Seven Matrix Vector Multiplies

Five Matrix Matrix Multiplies

Testing Multiplies

BuildingQ

```
QFun[v_] := Module[
  {m, w, Q},
  m = Length[v];
  w = v;
  w[[1]] += Norm[v];
  w = w / Norm[v];

  Q = IdentityMatrix[m] - ─────────────── Outer[Times, w, Conjugate[w]]
                          w.Conjugate[w]
]
```

Testing!

```
A = RandomReal[{-1.0, 1.0}, {4, 4}];
v = A[[All, 1]];
Q = QFun[v];
MatrixForm[Q.A]
MatrixForm[Q.Q]
```

$$\begin{pmatrix} -1.47235 & 0.569091 & 1.13512 & 0.0715938 \\ 0. & -0.805308 & 0.470433 & -0.451227 \\ -1.11022 \times 10^{-16} & -0.396143 & -0.310926 & -0.153725 \\ 2.498 \times 10^{-16} & 1.15558 & -0.470344 & 0.154824 \end{pmatrix}$$

$$\begin{pmatrix} 1. & 1.38778 \times 10^{-17} & -1.11022 \times 10^{-16} & 4.85723 \times 10^{-17} \\ 1.38778 \times 10^{-17} & 1. & -3.46945 \times 10^{-18} & 1.30104 \times 10^{-18} \\ -1.11022 \times 10^{-16} & -3.46945 \times 10^{-18} & 1. & -2.22045 \times 10^{-16} \\ 4.85723 \times 10^{-17} & 1.30104 \times 10^{-18} & -2.22045 \times 10^{-16} & 1. \end{pmatrix}$$

OK so Q zeros the first column of A (except the first entry) and it is real easy to invert. One more cool thing is all of Q's eigenvalues have magnitude 1

```
Eigenvalues[Q]
```

$\{-1., 1., 1., 1.\}$

Putting the Qs together for HW2

Here is a simple example of the procedure for a 4x4 matrix.

```
QFun[v_, i_] := Module[
    {m, w, Q},
    m = Length[v];
    w = v;
    w[[i]] += Norm[v];
    w = w / Norm[v];

    Q = IdentityMatrix[m] - ───────────── Outer[Times, w, Conjugate[w]]
                                2
                            ─────────────
                            w.Conjugate[w]

]
```

```
A = RandomReal[ {-1.0, 1.0}, {4, 4}];
v = A[[All, 1]];
Q1 = QFun[v, 1];
A1 = Q1.A
v = A1[[All, 2]];
v[[1]] = 0.0;
Q2 = QFun[v, 2];
A2 = Q2.A1
v = A2[[All, 3]];
v[[1]] = 0;
v[[2]] = 0;
Q3 = QFun[v, 3];
A3 = Q3.A2
```

$\{\{-1.24307, 0.35711, 0.24048, -0.0300433\}, \{1.60462 \times 10^{-17}, 0.608849, 0.45614, -0.951516\},$
$\{3.05745 \times 10^{-17}, 0.615602, -0.65626, -0.505339\}, \{0., -0.67129, -0.137456, -0.265171\}\}$

$\{\{-1.24307, 0.35711, 0.24048, -0.0300433\}, \{-2.60971 \times 10^{-17}, -1.09558, 0.0310357, 0.65026\},$
$\{1.53533 \times 10^{-17}, -1.11022 \times 10^{-16}, -0.809799, 0.0731876\},$
$\{1.65982 \times 10^{-17}, 1.11022 \times 10^{-16}, 0.0299714, -0.896032\}\}$

$\{\{-1.24307, 0.35711, 0.24048, -0.0300433\}, \{-2.60971 \times 10^{-17}, -1.09558, 0.0310357, 0.65026\},$
$\{1.47289 \times 10^{-17}, -1.15053 \times 10^{-16}, -0.810353, 0.106278\},$
$\{-1.71547 \times 10^{-17}, -1.0684 \times 10^{-16}, 2.18922 \times 10^{-15}, 0.892712\}\}$