$
    (c) $y = \sin(nx)$, $\lambda = 2n$, $n = 1, 2, 3, \ldots$
    (d) $y = \cos(nx)$, $\lambda = 2n$, $n = 1, 2, 3, \ldots$
    (e) none of the above

11. The eigenvalue problem $y'' + \lambda y = 0$, $y(0) = 0$, $y(1) = 0$ has the solution

    Select the correct answer.

    (a) $y = \sin(n\pi x)$, $\lambda = (n\pi)^2$, $n = 1, 2, 3, \ldots$
    (b) $y = \cos(n\pi x)$, $\lambda = (n\pi)^2$, $n = 1, 2, 3, \ldots$
    (c) $y = \sin(n\pi x)$, $\lambda = n\pi$, $n = 1, 2, 3, \ldots$
    (d) $y = \cos(n\pi x)$, $\lambda = n\pi$, $n = 1, 2, 3, \ldots$
    (e) none of the above

12. A model for a nonlinear mass-spring system is $\frac{d^2x}{dt^2} + 2x - 4x^3 = 0$, where x is displacement and m is mass. Solve for dx/dt by multiplying by dx/dt and integrating.

13. A pendulum of length 8 feet hangs from the ceiling. Let $g = 32$ represent the gravitational acceleration. The correct differential equation for the angle, θ, that the swinging pendulum makes with the vertical is

    Select the correct answer.

    (a) $\frac{d\theta}{dt} + 4\sin\theta = 0$
    (b) $\frac{d^2\theta}{dt^2} + 4\sin\theta = 0$
    (c) $\frac{d\theta}{dt} + \sin\theta/4 = 0$
    (d) $\frac{d^2\theta}{dt^2} + \sin\theta/4 = 0$
    (e) $\frac{d^2\theta}{dt^2} - 4\sin\theta = 0$

14. In the previous problem, the correct linearized differential equation for the angle, θ is

    Select the correct answer.

    (a) $\frac{d\theta}{dt} + \theta/4 = 0$
    (b) $\frac{d^2\theta}{dt^2} + \theta/4 = 0$
    (c) $\frac{d^2\theta}{dt^2} - 4\theta = 0$
    (d) $\frac{d\theta}{dt} + 4\theta = 0$
    (e) $\frac{d^2\theta}{dt^2} + 4\theta = 0$

15. The solution of the differential equation of the previous problem is

    Select the correct answer.

    (a) $\theta = c_1 \cos(2t) + c_2 \sin(2t)$
    (b) $\theta = c_1 \cos(4t) + c_2 \sin(4t)$
    (c) $\theta = ce^{2t}$
    (d) $\theta = ce^{-2t}$
    (e) $\theta = ce^{-t/2}$

16. If the pendulum of the previous three problems is started from rest at an angle of $50°$ from the vertical, the solution for θ is

    (a) $\theta = 50\cos(2t)$
    (b) $\theta = 50\sin(2t)$
    (c) $\theta = 50e^{2t}$
    (d) $\theta = 50e^{-2t}$
    (e) none of the above

17. A rocket with mass m is launched vertically upward from the surface of the earth with a velocity of 10 miles per second. Let $y(t)$ be the distance of the rocket from the center of the earth at time t. Write down the initial value problem for the position, $y(t)$, assuming that the only force acting on the rocket is gravity, which is inversely proportional to the square of the distance from the center of the earth.

18. Solve the differential equation of the previous problem for the velocity by multiplying by the velocity and integrating.

19. In the previous problem, what is the escape velocity?

20. The differential equation $\frac{d^2x}{dt^2} + xe^{-0.4x} = 0$ is a model for an undamped spring-mass system with a nonlinear forcing function. The initial conditions are $x(0) = 0$, $x'(0) = -0.1$. The solution of the linearized system is

Select the correct answer.

(a) $x = -0.1\cos t$

(b) $x = -0.1\sin t$

(c) $x = 0.1\cos t - 0.1\sin t$

(d) $x = 0.1(e^{-t} - e^{t})/2$

(e) $x = 0.1(e^{t} - e^{-t})/2$

ANSWER KEY

Zill Differential Equations 9e Chapter 5 Form G

1. $k = 32$ Newtons per meter
2. $\omega = 4\text{sec}^{-1}$
3. $\frac{d^2x}{dt^2} + 16x = 0$, $x(0) = 0.06$, $\frac{dx}{dt}(0) = 0$
4. $x = 0.06\cos(4t)$
5. c
6. a
7. $y'''' = x/(EI)$, $y(0) = 0$, $y'(0) = 0$, $y(L) = 0$, $y''(L) = 0$
8. $y = 7L^3x^2/(240EI) - 3L^2x^3/(80EI) + x^5/(120EI)$
9. c
10. e
11. a
12. $dx/dt = \sqrt{c - 2x^2 - 2x^4}$
13. b
14. e
15. a
16. e
17. $m\frac{d^2y}{dt^2} = -k/y^2$
18. $v = \sqrt{c + 2k/(my)}$
19. $v_0 = \sqrt{2k/(mR)}$
20. b

1. A spring attached to the ceiling is stretched 1.225 meters by a one kilogram mass. The value of the Hooke's Law spring constant, k is

   Select the correct answer.

   (a) 4 Newton-meters
   (b) 1/4 Newtons per meter
   (c) 1/4 Newton-meter
   (d) 8 Newtons per meter
   (e) none of the above

2. In the previous problem, if the mass is set in motion, the natural frequency, ω, is

   Select the correct answer.

   (a) $\sqrt{8}$ sec
   (b) $\sqrt{8}$ sec^{-1}
   (c) $\sqrt{32}$ sec
   (d) $\sqrt{32}$ sec^{-1}
   (e) 1/8 sec^{-1}

3. In the previous two problems, If the mass is set to motion in a medium that imparts a damping force numerically equal to 4 times the velocity, the correct differential equation for the position, $x(t)$, of the mass at a function of time, t, is

   Select the correct answer.

   (a) $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + x/4 = 0$
   (b) $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 2x = 0$
   (c) $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 0$
   (d) $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 8x = 0$
   (e) $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 32x = 0$

4. If the mass in the previous problem is pulled down two meters and released, the solution for the position is

   Select the correct answer.

   (a) $x = 2\cos(2\sqrt{2}t) + 2\sin(2\sqrt{2}t)$
   (b) $x = 2\sin(2\sqrt{2}t)$
   (c) $x = 2\cos(2\sqrt{2}t)$
   (d) $x = 2e^{-2t}\cos(2t)$
   (e) $x = 2e^{-2t}(\cos(2t) + \sin(2t))$

5. A 4 pound weight is attached to a spring and stretches it one foot. If the system is set into motion, what is the period of motion?

6. In the previous problem, if the system starts from rest in the equilibrium position, describe the subsequent motion if the system is subjected to an external force of $\sin(2t)$ pounds.

7. A vibrating spring problem has the solution $x = 5\cos 4t + 12\sin 4t$. What is the period? What is the amplitude? Write the solution as $x = A\sin(\omega t + \phi)$.

8. A beam of length L is embedded at the left end and simply supported at the right end. The weight density is $w(x) = x$. Let $y(x)$ represent the deflection at point x. The correct form of the boundary value problem for the deflection, $y(x)$, as a function of distance, x, from the embedded end is

   Select the correct answer.

   (a) $\frac{d^2y}{dx^2} = x/EI$, $y(0) = 0$, $y(L) = 0$
   (b) $\frac{d^4y}{dx^4} = x/EI$, $y(0) = 0$, $y'(0) = 0$, $y(L) = 0$, $y''(L) = 0$
   (c) $\frac{d^2y}{dx^2} = xEI$, $y(0) = 0$, $y(L) = 0$
   (d) $\frac{d^4y}{dx^4} = xEI$, $y(0) = 0$, $y''(0) = 0$, $y''(L) = 0y'''(L) = 0$
   (e) none of the above

9. The solution of the boundary value problem in the previous problem is

   Select the correct answer.

   (a) $y = EI(7L^3x/6 - 3L^2x^3/2 + x^5/3)/40$
   (b) $y = (x^3 - L^2x)/(6EI)$
   (c) $y = 1/(40EI)(7L^3x^2/6 - 3L^2x^3/2 + x^5/3)$
   (d) $y = EI(x^3 - L^2x)/6$
   (e) none of the above

10. Why does it make no physical sense to consider the mathematical problem of beam deflections for a beam that is simply supported at one end and free at the other end?

11. A thin, vertical, homogeneous column of length L is subjected to a constant load, P. It is hinged at both ends. Its flexural rigidity, EI, is given. Write down the correct boundary value problem for the deflection, $y(x)$, as a function of distance, x, from the top of the column.

12. Solve the boundary value problem in the previous problem. What is the first (Euler) buckling load?

13. Solve the eigenvalue problem $y'' + \lambda y = 0$, $y(0) = 0$, $y(1) = 0$.

14. Solve the eigenvalue problem $y'' + \lambda y = 0$, $y'(0) = 0$, $y'(1) = 0$.

15. A pendulum of length 4 feet hangs from the ceiling. Let $g = 32$ represent the gravitational acceleration. The correct linearized differential equation for the angle, θ, that the swinging pendulum makes with the vertical is

    Select the correct answer.

    (a) $\frac{d\theta}{dt} + \theta/8 = 0$
    (b) $\frac{d^2\theta}{dt^2} + \theta/8 = 0$
    (c) $\frac{d^2\theta}{dt^2} - 8\theta = 0$
    (d) $\frac{d\theta}{dt} + 8\theta = 0$
    (e) $\frac{d^2\theta}{dt^2} + 8\theta = 0$

16. The solution of the differential equation of the previous problem is

    Select the correct answer.

    (a) $\theta = c_1 \cos(8t) + c_2 \sin(8t)$
    (b) $\theta = c_1 \cos(\sqrt{8}t) + c_2 \sin(\sqrt{8}t)$
    (c) $\theta = ce^{8t}$
    (d) $\theta = ce^{-8t}$
    (e) $\theta = ce^{-t/8}$

17. The differential equation $\frac{d^2x}{dt^2} + xe^{-0.2x} = 0$ is a model for an undamped spring-mass system with a nonlinear restoring force. The initial conditions are $x(0) = 0.1$, $x'(0) = -0.2$. The solution of the linearized system is

    Select the correct answer.

    (a) $x = 0.2\cos t - 0.1\sin t$
    (b) $x = 0.1\cos t + 0.2\sin t$
    (c) $x = 0.1\cos t - 0.2\sin t$
    (d) $x = 0.05e^{t} - 0.15e^{-t})$
    (e) $x = -0.05e^{t} + 0.15e^{-t}$

18. A rocket with mass m is launched vertically upward from the surface of the earth with a velocity v_0. Let $y(t)$ be the distance of the rocket from the center of the earth at time t. Assuming that the only force acting on the rocket is gravity, which is inversely proportional to the square of the distance from the center of the earth, the correct differential equation for the position of the rocket is

    Select the correct answer.

    (a) $\frac{dy}{dt} = v_0 - k/y^2$
    (b) $\frac{d^2y}{dt^2} = -k/y^2$
    (c) $m\frac{dy}{dt} = v_0 - k/y^2$
    (d) $m\frac{d^2y}{dt^2} = -k/y^2$
    (e) none of the above

19. In the previous problem, the solution for the velocity, v, is

    Select the correct answer.

    (a) $v = \sqrt{c + 2k/(my)}$
    (b) $v = v_0 - k/y^2$
    (c) $v = v_0 + k/y^2$
    (d) $v = c + 2k/(my)$
    (e) none of the above

20. In the previous problem, if $y(0) = R$, the escape velocity is

    Select the correct answer.

    (a) $v_0 = \sqrt{2m/(kR)}$
    (b) $v_0 = 2k/(mR)$
    (c) $v_0 = \sqrt{2k/(mR)}$
    (d) $v_0 = 2m/(kR)$
    (e) none of the above

ANSWER KEY
Zill Differential Equations 9e Chapter 5 Form H

1. d
2. b
3. d
4. e
5. $T = \sqrt{2}\pi/4$
6. $x = (-\sqrt{2}\sin(4\sqrt{2}t) + 4\sin(2t))/14$
7. $T = \pi/2$, $A = 13$, $x = 13\sin(4t + \phi)$, where $\tan\phi = 5/12$
8. b
9. c
10. The beam would fall off of its support and would not deflect.
11. $y'' + Py/(EI) = 0$, $y(0) = 0$, $y(L) = 0$
12. $P_n = EI(n\pi/L)^2$, $y = \sin(n\pi x/L)$, $P_1 = EI(\pi/L)^2$
13. $\lambda_n = (n\pi)^2$, $y = \sin(n\pi x)$, $n = 1, 2, 3, \ldots$
14. $\lambda_n = (n\pi)^2$, $y = \cos(n\pi x)$, $n = 0, 1, 2, \ldots$ Note that $\lambda = 0$ is an eigenvalue.
15. e
16. b
17. c
18. d
19. a
20. c