1. The Wronskian of the vector functions
$$\mathbf{X}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t}$$
 and $\mathbf{X}_2 = \begin{pmatrix} 3 \\ 5 \end{pmatrix} e^{6t}$ is Select the correct answer.

- (a) $8e^{4t}$
- (b) $2e^{4t}$
- (c) $-8e^{4t}$
- (d) $-2e^{4t}$
- (e) $15e^{4t}$

2. The Wronskian of the vector functions
$$\mathbf{X}_1 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} e^t$$
, $\mathbf{X}_2 = \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix} e^{2t}$ and $\mathbf{X}_3 = \begin{pmatrix} -1 \\ 5 \\ 0 \end{pmatrix} e^{3t}$ is

- (a) $55e^{6t}$
- (b) $-55e^{6t}$
- (c) $45e^{6t}$
- (d) $-45e^{-6t}$
- (e) $45e^{-6t}$
- 3. If \mathbf{X}_1 , \mathbf{X}_2 , and \mathbf{X}_3 are solutions of the third order system $\mathbf{X}' = A\mathbf{X}$ and \mathbf{X}_p is a particular solution of $\mathbf{X}' = A\mathbf{X} + \mathbf{f}(t)$, then the general solution of $\mathbf{X}' = A\mathbf{X} + \mathbf{f}(t)$ is Select the correct answer.

(a)
$$c_1 \mathbf{X}_1 + c_2 \mathbf{X}_2 + c_3 \mathbf{X}_3$$

(b)
$$c_1 \mathbf{X}_1 + c_2 \mathbf{X}_2 + c_3 \mathbf{X}_3 + c_4 \mathbf{X}_p$$

(c)
$$c_1 \mathbf{X}_1 + c_2 \mathbf{X}_2 + c_3 \mathbf{X}_3 + \mathbf{X}_p$$

(d)
$$\mathbf{X}_1 + \mathbf{X}_2 + \mathbf{X}_3 + \mathbf{X}_p$$

(e)
$$\mathbf{X}_1 + \mathbf{X}_2 + \mathbf{X}_3 + c_4 \mathbf{X}_p$$

4. The characteristic equation of
$$A = \begin{pmatrix} 1 & -3 \\ 2 & -4 \end{pmatrix}$$
 is

Select the correct answer.

(a)
$$\lambda^2 + 3\lambda - 2 = 0$$

(b)
$$\lambda^2 + 5\lambda + 2 = 0$$

(c)
$$\lambda^2 - 5\lambda + 2 = 0$$

$$(d) \lambda^2 + 3\lambda + 2 = 0$$

(e)
$$\lambda^2 - 3\lambda + 2 = 0$$

5. The solution of the system $\mathbf{X}' = \begin{pmatrix} 1 & -3 \\ 2 & -4 \end{pmatrix} \mathbf{X}$ is

Select the correct answer.

(a)
$$\mathbf{X} = c_1 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-2t}$$

(b)
$$\mathbf{X} = c_1 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$$

(c)
$$\mathbf{X} = c_1 \begin{pmatrix} 2 \\ 3 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-2t}$$

(d)
$$\mathbf{X} = c_1 \begin{pmatrix} 2 \\ 3 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$$

- (e) none of the above
- 6. The characteristic equation of $A = \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}$ is

Select the correct answer.

(a)
$$\lambda^2 + 3\lambda - 4 = 0$$

(b)
$$\lambda^2 + 3\lambda + 4 = 0$$

(c)
$$\lambda^2 - 3\lambda = 0$$

(d)
$$\lambda^2 + 3\lambda = 0$$

(e)
$$\lambda^2 - 3\lambda - 4 = 0$$

7. The eigenvalues of $A = \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}$ are

Select the correct answer.

(a)
$$1 \pm \sqrt{3}i$$

(b)
$$-1 \pm \sqrt{3}i$$

(c)
$$0, 3$$

(e)
$$0, -1$$

8. The solution of the system
$$\mathbf{X}' = \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix} \mathbf{X}$$
 is

(a)
$$\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-3t}$$

(b)
$$\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{3t}$$

(c)
$$\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-3t}$$

(d)
$$\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{3t}$$

- (e) none of the above
- 9. The characteristic equation of $A = \begin{pmatrix} -3 & -1 \\ 1 & -1 \end{pmatrix}$ is

Select the correct answer.

(a)
$$\lambda^2 + 4\lambda - 4 = 0$$

(b)
$$\lambda^2 + 4\lambda + 4 = 0$$

(c)
$$\lambda^2 - 2\lambda = 0$$

(d)
$$\lambda^2 - 2\lambda = 0$$

(e)
$$\lambda^2 - 2\lambda - 4 = 0$$

10. The solution of the system
$$\mathbf{X}' = \begin{pmatrix} -3 & -1 \\ 1 & -1 \end{pmatrix} \mathbf{X}$$
 is

Select the correct answer.

(a)
$$\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-2t} + c_2 \left[\begin{pmatrix} -1 \\ 1 \end{pmatrix} t e^{-2t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-2t} \right]$$

(b)
$$\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} t e^{-2t} + c_3 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-2t}$$

(c)
$$\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t}$$

(d)
$$\mathbf{X} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-t}$$

(e) none of the above

11. The characteristic equation for the matrix
$$A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 3 \end{pmatrix}$$
 is

(a)
$$-\lambda^3 - 7\lambda^2 - 16\lambda + 12 = 0$$

(b)
$$-\lambda^3 + 7\lambda^2 + 14\lambda + 6 = 0$$

(c)
$$-\lambda^3 + 7\lambda^2 - 14\lambda + 6 = 0$$

(d)
$$-\lambda^3 + 7\lambda^2 + 16\lambda + 12 = 0$$

(e)
$$-\lambda^3 + 7\lambda^2 - 16\lambda + 12 = 0$$

12. The eigenvalues of the matrix A of the previous problem are Select the correct answer.

(a)
$$-2, -2, 3$$

(c)
$$1 \pm i$$
, 3

(d)
$$-1 \pm i$$
, 3

(e)
$$1 \pm \sqrt{3}i$$
, 3

13. Let A be the matrix of the previous two problems. The solution of $\mathbf{X}' = A\mathbf{X}$ is Select the correct answer.

(a)
$$c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{2t} + c_3 \left[\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} t e^{2t} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{2t} \right]$$

(b)
$$c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{-2t} + c_3 \left[\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} t e^{-2t} + \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} e^{-2t} \right]$$

(c)
$$c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{-2t} + c_3 \begin{bmatrix} 0 \\ 1 \\ -1 \end{pmatrix} t e^{-2t} + \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} e^{-2t}$$

(d)
$$c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{2t} + c_3 \begin{bmatrix} 0 \\ 1 \\ -1 \end{pmatrix} t e^{2t} + \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} e^{2t}$$

(e)
$$c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{2t} + c_3 \left[\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} t e^{2t} + \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} e^{2t} \right]$$

14. The characteristic equation of
$$A = \begin{pmatrix} 2 & -2 \\ 2 & 2 \end{pmatrix}$$
 is

(a)
$$\lambda^2 - 4\lambda = 0$$

(b)
$$\lambda^2 + 4\lambda = 0$$

(c)
$$\lambda^2 + 4\lambda - 8 = 0$$

(d)
$$\lambda^2 - 4\lambda + 8 = 0$$

(e)
$$\lambda^2 - 4\lambda - 8 = 0$$

15. The eigenvalues of
$$A = \begin{pmatrix} 2 & -2 \\ 2 & 2 \end{pmatrix}$$
 are

Select the correct answer.

(a)
$$0, 4$$

(b)
$$0, -4$$

(c)
$$2, 2$$

(d)
$$2 \pm 2i$$

(e)
$$-2 \pm 2i$$

16. The solution of the system
$$\mathbf{X}' = \begin{pmatrix} 2 & -2 \\ 2 & 2 \end{pmatrix} \mathbf{X}$$
 is

Select the correct answer.

(a)
$$\mathbf{X} = c_1 e^{2t} \left[\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cos(2t) - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin(2t) \right] + c_2 e^{2t} \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos(2t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sin(2t) \right]$$

(b)
$$\mathbf{X} = c_1 e^{-2t} \left[\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cos(2t) - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin(2t) \right] + c_2 e^{-2t} \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos(2t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sin(2t) \right]$$

(c)
$$\mathbf{X} = c_1 e^{2t} \left[\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cos t + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin t \right] + c_2 e^{2t} \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos t + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sin t \right]$$

(d)
$$\mathbf{X} = c_1 e^{-2t} \left[\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cos t - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin t \right] + c_2 e^{-2t} \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos t + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sin t \right]$$