1. Write the system in matrix form:
$$\frac{dx}{dt} = 4x - 3y$$
$$\frac{dy}{dt} = x + 2y.$$

2. Write the system in matrix form:
$$\frac{dx}{dt} = 2x + 4y - z$$
$$\frac{dy}{dt} = -5x + 2y + 6z$$
$$\frac{dz}{dt} = x + 2y + 3z$$

3. Write the system without matrices:
$$\mathbf{X}' = \begin{pmatrix} 5 & -1 \\ 2 & 2 \end{pmatrix} \mathbf{X}.$$

4. Verify that $\mathbf{X} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t}$ is a solution of $\mathbf{X}' = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix} \mathbf{X}$.

5. The vector functions $\mathbf{X}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t}$ and $\mathbf{X}_2 = \begin{pmatrix} 3 \\ 5 \end{pmatrix} e^{6t}$ are solutions of the system $\mathbf{X}' = \begin{pmatrix} 1 & 3 \\ 5 & 3 \end{pmatrix} \mathbf{X}$. Are they linearly independent on $(-\infty, \infty)$? Explain.

6. What is the characteristic equation for the matrix $\begin{pmatrix} 5 & -1 \\ 2 & 2 \end{pmatrix}$? What are the eigenvalues?

7. Solve the system $\mathbf{X}' = \begin{pmatrix} 5 & -1 \\ 2 & 2 \end{pmatrix} \mathbf{X}$.

8. Solve the system $\mathbf{X}' = \begin{pmatrix} 5 & -1 \\ 2 & 2 \end{pmatrix} \mathbf{X} + \begin{pmatrix} 2 \\ e^t \end{pmatrix}$.

9. What is the characteristic equation for the matrix $\begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix}$? What are the eigenvalues?

10. Solve the system $\mathbf{X}' = \begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix} \mathbf{X}$.

11. What is the characteristic equation for the matrix $\begin{pmatrix} -3 & -1 \\ 1 & -5 \end{pmatrix}$? What are the eigenvalues?

12. Solve the system $\mathbf{X}' = \begin{pmatrix} -3 & -1 \\ 1 & -5 \end{pmatrix} \mathbf{X}$.

13. Solve the system $\mathbf{X}' = \begin{pmatrix} -3 & -1 \\ 1 & -5 \end{pmatrix} \mathbf{X} + \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

14. What is the characteristic equation for the matrix $\begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$? What are the eigenvalues?

15. Solve the system $\mathbf{X}' = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix} \mathbf{X}$.

16. What is the characteristic equation for the matrix $\begin{pmatrix} 3 & -2 \\ 1 & 5 \end{pmatrix}$? What are the eigenvalues?

17. Solve the system $\mathbf{X}' = \begin{pmatrix} 3 & -2 \\ 1 & 5 \end{pmatrix} \mathbf{X}$.

18. Solve the system $\mathbf{X}' = \begin{pmatrix} 3 & -2 \\ 1 & 5 \end{pmatrix} \mathbf{X} + \begin{pmatrix} t \\ 1 \end{pmatrix}$.

19. What is the characteristic equation for the matrix $\begin{pmatrix} 2 & 1 & 2 \\ 3 & 0 & 6 \\ -4 & 0 & -3 \end{pmatrix}$? What are the eigenvalues?

20. Solve the system $\mathbf{X}' = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 0 & 6 \\ -4 & 0 & -3 \end{pmatrix} \mathbf{X}$.

ANSWER KEY
Zill Differential Equations 9e Chapter 8 Form A

1. $\mathbf{X}' = \begin{pmatrix} 4 & -3 \\ 1 & 2 \end{pmatrix} \mathbf{X}.$

2. $\mathbf{X}' = \begin{pmatrix} 2 & 4 & -1 \\ -5 & 2 & 6 \\ 1 & 2 & 3 \end{pmatrix} \mathbf{X}.$

3. $\dfrac{dx}{dt} = 5x - y$
   $\dfrac{dy}{dt} = 2x + 2y.$

4. $\mathbf{X}' = \begin{pmatrix} 4 \\ 4 \end{pmatrix} e^{4t}, \ \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix} \mathbf{X} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} e^{4t} = \mathbf{X}'.$

5. Yes, because their Wronskian is $8e^{4t} \neq 0$

6. $\lambda^2 - 7\lambda + 12 = 0$, eigenvalues are 3, 4

7. $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{3t}$

8. $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{3t} + \begin{pmatrix} (-2 - e^t)/6 \\ (1 - 2e^t)/3 \end{pmatrix}$

9. $\lambda^2 - 6\lambda + 8 = 0$, eigenvalues are 2, 4

10. $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{4t}$

11. $\lambda^2 + 8\lambda + 16 = 0$, eigenvalues are -4, -4

12. $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-4t} + c_2 \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-4t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-4t} \right]$

13. $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-4t} + c_2 \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-4t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-4t} \right] + \begin{pmatrix} 9/16 \\ 5/16 \end{pmatrix}$

14. $-\lambda^3 + 6\lambda^2 + 15\lambda + 8 = 0$, eigenvalues are -1, -1, 8

15. $\mathbf{X} = c_1 \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} e^{8t} + c_2 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} e^{-t} + c_3 \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} e^{-t}$

16. $\lambda^2 - 8\lambda + 17 = 0$, eigenvalues are $4 \pm i$

17. $\mathbf{X} = c_1 \left[\begin{pmatrix} -1 \\ 1 \end{pmatrix} \cos t - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin t \right] e^{4t} + c_2 \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos t + \begin{pmatrix} -1 \\ 1 \end{pmatrix} \sin t \right] e^{4t}$

18. $\mathbf{X} = c_1 \left[\begin{pmatrix} -1 \\ 1 \end{pmatrix} \cos t - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin t \right] e^{4t} + c_2 \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos t + \begin{pmatrix} -1 \\ 1 \end{pmatrix} \sin t \right] e^{4t} +$
    $\begin{pmatrix} (-85t - 57)/289 \\ (17t - 43)/289 \end{pmatrix}$

19. $-\lambda^3 - \lambda^2 + \lambda - 15 = 0$, eigenvalues are -3, $1 \pm 2i$

20. $$\mathbf{X} = c_1 \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} e^{-3t} + c_2 e^t \left[\begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} \cos(2t) - \begin{pmatrix} -1 \\ -3 \\ 0 \end{pmatrix} \sin(2t) \right] +$$
$$c_3 e^t \left[\begin{pmatrix} -1 \\ -3 \\ 0 \end{pmatrix} \cos t + \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} \right]$$

1. Write the system in matrix form:
$\frac{dx}{dt} = 5x - 2y$
$\frac{dy}{dt} = -x + 3y.$

2. Write the system in matrix form:
$\frac{dx}{dt} = 5x + 3y - z$
$\frac{dy}{dt} = -2x - 4y + 8z$
$\frac{dz}{dt} = 2x + 2y - 5z$

3. Write the system without matrices:
$\mathbf{X}' = \begin{pmatrix} 1 & 5 \\ 2 & -3 \end{pmatrix} \mathbf{X}.$

4. Verify that $\mathbf{X} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} e^t$ is a solution of $\mathbf{X}' = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix} \mathbf{X}.$

5. The vector functions $\mathbf{X}_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^t$ and $\mathbf{X}_2 = \begin{pmatrix} 4 \\ -4 \end{pmatrix} te^t$ are solutions of the system $\mathbf{X}' = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{X}$. Are they linearly independent on $(-\infty, \infty)$? Explain.

6. What is the characteristic equation for the matrix $\begin{pmatrix} 5 & -4 \\ 2 & -1 \end{pmatrix}$? What are the eigenvalues?

7. Solve the system $\mathbf{X}' = \begin{pmatrix} 5 & -4 \\ 2 & -1 \end{pmatrix} \mathbf{X}.$

8. What is the characteristic equation for the matrix $\begin{pmatrix} -4 & 1 \\ 3 & -2 \end{pmatrix}$? What are the eigenvalues?

9. Solve the system $\mathbf{X}' = \begin{pmatrix} -4 & 1 \\ 3 & -2 \end{pmatrix} \mathbf{X}.$

10. What is the characteristic equation for the matrix $\begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}$? What are the eigenvalues?

11. Solve the system $\mathbf{X}' = \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{X}.$

12. What is the characteristic equation for the matrix $\begin{pmatrix} 1 & -2 & 2 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$? What are the eigenvalues?

13. Solve the system $\mathbf{X}' = \begin{pmatrix} 1 & -2 & 2 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix} \mathbf{X}.$

14. What is the characteristic equation for the matrix $\begin{pmatrix} 2 & -2 \\ 1 & 4 \end{pmatrix}$? What are the eigenvalues?

15. Solve the system $\mathbf{X}' = \begin{pmatrix} 2 & -2 \\ 1 & 4 \end{pmatrix} \mathbf{X}$.

16. Solve the system $\mathbf{X}' = \begin{pmatrix} 2 & -2 \\ 1 & 4 \end{pmatrix} \mathbf{X} + \begin{pmatrix} t \\ 1 \end{pmatrix}$.

17. What is the characteristic equation for the matrix $\begin{pmatrix} 2 & 4 & 4 \\ -1 & -2 & 0 \\ -1 & 0 & -2 \end{pmatrix}$? What are the eigenvalues?

18. Solve the system $\mathbf{X}' = \begin{pmatrix} 2 & 4 & 4 \\ -1 & -2 & 0 \\ -1 & 0 & -2 \end{pmatrix} \mathbf{X}$.

19. Let $A = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & 3 & 0 \end{pmatrix}$. Compute e^{At}.

20. Let $A = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & 3 & 0 \end{pmatrix}$. Solve the system $\mathbf{X}' = A\mathbf{X}$.

1. $\mathbf{X}' = \begin{pmatrix} 5 & -2 \\ -1 & 3 \end{pmatrix} \mathbf{X}.$

2. $\mathbf{X}' = \begin{pmatrix} 5 & 3 & -1 \\ -2 & -4 & 8 \\ 2 & 2 & -5 \end{pmatrix} \mathbf{X}.$

3. $\frac{dx}{dt} = x + 5y$
   $\frac{dy}{dt} = 2x - 3y.$

4. $\mathbf{X}' = \begin{pmatrix} 2 \\ -1 \end{pmatrix} e^t$

   $\begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix} \mathbf{X} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} e^t = \mathbf{X}'.$

5. Yes, their Wronskian is $-16te^{2t} \neq 0$ (except at $t = 0$)

6. $\lambda^2 - 4\lambda + 3 = 0$, the eigenvalues are 1, 3

7. $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{3t}$

8. $\lambda^2 + 6\lambda + 5 = 0$, the eigenvalues are -1, -5

9. $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-5t}$

10. $\lambda^2 - 4\lambda + 4 = 0$, the eigenvalues are 2, 2

11. $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} + c_2 \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{2t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{2t} \right]$

12. $-\lambda^3 + 3\lambda^2 + 9\lambda + 5 = 0$, the eigenvalues are -1, -1, 5

13. $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} e^{-t} + c_3 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} e^{5t}$

14. $\lambda^2 - 6\lambda + 10 = 0$, the eigenvalues are $3 \pm i$

15. $\mathbf{X} = c_1 e^{3t} \left[\begin{pmatrix} -1 \\ 1 \end{pmatrix} \cos t - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin t \right] + c_2 e^{3t} \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos t + \begin{pmatrix} -1 \\ 1 \end{pmatrix} \sin t \right]$

16. $\mathbf{X} = c_1 e^{3t} \left[\begin{pmatrix} -1 \\ 1 \end{pmatrix} \cos t - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin t \right] + c_2 e^{3t} \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos t + \begin{pmatrix} -1 \\ 1 \end{pmatrix} \sin t \right] +$
    $\begin{pmatrix} (-20t - 17)/50 \\ (5t - 7)/50 \end{pmatrix}$

17. $-\lambda^3 - 2\lambda^2 - 4\lambda - 8 = 0$, the eigenvalues are -2, $\pm 2i$

18. $\mathbf{X} = c_1 \left[\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \cos(2t) - \begin{pmatrix} -2 \\ 0 \\ 0 \end{pmatrix} \sin(2t) \right] + c_2 \left[\begin{pmatrix} -2 \\ 0 \\ 0 \end{pmatrix} \cos(2t) + \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \sin(2t) \right] +$
$c_3 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{-2t}$

19. $e^{At} = \begin{pmatrix} 1 & 0 & 0 \\ t & 1 & 0 \\ 2t + 3t^2/2 & 3t & 1 \end{pmatrix}$

20. $\mathbf{X} = e^{At}C$, where e^{At} was calculated in the previous problem, and C is a three dimensional column vector

1. The Wronskian of the vector functions $\mathbf{X}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t}$ and $\mathbf{X}_2 = \begin{pmatrix} 3 \\ 5 \end{pmatrix} e^{6t}$ is

   Select the correct answer.

   (a) $8e^{4t}$
   (b) $2e^{4t}$
   (c) $-8e^{4t}$
   (d) $-2e^{4t}$
   (e) $15e^{4t}$

2. The Wronskian of the vector functions $\mathbf{X}_1 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} e^{t}$, $\mathbf{X}_2 = \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix} e^{2t}$ and $\mathbf{X}_3 = \begin{pmatrix} -1 \\ 5 \\ 0 \end{pmatrix} e^{3t}$ is

   Select the correct answer.

   (a) $55e^{6t}$
   (b) $-55e^{6t}$
   (c) $45e^{6t}$
   (d) $-45e^{-6t}$
   (e) $45e^{-6t}$

3. If $\mathbf{X}_1$, $\mathbf{X}_2$, and $\mathbf{X}_3$ are solutions of the third order system $\mathbf{X}' = A\mathbf{X}$ and $\mathbf{X}_p$ is a particular solution of $\mathbf{X}' = A\mathbf{X} + \mathbf{f}(t)$, then the general solution of $\mathbf{X}' = A\mathbf{X} + \mathbf{f}(t)$ is

   Select the correct answer.

   (a) $c_1\mathbf{X}_1 + c_2\mathbf{X}_2 + c_3\mathbf{X}_3$
   (b) $c_1\mathbf{X}_1 + c_2\mathbf{X}_2 + c_3\mathbf{X}_3 + c_4\mathbf{X}_p$
   (c) $c_1\mathbf{X}_1 + c_2\mathbf{X}_2 + c_3\mathbf{X}_3 + \mathbf{X}_p$
   (d) $\mathbf{X}_1 + \mathbf{X}_2 + \mathbf{X}_3 + \mathbf{X}_p$
   (e) $\mathbf{X}_1 + \mathbf{X}_2 + \mathbf{X}_3 + c_4\mathbf{X}_p$

4. The characteristic equation of $A = \begin{pmatrix} 1 & -3 \\ 2 & -4 \end{pmatrix}$ is

   Select the correct answer.

   (a) $\lambda^2 + 3\lambda - 2 = 0$
   (b) $\lambda^2 + 5\lambda + 2 = 0$
   (c) $\lambda^2 - 5\lambda + 2 = 0$
   (d) $\lambda^2 + 3\lambda + 2 = 0$
   (e) $\lambda^2 - 3\lambda + 2 = 0$

5. The solution of the system $\mathbf{X}' = \begin{pmatrix} 1 & -3 \\ 2 & -4 \end{pmatrix} \mathbf{X}$ is

   Select the correct answer.

   (a) $\mathbf{X} = c_1 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-2t}$
   (b) $\mathbf{X} = c_1 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$
   (c) $\mathbf{X} = c_1 \begin{pmatrix} 2 \\ 3 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-2t}$
   (d) $\mathbf{X} = c_1 \begin{pmatrix} 2 \\ 3 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$
   (e) none of the above

6. The characteristic equation of $A = \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}$ is

   Select the correct answer.

   (a) $\lambda^2 + 3\lambda - 4 = 0$
   (b) $\lambda^2 + 3\lambda + 4 = 0$
   (c) $\lambda^2 - 3\lambda = 0$
   (d) $\lambda^2 + 3\lambda = 0$
   (e) $\lambda^2 - 3\lambda - 4 = 0$

7. The eigenvalues of $A = \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}$ are

   Select the correct answer.

   (a) $1 \pm \sqrt{3}i$
   (b) $-1 \pm \sqrt{3}i$
   (c) 0, 3
   (d) 0, 1
   (e) 0, -1

8. The solution of the system $\mathbf{X}' = \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix} \mathbf{X}$ is

   Select the correct answer.

   (a) $\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-3t}$

   (b) $\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{3t}$

   (c) $\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-3t}$

   (d) $\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{3t}$

   (e) none of the above

9. The characteristic equation of $A = \begin{pmatrix} -3 & -1 \\ 1 & -1 \end{pmatrix}$ is

   Select the correct answer.

   (a) $\lambda^2 + 4\lambda - 4 = 0$
   (b) $\lambda^2 + 4\lambda + 4 = 0$
   (c) $\lambda^2 - 2\lambda = 0$
   (d) $\lambda^2 - 2\lambda = 0$
   (e) $\lambda^2 - 2\lambda - 4 = 0$

10. The solution of the system $\mathbf{X}' = \begin{pmatrix} -3 & -1 \\ 1 & -1 \end{pmatrix} \mathbf{X}$ is

    Select the correct answer.

    (a) $\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-2t} + c_2 \left[\begin{pmatrix} -1 \\ 1 \end{pmatrix} te^{-2t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-2t} \right]$

    (b) $\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} te^{-2t} + c_3 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-2t}$

    (c) $\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t}$

    (d) $\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-t}$

    (e) none of the above

11. The characteristic equation for the matrix $A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 3 \end{pmatrix}$ is

    Select the correct answer.

    (a) $-\lambda^3 - 7\lambda^2 - 16\lambda + 12 = 0$
    (b) $-\lambda^3 + 7\lambda^2 + 14\lambda + 6 = 0$
    (c) $-\lambda^3 + 7\lambda^2 - 14\lambda + 6 = 0$
    (d) $-\lambda^3 + 7\lambda^2 + 16\lambda + 12 = 0$
    (e) $-\lambda^3 + 7\lambda^2 - 16\lambda + 12 = 0$

12. The eigenvalues of the matrix A of the previous problem are

    Select the correct answer.

    (a) $-2, -2, 3$
    (b) $2, 2, 3$
    (c) $1 \pm i, 3$
    (d) $-1 \pm i, 3$
    (e) $1 \pm \sqrt{3}i, 3$

13. Let A be the matrix of the previous two problems. The solution of $\mathbf{X}' = A\mathbf{X}$ is

    Select the correct answer.

    (a) $c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{2t} + c_3 \left[\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} te^{2t} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{2t} \right]$

    (b) $c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{-2t} + c_3 \left[\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} te^{-2t} + \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} e^{-2t} \right]$

    (c) $c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{-2t} + c_3 \left[\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} te^{-2t} + \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} e^{-2t} \right]$

    (d) $c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{2t} + c_3 \left[\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} te^{2t} + \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} e^{2t} \right]$

    (e) $c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{2t} + c_3 \left[\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} te^{2t} + \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} e^{2t} \right]$

14. The characteristic equation of $A = \begin{pmatrix} 2 & -2 \\ 2 & 2 \end{pmatrix}$ is

    Select the correct answer.

    (a) $\lambda^2 - 4\lambda = 0$
    (b) $\lambda^2 + 4\lambda = 0$
    (c) $\lambda^2 + 4\lambda - 8 = 0$
    (d) $\lambda^2 - 4\lambda + 8 = 0$
    (e) $\lambda^2 - 4\lambda - 8 = 0$

15. The eigenvalues of $A = \begin{pmatrix} 2 & -2 \\ 2 & 2 \end{pmatrix}$ are

    Select the correct answer.

    (a) 0, 4
    (b) 0, -4
    (c) 2, 2
    (d) $2 \pm 2i$
    (e) $-2 \pm 2i$

16. The solution of the system $\mathbf{X}' = \begin{pmatrix} 2 & -2 \\ 2 & 2 \end{pmatrix} \mathbf{X}$ is

    Select the correct answer.

    (a) $\mathbf{X} = c_1 e^{2t} \left[\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cos(2t) - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin(2t) \right] + c_2 e^{2t} \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos(2t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sin(2t) \right]$
    (b) $\mathbf{X} = c_1 e^{-2t} \left[\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cos(2t) - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin(2t) \right] + c_2 e^{-2t} \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos(2t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sin(2t) \right]$
    (c) $\mathbf{X} = c_1 e^{2t} \left[\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cos t + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin t \right] + c_2 e^{2t} \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos t + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sin t \right]$
    (d) $\mathbf{X} = c_1 e^{-2t} \left[\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cos t - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin t \right] + c_2 e^{-2t} \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos t + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sin t \right]$
    (e) none of the above

17. A particular solution of $\mathbf{X}' = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\mathbf{X} + \begin{pmatrix} 1 \\ t \end{pmatrix}$ is

Select the correct answer.

(a) $\begin{pmatrix} t^2 \\ 0 \end{pmatrix}$

(b) $\begin{pmatrix} 0 \\ t \end{pmatrix}$

(c) $\begin{pmatrix} 1 \\ t \end{pmatrix}$

(d) $\begin{pmatrix} t \\ 0 \end{pmatrix}$

(e) $\begin{pmatrix} t \\ 1 \end{pmatrix}$

18. A particular solution of $\mathbf{X}' = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\mathbf{X} + \begin{pmatrix} 1 \\ e^t \end{pmatrix}$ is

Select the correct answer.

(a) $\begin{pmatrix} e^t/2 \\ -e^t/2 + 1 \end{pmatrix}$

(b) $\begin{pmatrix} -e^t/2 \\ e^t/2 - 1 \end{pmatrix}$

(c) $\begin{pmatrix} e^t/2 \\ -e^t/2 - 1 \end{pmatrix}$

(d) $\begin{pmatrix} e^t/2 \\ e^t/2 + 1 \end{pmatrix}$

(e) $\begin{pmatrix} e^t/2 \\ e^t/2 - 1 \end{pmatrix}$

19. Let $A = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{pmatrix}$. Then $e^{At} =$

Select the correct answer.

(a) $\begin{pmatrix} 1 & t & 2t \\ 0 & 1 & 4t \\ 0 & 0 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & -t & 2t + 2t^2 \\ 0 & 1 & -4t \\ 0 & 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & t & 2t + 2t^2 \\ 0 & 1 & 4t \\ 0 & 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & t & 2t + 4t^2 \\ 0 & 1 & 4t \\ 0 & 0 & 1 \end{pmatrix}$

(e) none of the above

20. Let $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. Then $e^{At} =$

Select the correct answer.

(a) $I \cos t + A \sin t$

(b) $I \cos t - A \sin t$

(c) $Ie^{t} + Ae^{-t}$

(d) $Ie^{-} + Ae^{-t}$

(e) $I \cosh t + A \sinh t$

ANSWER KEY

Zill Differential Equations 9e Chapter 8 Form C

1. a
2. b
3. c
4. d
5. a
6. c
7. c
8. b
9. b
10. a
11. e
12. b
13. e
14. d
15. d
16. a
17. d
18. e
19. c
20. a

1. The Wronskian of the vector functions $\mathbf{X}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t}$ and $\mathbf{X}_2 = \begin{pmatrix} 3 \\ 5 \end{pmatrix} e^{6t}$ is

   Select the correct answer.

   (a) $8e^{4t}$
   (b) $2e^{4t}$
   (c) $-8e^{4t}$
   (d) $-2e^{4t}$
   (e) $15e^{4t}$

2. The Wronskian of the vector functions $\mathbf{X}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} e^{t}$, $\mathbf{X}_2 = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix} e^{-2t}$ and $\mathbf{X}_3 = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} e^{4t}$is

   Select the correct answer.

   (a) $14e^{3t}$
   (b) $10e^{3t}$
   (c) $-10e^{3t}$
   (d) $2e^{3t}$
   (e) $-2e^{3t}$

3. If $\mathbf{X}_1$ and $\mathbf{X}_2$ are solutions of the second order system $\mathbf{X}' = A\mathbf{X}$ and $\mathbf{X}_p$ is a particular solution of $\mathbf{X}' = A\mathbf{X} + \mathbf{f}(t)$, then the general solution of $\mathbf{X}' = A\mathbf{X} + \mathbf{f}(t)$ is

   Select the correct answer.

   (a) $\mathbf{X}_1 + \mathbf{X}_2 + \mathbf{X}_p$
   (b) $\mathbf{X}_1 + \mathbf{X}_2 + c_3\mathbf{X}_p$
   (c) $c_1\mathbf{X}_1 + c_2\mathbf{X}_2$
   (d) $c_1\mathbf{X}_1 + c_2\mathbf{X}_2 + c_3\mathbf{X}_p$
   (e) $c_1\mathbf{X}_1 + c_2\mathbf{X}_2 + \mathbf{X}_p$

4. The characteristic equation of $A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$ is

   Select the correct answer.

   (a) $\lambda^2 - 1 = 0$
   (b) $\lambda^2 + 4\lambda + 5 = 0$
   (c) $\lambda^2 - 4\lambda + 5 = 0$
   (d) $\lambda^2 + 4\lambda + 3 = 0$
   (e) $\lambda^2 - 4\lambda + 3 = 0$

5. The eigenvalues of $A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$ are

   Select the correct answer.

   (a) ± 1
   (b) $-2 \pm i$
   (c) $2 \pm i$
   (d) 1, 3
   (e) -1, -3

6. The solution of the system $\mathbf{X}' = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \mathbf{X}$ is

   Select the correct answer.

   (a) $\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-3t}$
   (b) $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-3t}$
   (c) $\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{3t}$
   (d) $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{3t}$
   (e) none of the above

7. The characteristic equation of $A = \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}$ is

   Select the correct answer.

   (a) $\lambda^2 - 3\lambda + 4 = 0$
   (b) $\lambda^2 - 3\lambda - 4 = 0$
   (c) $\lambda^2 - 3\lambda = 0$
   (d) $\lambda^2 + 3\lambda = 0$
   (e) $\lambda^2 + 3\lambda + 4 = 0$

8. The eigenvalues of $A = \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}$ are

   Select the correct answer.

   (a) 0, 3
   (b) 0, -3
   (c) $(3 \pm \sqrt{7}i)/2$
   (d) $(-3 \pm \sqrt{7}i)/2$
   (e) -1, 4

9. The solution of the system $\mathbf{X}' = \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix} \mathbf{X}$ is

   Select the correct answer.

   (a) $\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-3t}$
   (b) $\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{3t}$
   (c) $\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-3t}$
   (d) $\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{3t}$
   (e) none of the above

10. The characteristic equation of $A = \begin{pmatrix} -3 & -2 \\ 2 & 1 \end{pmatrix}$ is

    Select the correct answer.

    (a) $\lambda^2 - 2\lambda - 4 = 0$
    (b) $\lambda^2 - 2\lambda - 1 = 0$
    (c) $\lambda^2 - 2\lambda + 1 = 0$
    (d) $\lambda^2 + 2\lambda - 1 = 0$
    (e) $\lambda^2 + 2\lambda + 1 = 0$

11. The eigenvalues of $A = \begin{pmatrix} -3 & -2 \\ 2 & 1 \end{pmatrix}$ are

    Select the correct answer.

    (a) 1, 1
    (b) -1, -1
    (c) $1 \pm \sqrt{5}$
    (d) $1 \pm \sqrt{2}$
    (e) $-1 \pm \sqrt{2}$

12. The solution of the system $\mathbf{X}' = \begin{pmatrix} -3 & -2 \\ 2 & 1 \end{pmatrix} \mathbf{X}$ is

    Select the correct answer.

    (a) $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t} + c_2 \left[\begin{pmatrix} 1 \\ -1 \end{pmatrix} te^{-t} + \begin{pmatrix} -1/2 \\ 0 \end{pmatrix} e^{-t} \right]$
    (b) $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{t} + c_2 \left[\begin{pmatrix} 1 \\ -1 \end{pmatrix} te^{t} + \begin{pmatrix} -1/2 \\ 0 \end{pmatrix} e^{t} \right]$
    (c) $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{t}$
    (d) $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{t}$
    (e) none of the above

13. The characteristic equation of $A = \begin{pmatrix} -3 & -5 \\ 1 & -1 \end{pmatrix}$ is

    Select the correct answer.

    (a) $\lambda^2 + 4\lambda - 2 = 0$
    (b) $\lambda^2 - 4\lambda - 2 = 0$
    (c) $\lambda^2 - 4\lambda - 8 = 0$
    (d) $\lambda^2 + 4\lambda + 8 = 0$
    (e) $\lambda^2 + 4\lambda + 2 = 0$

14. The eigenvalues of $A = \begin{pmatrix} -3 & -5 \\ 1 & -1 \end{pmatrix}$ are

    Select the correct answer.

    (a) $-2 \pm \sqrt{2}$
    (b) $2 \pm \sqrt{2}$
    (c) $-2 \pm 2i$
    (d) $2 \pm 2i$
    (e) $2 \pm 2\sqrt{3}$

15. The characteristic equation of $A = \begin{pmatrix} 1 & -12 & -14 \\ 1 & 2 & -3 \\ 1 & 1 & -2 \end{pmatrix}$ is

    Select the correct answer.

    (a) $-\lambda^3 + \lambda^2 + 25\lambda - 25 = 0$
    (b) $-\lambda^3 + \lambda^2 - 25\lambda + 25 = 0$
    (c) $-\lambda^3 - \lambda^2 + 25\lambda - 25 = 0$
    (d) $-\lambda^3 - \lambda^2 - 25\lambda + 25 = 0$
    (e) none of the above

16. The eigenvalues of $A = \begin{pmatrix} 1 & -12 & -14 \\ 1 & 2 & -3 \\ 1 & 1 & -2 \end{pmatrix}$ are

    Select the correct answer.

    (a) $1, \pm 5i$
    (b) $-1, \pm 5i$
    (c) $1, 4, -6$
    (d) $1, -4, 6$
    (e) none of the above

17. A particular solution of $\mathbf{X}' = \begin{pmatrix} 2 & 3 \\ 2 & 1 \end{pmatrix}\mathbf{X} + \begin{pmatrix} t \\ 1 \end{pmatrix}$ is

    Select the correct answer.

    (a) $\begin{pmatrix} t/4 + 19/16 \\ -t/2 + 7/8 \end{pmatrix}$

    (b) $\begin{pmatrix} t/4 - 19/16 \\ -t/2 + 7/8 \end{pmatrix}$

    (c) $\begin{pmatrix} t/4 + 1/8 \\ -t/2 - 7/8 \end{pmatrix}$

    (d) $\begin{pmatrix} t/4 - 1/8 \\ -t/2 - 7/8 \end{pmatrix}$

    (e) none of the above

18. The general solution of $\mathbf{X}' = \begin{pmatrix} 2 & 3 \\ 2 & 1 \end{pmatrix}\mathbf{X} + \begin{pmatrix} t \\ 1 \end{pmatrix}$ is

    Select the correct answer.

    (a) $c_1\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{t} + c_2\begin{pmatrix} 3 \\ 2 \end{pmatrix}e^{-4t} + \begin{pmatrix} t/4 + 19/16 \\ -t/2 + 7/8 \end{pmatrix}$

    (b) $c_1\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{t} + c_2\begin{pmatrix} 3 \\ 2 \end{pmatrix}e^{-4t} + \begin{pmatrix} t/4 + 19/16 \\ -t/2 + 7/8 \end{pmatrix}$

    (c) $c_1\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{-t} + c_2\begin{pmatrix} 3 \\ 2 \end{pmatrix}e^{4t} + \begin{pmatrix} t/4 - 19/16 \\ -t/2 + 7/8 \end{pmatrix}$

    (d) $c_1\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{t} + c_2\begin{pmatrix} 3 \\ 2 \end{pmatrix}e^{-4t} + \begin{pmatrix} t/4 - 19/16 \\ -t/2 + 7/8 \end{pmatrix}$

    (e) none of the above

19. The solution of the previous problem that satisfies the initial condition $\mathbf{X}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is

    Select the correct answer.

    (a) $\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{-t} + \begin{pmatrix} 3 \\ 2 \end{pmatrix}e^{4t}/16 + \begin{pmatrix} t/4 - 19/16 \\ -t/2 + 7/8 \end{pmatrix}$

    (b) $\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{-t}/16 + \begin{pmatrix} 3 \\ 2 \end{pmatrix}e^{4t} + \begin{pmatrix} t/4 + 19/16 \\ -t/2 + 7/8 \end{pmatrix}$

    (c) $\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{-t} + \begin{pmatrix} 3 \\ 2 \end{pmatrix}e^{4t}/16 + \begin{pmatrix} t/4 + 19/16 \\ -t/2 + 7/8 \end{pmatrix}$

    (d) $\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{-t}/16 + \begin{pmatrix} 3 \\ 2 \end{pmatrix}e^{4t} + \begin{pmatrix} t/4 - 19/16 \\ -t/2 + 7/8 \end{pmatrix}$

    (e) none of the above

20. A particular solution of $\mathbf{X}' = \begin{pmatrix} 2 & 3 \\ 2 & 1 \end{pmatrix} \mathbf{X} + \begin{pmatrix} e^{-t} \\ 0 \end{pmatrix}$ is

Select the correct answer.

(a) $\begin{pmatrix} 2te^t/5 + 3e^t \\ -2te^t/5 + 2e^t \end{pmatrix}$

(b) $\begin{pmatrix} 2te^{-t}/5 + 3e^{-t} \\ -2te^{-t}/5 + 2e^{-t} \end{pmatrix}$

(c) $\begin{pmatrix} 2te^{-t}/5 - 3e^{-t} \\ -2te^{-t}/5 - 2e^{-t} \end{pmatrix}$

(d) $\begin{pmatrix} 2te^t/5 - 3e^t \\ -2te^t/5 - 2e^t \end{pmatrix}$

(e) none of the above

ANSWER KEY

Zill Differential Equations 9e Chapter 8 Form D

1. a
2. d
3. e
4. e
5. d
6. d
7. c
8. a
9. e
10. e
11. b
12. a
13. d
14. c
15. b
16. a
17. b
18. c
19. a
20. e

1. Write the system in matrix form:
$$\frac{dx}{dt} = -4x + 2y$$
$$\frac{dy}{dt} = x - 3y.$$

2. Write the system in matrix form:
$$\frac{dx}{dt} = x + 2y + z$$
$$\frac{dy}{dt} = x + 3y - z$$
$$\frac{dz}{dt} = y - z$$

3. Write the system without matrices:
$$\mathbf{X}' = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix} \mathbf{X}.$$

4. The Wronskian of the vector functions $\mathbf{X}_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-4t}$ and $\mathbf{X}_2 = \begin{pmatrix} 3 \\ 6 \end{pmatrix} e^{5t}$ is

   Select the correct answer.

   (a) $-12e^t$
   (b) $12e^t$
   (c) 0
   (d) e^t
   (e) $-e^t$

5. The Wronskian of the vector functions $\mathbf{X}_1 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} e^t$, $\mathbf{X}_2 = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} e^{2t}$, and $\mathbf{X}_3 = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} e^{3t}$is

   Select the correct answer.

   (a) 0
   (b) $16e^{6t}$
   (c) $-16e^{6t}$
   (d) $4e^{6t}$
   (e) $-4e^{-6t}$

6. What is the characteristic equation for the matrix $\begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix}$? What are the eigenvalues?

7. Solve the system $\mathbf{X}' = \begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix} \mathbf{X}$.

8. If $\mathbf{X}_1$, $\mathbf{X}_2$, and $\mathbf{X}_3$ are solutions of the third order system $\mathbf{X}' = A\mathbf{X}$ and $\mathbf{X}_p$ is a particular solution of $\mathbf{X}' = A\mathbf{X} + \mathbf{f}(t)$, then the general solution of $\mathbf{X}' = A\mathbf{X} + \mathbf{f}(t)$ is

   Select the correct answer.

   (a) $\mathbf{X}_1 + \mathbf{X}_2 + \mathbf{X}_3 + \mathbf{X}_p$
   (b) $\mathbf{X}_1 + \mathbf{X}_2 + \mathbf{X}_3 + c_4\mathbf{X}_p$
   (c) $c_1\mathbf{X}_1 + c_2\mathbf{X}_2 + c_3\mathbf{X}_3$
   (d) $c_1\mathbf{X}_1 + c_2\mathbf{X}_2 + c_3\mathbf{X}_3 + \mathbf{X}_p$
   (e) $c_1\mathbf{X}_1 + c_2\mathbf{X}_2 + c_3\mathbf{X}_3 + c_4\mathbf{X}_p$

9. The characteristic equation for the matrix $\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$ is

   Select the correct answer

   (a) $\lambda^2 - 6\lambda + 13 = 0$
   (b) $\lambda^2 - 6\lambda + 5 = 0$
   (c) $\lambda^2 - 6\lambda - 5 = 0$
   (d) $\lambda^2 + 6\lambda + 13 = 0$
   (e) $\lambda^2 + 6\lambda + 5 = 0$

10. The solution of the system $\mathbf{X}' = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}\mathbf{X}$ is

    Select the correct answer

    (a) $c_1\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^t + c_2\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{5t}$
    (b) $c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^t + c_2\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{5t}$
    (c) $c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^t + c_2\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{5t}$
    (d) $c_1\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^t + c_2\begin{pmatrix} -1 \\ 1 \end{pmatrix}e^{5t}$
    (e) none of the above

11. What is the characteristic equation for the matrix $\begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix}$? What are the eigenvalues?

12. Solve the system $\mathbf{X}' = \begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix}\mathbf{X}$.

13. Find a particular solution of $\mathbf{X}' = \begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix}\mathbf{X} + \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

14. Find a particular solution of $\mathbf{X}' = \begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix} \mathbf{X} + \begin{pmatrix} e^{3t} \\ 0 \end{pmatrix}$

15. The characteristic equation for the matrix $\begin{pmatrix} 1 & -2 & 2 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$ is

Select the correct answer

(a) $-\lambda^3 + 3\lambda^2 + 9\lambda + 5 = 0$
(b) $-\lambda^3 + 3\lambda^2 - 9\lambda + 5 = 0$
(c) $-\lambda^3 + 3\lambda^2 - 9\lambda - 5 = 0$
(d) $-\lambda^3 - 3\lambda^2 + 9\lambda - 5 = 0$
(e) $-\lambda^3 - 3\lambda^2 + 9\lambda + 5 = 0$

16. The solution of the system $\mathbf{X}' = \begin{pmatrix} 1 & -2 & 2 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix} \mathbf{X}$ is

Select the correct answer

(a) $c_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} e^{-t} + c_3 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} e^{5t}$
(b) $c_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} e^{-t} + c_3 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} e^{5t}$
(c) $c_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} e^{-t} + c_3 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} e^{5t}$
(d) $c_1 \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} e^{-t} + c_3 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} e^{5t}$
(e) $c_1 \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} e^{-t} + c_3 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} e^{5t}$

17. The characteristic equation of $A = \begin{pmatrix} 2 & -2 \\ 1 & 4 \end{pmatrix}$ is

Select the correct answer

(a) $\lambda^2 - 6\lambda + 6 = 0$
(b) $\lambda^2 + 6\lambda - 6 = 0$
(c) $\lambda^2 + 6\lambda + 10 = 0$
(d) $\lambda^2 - 6\lambda - 10 = 0$
(e) $\lambda^2 - 6\lambda + 10 = 0$

18. The eigenvalues of $A = \begin{pmatrix} 2 & -2 \\ 1 & 4 \end{pmatrix}$ are

    Select the correct answer

    (a) 1, 6
    (b) -1, -6
    (c) $-3 \pm i$
    (d) $3 \pm i$
    (e) $3 \pm \sqrt{19}$

19. Let $A = \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix}$. Find e^{At}.

20. Let $A = \begin{pmatrix} 0 & -2 \\ 0 & 0 \end{pmatrix}$. Then $e^{At} =$

    Select the correct answer.

    (a) $A + At$
    (b) $A - At$
    (c) $I + At$
    (d) $I - At$
    (e) none of the above

ANSWER KEY
Zill Differential Equations 9e Chapter 8 Form E

1. $\mathbf{X}' = \begin{pmatrix} -4 & 2 \\ 1 & -3 \end{pmatrix} \mathbf{X}.$

2. $\mathbf{X}' = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 3 & -1 \\ 0 & 1 & -1 \end{pmatrix} \mathbf{X}.$

3. $\dfrac{dx}{dt} = 2x + 4y$
$\dfrac{dy}{dt} = x + 3y.$

4. b

5. a

6. $\lambda^2 - 3\lambda + 2 = 0$, the eigenvalues are 1, 2

7. $\mathbf{X} = c_1 \begin{pmatrix} 2 \\ 3 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$

8. d

9. b

10. a

11. $\lambda^2 - 6\lambda + 9 = 0$, the eigenvalues are 3, 3

12. $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{3t} + c_2 \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{3t} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} \right]$

13. $\mathbf{X}_p = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

14. $\mathbf{X}_p = \begin{pmatrix} t - t^2/2 \\ t^2/2 \end{pmatrix} e^{3t}$

15. a

16. b

17. e

18. d

19. $e^{At} = \begin{pmatrix} e^{2t} & 0 \\ 0 & e^{4t} \end{pmatrix}$

20. c

1. The Wronskian of the vector functions $\mathbf{X}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$ and $\mathbf{X}_2 = \begin{pmatrix} 2 \\ 4 \end{pmatrix} e^{3t}$ is

   Select the correct answer.

   (a) $6e^{-5t}$
   (b) $-6e^{-5t}$
   (c) $2e^{-5t}$
   (d) $2e^{5t}$
   (e) $-2e^{5t}$

2. The Wronskian of the vector functions $\mathbf{X}_1 = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} e^{t}$, $\mathbf{X}_2 = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} e^{-3t}$, and $\mathbf{X}_3 = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix} e^{5t}$ is

   Select the correct answer.

   (a) $18e^{-3t}$
   (b) $-18e^{-3t}$
   (c) $18e^{3t}$
   (d) $6e^{3t}$
   (e) $-6e^{3t}$

3. Write the system in matrix form:
   $$\frac{dx}{dt} = -2x + 3y$$
   $$\frac{dy}{dt} = 5x.$$

4. Write the system in matrix form:
   $$\frac{dx}{dt} = -2x + 4y - z + 2e^t$$
   $$\frac{dy}{dt} = 5x + 2y + 3z + t^3$$
   $$\frac{dz}{dt} = 3x + 4y + 3z - 1.$$

5. Write the system without matrices:
   $$\mathbf{X}' = \begin{pmatrix} 3 & 1 \\ -2 & 2 \end{pmatrix} \mathbf{X}.$$

6. The characteristic equation of $A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$ is

   Select the correct answer.

   (a) $\lambda^2 + 4\lambda + 5 = 0$
   (b) $\lambda^2 - 4\lambda + 5 = 0$
   (c) $\lambda^2 - 4\lambda - 3 = 0$
   (d) $\lambda^2 - 4\lambda + 3 = 0$
   (e) $\lambda^2 + 4\lambda + 3 = 0$

7. The eigenvalues of the matrix of the previous problem are

   Select the correct answer.

   (a) $2 \pm \sqrt{7}$
   (b) $-2 \pm i$
   (c) $2 \pm i$
   (d) $-1,\ -3$
   (e) $1,\ 3$

8. The solution of the system $\mathbf{X}' = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \mathbf{X}$ is

   Select the correct answer.

   (a) $\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-3t}$
   (b) $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-3t}$
   (c) $\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{3t}$
   (d) $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{3t}$
   (e) none of the above

9. The characteristic equation of $A = \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}$ is

   Select the correct answer.

   (a) $\lambda^2 + 3\lambda + 4 = 0$
   (b) $\lambda^2 - 3\lambda + 4 = 0$
   (c) $\lambda^2 - 3\lambda = 0$
   (d) $\lambda^2 - 3\lambda - 4 = 0$
   (e) $\lambda^2 + 3\lambda - 4 = 0$

10. The eigenvalues of the matrix of the previous problem are

    Select the correct answer.

    (a) $(3 \pm \sqrt{7}i)/2$
    (b) $(-3 \pm \sqrt{7}i)/2$
    (c) 0, 3
    (d) 1, −4
    (e) −1, 4

11. The solution of the system $\mathbf{X}' = \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix} \mathbf{X}$ is

    Select the correct answer.

    (a) $\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-3t}$
    (b) $\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{3t}$
    (c) $\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-3t}$
    (d) $\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{3t}$
    (e) none of the above

12. Find the characteristic equation of $A = \begin{pmatrix} -4 & 3 \\ -3 & 2 \end{pmatrix}$. What are the eigenvalues?

13. Solve the system $\mathbf{X}' = \begin{pmatrix} -4 & 3 \\ -3 & 2 \end{pmatrix} \mathbf{X}$.

14. Solve the system of the previous problem if $\mathbf{X}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

15. Find the characteristic equation of $A = \begin{pmatrix} -4 & 5 \\ -2 & 2 \end{pmatrix}$. What are the eigenvalues?

16. Solve the system $\mathbf{X}' = \begin{pmatrix} -4 & 5 \\ -2 & 2 \end{pmatrix} \mathbf{X}$.

17. Find the solution of $\mathbf{X}' = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 0 & 6 \\ -4 & 0 & -3 \end{pmatrix} \mathbf{X}$.

18. Find a particular solution of $\mathbf{X}' = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 0 & 6 \\ -4 & 0 & -3 \end{pmatrix} \mathbf{X} + \begin{pmatrix} t \\ -1 \\ 0 \end{pmatrix}$.

19. Let $A = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}$. Then $e^{At} =$

Select the correct answer.

(a) $I\cos(2t) - A\sin(2t)/2$
(b) $I\cos(2t) + A\sin(2t)/2$
(c) $Ie^{2t} - Ae^{2t}/2$
(d) $Ie^{2t} + Ae^{2t}/2$
(e) none of the above

20. The characteristic equation of $A = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 0 & 6 \\ -4 & 0 & -3 \end{pmatrix}$ is

Select the correct answer.

(a) $-\lambda^3 - \lambda^2 + \lambda - 33 = 0$
(b) $-\lambda^3 - \lambda^2 - \lambda + 33 = 0$
(c) $-\lambda^3 - \lambda^2 + \lambda - 15 = 0$
(d) $-\lambda^3 - \lambda^2 + \lambda + 15 = 0$
(e) none of the above

1. d
2. e
3. $\mathbf{X}' = \begin{pmatrix} -2 & 3 \\ 5 & 0 \end{pmatrix} \mathbf{X}$
4. $\mathbf{X}' = \begin{pmatrix} -2 & 4 & -1 \\ 5 & 2 & 3 \\ 3 & 4 & 3 \end{pmatrix} \mathbf{X} + \begin{pmatrix} 2e^t \\ t^3 \\ -1 \end{pmatrix}$
5. $\frac{dx}{dt} = 3x + y$
   $\frac{dy}{dt} = -2x + 2y.$
6. d
7. e
8. d
9. c
10. c
11. e
12. $\lambda^2 + 2\lambda + 1 = 0$, eigenvalues are -1, -1
13. $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + c_2 \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} -1/3 \\ 0 \end{pmatrix} e^{-t} \right]$
14. $\mathbf{X} = -3 \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} -1/3 \\ 0 \end{pmatrix} e^{-t} \right]$
15. $\lambda^2 + 2\lambda + 2 = 0$, eigenvalues are $-1 \pm i$
16. $\mathbf{X} = c_1 e^{-t} \left[\begin{pmatrix} 5 \\ 3 \end{pmatrix} \cos t - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sin t \right] + c_2 e^{-t} \left[\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cos t + \begin{pmatrix} 5 \\ 3 \end{pmatrix} \sin t \right]$
17. $\mathbf{X} = c_1 \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} e^{-3t} + c_2 e^t \left[\begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} \cos(2t) - \begin{pmatrix} -1 \\ -3 \\ 0 \end{pmatrix} \sin(2t) \right] +$
    $c_3 e^t \left[\begin{pmatrix} -1 \\ -3 \\ 0 \end{pmatrix} \cos(2t) + \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} \sin(2t) \right]$
18. $\mathbf{X}_p = \begin{pmatrix} 0 \\ -t \\ 0 \end{pmatrix}$
19. b
20. c

1. Write the system in matrix form:
$$\frac{dx}{dt} = -4x + 2y$$
$$\frac{dy}{dt} = x - 3y.$$

2. Write the system in matrix form:
$$\frac{dx}{dt} = x + 2y + z + e^{2t}$$
$$\frac{dy}{dt} = x + 3y - z + t + 1$$
$$\frac{dz}{dt} = x + y - z - t^2$$

3. Write the system without matrices:
$$\mathbf{X}' = \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix} \mathbf{X} + \begin{pmatrix} \sin t \\ \cos t \end{pmatrix}.$$

4. The Wronskian of the vector functions $\mathbf{X}_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-4t}$ and $\mathbf{X}_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^{4t}$ is

   Select the correct answer.

   (a) $-8e^{8t}$
   (b) $8e^{8t}$
   (c) 0
   (d) 8
   (e) -8

5. The Wronskian of the vector functions $\mathbf{X}_1 = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} e^{t}$, $\mathbf{X}_2 = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} e^{-2t}$, and $\mathbf{X}_3 = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} e^{3t}$ is

   Select the correct answer.

   (a) 0
   (b) $-32e^{2t}$
   (c) $32e^{2t}$
   (d) $24e^{2t}$
   (e) $-24e^{2t}$

6. What is the characteristic equation for the matrix $\begin{pmatrix} 3 & 1 \\ 3 & 5 \end{pmatrix}$? What are the eigenvalues?

7. Solve the system $\mathbf{X}' = \begin{pmatrix} 3 & 1 \\ 3 & 5 \end{pmatrix} \mathbf{X}$.

8. What is the characteristic equation for the matrix $\begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix}$? What are the eigenvalues?

9. Solve the system $\mathbf{X}' = \begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix}\mathbf{X}$.

10. The characteristic equation for the matrix $A = \begin{pmatrix} 4 & -1 \\ 1 & 2 \end{pmatrix}$ is

    Select the correct answer.

    (a) $\lambda^2 + 6\lambda + 9 = 0$
    (b) $\lambda^2 + 6\lambda + 7 = 0$
    (c) $\lambda^2 - 6\lambda + 9 = 0$
    (d) $\lambda^2 - 6\lambda + 7 = 0$
    (e) $\lambda^2 - 6\lambda - 9 = 0$

11. The eigenvalues of the matrix $A = \begin{pmatrix} 4 & -1 \\ 1 & 2 \end{pmatrix}$ are

    Select the correct answer.

    (a) $-3, -3$
    (b) $3, 3$
    (c) $3 \pm \sqrt{2}$
    (d) $-3 \pm \sqrt{2}$
    (e) $3 \pm 3\sqrt{2}$

12. The solution of $\mathbf{X}' = \begin{pmatrix} 4 & -1 \\ 1 & 2 \end{pmatrix}\mathbf{X}$ is

    Select the correct answer.

    (a) $\mathbf{X} = c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{3t} + c_2\left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}te^{3t} + \begin{pmatrix} -1 \\ 0 \end{pmatrix}e^{3t}\right]$
    (b) $\mathbf{X} = c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{-3t} + c_2\left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}te^{-3t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}e^{-3t}\right]$
    (c) $\mathbf{X} = c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{-3t} + c_2\left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}te^{-3t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}e^{-3t}\right]$
    (d) $\mathbf{X} = c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{3t} + c_2\left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}te^{3t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}e^{3t}\right]$
    (e) none of the above

13. The characteristic equation for the matrix $A = \begin{pmatrix} 4 & -5 \\ 1 & 2 \end{pmatrix}$ is

Select the correct answer.

(a) $\lambda^2 + 6\lambda + 13 = 0$
(b) $\lambda^2 + 6\lambda + 3 = 0$
(c) $\lambda^2 - 6\lambda + 13 = 0$
(d) $\lambda^2 - 6\lambda + 3 = 0$
(e) $\lambda^2 - 6\lambda - 3 = 0$

14. The eigenvalues of the matrix $A = \begin{pmatrix} 4 & -5 \\ 1 & 2 \end{pmatrix}$ are

Select the correct answer.

(a) $3 \pm \sqrt{6}$
(b) $-3 \pm \sqrt{6}$
(c) $3 \pm 2\sqrt{3}$
(d) $3 \pm 2i$
(e) $-3 \pm 2i$

15. The solution of $\mathbf{X}' = \begin{pmatrix} 4 & -5 \\ 1 & 2 \end{pmatrix} \mathbf{X}$ is

Select the correct answer.

(a) $\mathbf{X} = c_1 e^{3t} \left[\begin{pmatrix} 5 \\ 1 \end{pmatrix} \cos(2t) - \begin{pmatrix} 0 \\ -2 \end{pmatrix} \sin(2t) \right] + c_2 e^{3t} \left[\begin{pmatrix} 0 \\ -2 \end{pmatrix} \cos(2t) + \begin{pmatrix} 5 \\ 1 \end{pmatrix} \sin(2t) \right]$

(b) $\mathbf{X} = c_1 e^{-3t} \left[\begin{pmatrix} 5 \\ 1 \end{pmatrix} \cos(2t) - \begin{pmatrix} 0 \\ -2 \end{pmatrix} \sin(2t) \right] + c_2 e^{-3t} \left[\begin{pmatrix} 0 \\ -2 \end{pmatrix} \cos(2t) + \begin{pmatrix} 5 \\ 1 \end{pmatrix} \sin(2t) \right]$

(c) $\mathbf{X} = c_1 e^{3t} \left[\begin{pmatrix} 1 \\ 5 \end{pmatrix} \cos(2t) - \begin{pmatrix} -2 \\ 0 \end{pmatrix} \sin(2t) \right] + c_2 e^{3t} \left[\begin{pmatrix} -2 \\ 0 \end{pmatrix} \cos(2t) + \begin{pmatrix} 1 \\ 5 \end{pmatrix} \sin(2t) \right]$

(d) $\mathbf{X} = c_1 e^{-3t} \left[\begin{pmatrix} 1 \\ 5 \end{pmatrix} \cos(2t) - \begin{pmatrix} -2 \\ 0 \end{pmatrix} \sin(2t) \right] + c_2 e^{-3t} \left[\begin{pmatrix} -2 \\ 0 \end{pmatrix} \cos(2t) + \begin{pmatrix} 1 \\ 5 \end{pmatrix} \sin(2t) \right]$

(e) none of the above

16. Find a particular solution of $\mathbf{X}' = \begin{pmatrix} 4 & -5 \\ 1 & 2 \end{pmatrix}\mathbf{X} + \begin{pmatrix} t \\ 1 \end{pmatrix}$.

17. Find the general solution of $\mathbf{X}' = \begin{pmatrix} 4 & -5 \\ 1 & 2 \end{pmatrix}\mathbf{X} + \begin{pmatrix} t \\ 1 \end{pmatrix}$.

18. Let $A = \begin{pmatrix} 0 & -5 & 2 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$. Find e^{At}.

19. Let $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$. Find P and D, so that $A = PDP^{-1}$, where D is a diagonal matrix.

20. If $\mathbf{X}_1$ and $\mathbf{X}_2$ are solutions of the second order system $\mathbf{X}' = A\mathbf{X}$ and $\mathbf{X}_p$ is a particular solution of $\mathbf{X}' = A\mathbf{X} + \mathbf{f}(t)$, then the general solution of $\mathbf{X}' = A\mathbf{X} + \mathbf{f}(t)$ is

    Select the correct answer.

    (a) $\mathbf{X} = \mathbf{X}_1 + \mathbf{X}_2 + \mathbf{X}_p$
    (b) $\mathbf{X} = \mathbf{X}_1 + \mathbf{X}_2 + c_3\mathbf{X}_p$
    (c) $\mathbf{X} = c_1\mathbf{X}_1 + c_2\mathbf{X}_2 + \mathbf{X}_p$
    (d) $\mathbf{X} = c_1\mathbf{X}_1 + c_2\mathbf{X}_2$
    (e) $\mathbf{X} = c_1\mathbf{X}_1 + c_2\mathbf{X}_2 + c_3\mathbf{X}_p$

1. $\mathbf{X}' = \begin{pmatrix} -4 & 2 \\ 1 & -3 \end{pmatrix} \mathbf{X}$

2. $\mathbf{X}' = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 3 & -1 \\ 1 & 1 & -1 \end{pmatrix} \mathbf{X} + \begin{pmatrix} e^{2t} \\ t+1 \\ -t^2 \end{pmatrix}$

3. $\dfrac{dx}{dt} = x + \sin t$
   $\dfrac{dy}{dt} = -x + 2y + \cos t.$

4. d

5. c

6. $\lambda^2 - 8\lambda + 12 = 0$, eigenvalues are 2, 6

7. $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{6t}$

8. $\lambda^2 - 6\lambda + 8 = 0$, eigenvalues are 2, 4

9. $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{4t}$

10. c

11. b

12. d

13. c

14. d

15. a

16. $\mathbf{X}_p = \begin{pmatrix} -2t/13 - 64/169 \\ t/13 - 46/169 \end{pmatrix}$

17. $\mathbf{X} = c_1 e^{3t} \left[\begin{pmatrix} 5 \\ 1 \end{pmatrix} \cos(2t) - \begin{pmatrix} 0 \\ -2 \end{pmatrix} \sin(2t) \right] +$
    $c_2 e^{3t} \left[\begin{pmatrix} 0 \\ -2 \end{pmatrix} \cos(2t) + \begin{pmatrix} 5 \\ 1 \end{pmatrix} \sin(2t) \right] + \begin{pmatrix} -2t/13 - 64/169 \\ t/13 - 46/169 \end{pmatrix}$

18. $e^{At} = \begin{pmatrix} 1 & -5t & 2t - 5t^2 \\ 0 & 1 & 2t \\ 0 & 0 & 1 \end{pmatrix}$

19. $D = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$, $P = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

20. c

1. The Wronskian of the vector functions $\mathbf{X}_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix} e^{5t}$ and $\mathbf{X}_2 = \begin{pmatrix} 3 \\ 4 \end{pmatrix} e^{-3t}$ is

   Select the correct answer.

   (a) 0
   (b) e^{2t}
   (c) $-e^{2t}$
   (d) $-e^{-2t}$
   (e) e^{-2t}

2. The Wronskian of the vector functions $\mathbf{X}_1 = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} e^{-t}$, $\mathbf{X}_2 = \begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix} e^{2t}$, and $\mathbf{X}_3 = \begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix} e^{3t}$ is

   Select the correct answer.

   (a) $30e^{4t}$
   (b) $-30e^{4t}$
   (c) $16e^{4t}$
   (d) $-16e^{4t}$
   (e) 0

3. Write the system in matrix form:
   $$\frac{dx}{dt} = 5x - 3y + 2$$
   $$\frac{dy}{dt} = 2x + 2y - \sin t.$$

4. Write the system in matrix form:
   $$\frac{dx}{dt} = 2x + 4y - z$$
   $$\frac{dy}{dt} = -5x + 2y + 6z$$
   $$\frac{dz}{dt} = x + 2y + 3z$$

5. Write the system without matrices:
   $$\mathbf{X}' = \begin{pmatrix} 5 & 1 \\ 2 & 3 \end{pmatrix} \mathbf{X}.$$

6. Solve the equation $\mathbf{X}' = \begin{pmatrix} -1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{pmatrix} \mathbf{X}$

7. The characteristic equation of $\begin{pmatrix} -6 & 2 \\ -1 & -3 \end{pmatrix}$ is

   Select the correct answer.

   (a) $\lambda^2 + 9\lambda - 20 = 0$
   (b) $\lambda^2 - 9\lambda + 16 = 0$
   (c) $\lambda^2 - 9\lambda + 20 = 0$
   (d) $\lambda^2 + 9\lambda + 16 = 0$
   (e) $\lambda^2 + 9\lambda + 20 = 0$

8. The eigenvalues of $\begin{pmatrix} -6 & 2 \\ -1 & -3 \end{pmatrix}$ are

   Select the correct answer.

   (a) 4, 5
   (b) -4, -5
   (c) $(9 \pm 3\sqrt{3})/2$
   (d) $(-9 \pm 3\sqrt{3})/2$
   (e) $(9 \pm \sqrt{161})/2$

9. The solution of the system $\mathbf{X}' = \begin{pmatrix} -6 & 2 \\ -1 & -3 \end{pmatrix} \mathbf{X}$ is

   Select the correct answer.

   (a) $\mathbf{X} = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-5t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-4t}$
   (b) $\mathbf{X} = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{5t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t}$
   (c) $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-5t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-4t}$
   (d) $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{5t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t}$
   (e) none of the above

10. The characteristic equation of $\begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix}$ is

    Select the correct answer.

    (a) $\lambda^2 + 6\lambda - 8 = 0$
    (b) $\lambda^2 - 6\lambda + 2 = 0$
    (c) $\lambda^2 - 6\lambda + 8 = 0$
    (d) $\lambda^2 + 6\lambda + 2 = 0$
    (e) $\lambda^2 + 6\lambda + 8 = 0$

11. The eigenvalues of $\begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix}$ are

    Select the correct answer.

    (a) 2, 4
    (b) −2, −4
    (c) $(3 \pm \sqrt{7})/2$
    (d) $(-3 \pm \sqrt{7})/2$
    (e) $(-3 \pm \sqrt{17})/2$

12. The solution of the system $\mathbf{X}' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \mathbf{X}$ is

    Select the correct answer.

    (a) $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-4t} + c_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{-2t}$
    (b) $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t} + c_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{2t}$
    (c) $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-4t} + c_2 \begin{pmatrix} 3 \\ 1 \end{pmatrix} e^{-2t}$
    (d) $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t} + c_2 \begin{pmatrix} 3 \\ 1 \end{pmatrix} e^{2t}$
    (e) none of the above

13. Find the characteristic equation of $\begin{pmatrix} 6 & -2 \\ 4 & 2 \end{pmatrix}$. What are the eigenvalues?

14. Solve the equation $\mathbf{X}' = \begin{pmatrix} 6 & -2 \\ 4 & 2 \end{pmatrix} \mathbf{X}$.

15. Find the characteristic equation of $\begin{pmatrix} 3 & -2 \\ 5 & 5 \end{pmatrix}$. What are the eigenvalues?

16. Solve the equation $\mathbf{X}' = \begin{pmatrix} 3 & -2 \\ 5 & 5 \end{pmatrix}\mathbf{X}$.

17. Let $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$. The eigenvalues of A are 1 and 3. Let $D = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$. Then $A = PDP^{-1}$, where $P =$

    Select the correct answer.

    (a) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

    (b) $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

    (c) $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

    (d) $\begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix}$

    (e) $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

18. Find a fundamental solution matrix for the equation $\mathbf{X}' = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}\mathbf{X}$

19. Solve the equation $\mathbf{X}' = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}\mathbf{X} + \begin{pmatrix} e^{-t} \\ 0 \end{pmatrix}$

20. Let $A = \begin{pmatrix} -3 & 0 \\ 0 & 2 \end{pmatrix}$. Find e^{At}.

ANSWER KEY

Zill Differential Equations 9e Chapter 8 Form H

1. c
2. e
3. $\mathbf{X}' = \begin{pmatrix} 5 & -3 \\ 2 & 2 \end{pmatrix}\mathbf{X} + \begin{pmatrix} 2 \\ -\sin t \end{pmatrix}$
4. $\mathbf{X}' = \begin{pmatrix} 2 & 4 & -1 \\ -5 & 2 & 6 \\ 1 & 2 & 3 \end{pmatrix}\mathbf{X}$
5. $\dfrac{dx}{dt} = 5x + y$
   $\dfrac{dy}{dt} = 2x + 3y.$
6. $\mathbf{X} = c_1\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}e^{-t} + c_2\begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}e^{3t} + c_3\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}e^{-2t}$
7. e
8. b
9. a
10. c
11. a
12. b
13. $\lambda^2 - 8\lambda + 20 = 0$, eigenvalues are $4 \pm 2i$
14. $\mathbf{X} = c_1e^{4t}\left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}\cos(2t) - \begin{pmatrix} 0 \\ -1 \end{pmatrix}\sin(2t)\right] + c_2e^{4t}\left[\begin{pmatrix} 0 \\ -1 \end{pmatrix}\cos(2t) + \begin{pmatrix} 1 \\ 1 \end{pmatrix}\sin(2t)\right]$
15. $\lambda^2 - 8\lambda + 25 = 0$, eigenvalues are $4 \pm 3i$
16. $\mathbf{X} = c_1e^{4t}\left[\begin{pmatrix} 2 \\ -1 \end{pmatrix}\cos(3t) - \begin{pmatrix} 0 \\ -3 \end{pmatrix}\sin(3t)\right] + c_2e^{4t}\left[\begin{pmatrix} 0 \\ -3 \end{pmatrix}\cos(3t) + \begin{pmatrix} 2 \\ -1 \end{pmatrix}\sin(3t)\right]$
17. e
18. $\Phi = \begin{pmatrix} e^{-t} & 2e^{4t} \\ -e^{-t} & 3e^{4t} \end{pmatrix}$
19. $\mathbf{X} = \Phi C + \begin{pmatrix} 3te^{-t}/5 - 2e^{-t}/25 \\ -3te^{-t}/5 - 3e^{-t}/25 \end{pmatrix}$ where Φ is the fundamental solution matrix of the previous problem and C is any constant vector
20. $A = \begin{pmatrix} e^{-3t} & 0 \\ 0 & e^{2t} \end{pmatrix}$