4620 Homework 1

The following programming assignments should be fairly straight forward. You can use any programing language or tool that you want.

1. Solve the linear system $\begin{pmatrix} -2 & 1 & 0 & 0 & 1 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 1 & 0 & 0 & 1 & -2 \end{pmatrix} \vec{x} = \begin{pmatrix} 1.0 \\ 2.0 \\ 3.0 \\ 4.0 \\ -10.0 \end{pmatrix}$ for the vector $\vec{x}$.

2. Solve (for $n = 30$) the linear system $A\,\vec{x} = \vec{b}$ for the vector $\vec{x}$ where b is an n vector with $b_i = i$ (for $1 \le i < n$) and $b_n = -\sum_{i=1}^{n-1} i$ while A is is an $n \times n$ matrix with all zero entries except $A_{i,i} = -2$ (for $1 \le i \le n$), $A_{i+1,i} = A_{i,i+1} = 1$ (for $1 \le i < n$) and $A_{1,n} = A_{n,1} = 1$. Plot the result. Explain (using language from linear algebra) the significance of the seperate formula for b_n.

3. Solve (for $n = 30$) the linear system $A\,\vec{x} = \vec{b}$ for the vector $\vec{x}$ where b is an n vector with $b_i = i$ (for $1 < i < n$) while A is is an $n \times n$ matrix with all zero entries except $A_{i,i} = -2$ (for $1 < i < n$), $A_{i+1,i} = A_{i,i+1} = 1$ (for $1 \le i < n$) and $A_{1,n} = A_{n,1} = 1$ while $A_{1,1} = A_{n,n} = b_1 = b_n = 10^6$. Comment on the values x_1 and x_n.

4. Compute the % of zero entries in the matrix A from Q2. Draw a picture (manually if you need to) of the locations of the zeroes in this matrix.

5. Define the matrix A as follows: $\begin{cases} A_{i,i} = -4 & \text{for} \quad 1 \le i \le n * m \\ A_{i,i+1} = 1 & \text{for} \quad 1 \le i < n * m \\ A_{i+1,i} = 1 & \text{for} \quad 1 \le i < n * m \\ A_{i,i+m} = 1 & \text{for} \quad 1 \le i \le (n-1) * m \\ A_{i+m,i} = 1 & \text{for} \quad 1 \le i \le (n-1) * m \end{cases}$. Solve the linear system $A.\vec{x} = \vec{b}$ with $b_i = 1$ for $n = 12$ and m = 13. Comment on the solution and explain how you know it is correct. Plot and describe the locations of the zeros of the matrix A.

6. For the matrix A in 5 solve the linear system with the same n and m but $b_i = 0$ for $i \neq 36$ and $b_{36} = 1$. Report the value of the x_{36} to six significant figures. Run the Jacobi Iteration $\vec{w}^{n+1} = D^{-1}\left(-(L+U)\,\vec{w}^{n} + \vec{b}\right)$ with $\vec{w}^{0} = \vec{0}$ to compute $\vec{w}^{5}$. Plot $\vec{w}^{5}$ and report the value $(\vec{w}^{5})_{36}$ to six significant figures. In the Jacobi Iteration D is the diagonal, L the lower triangular, and U the upper triangular portion of A i.e. $A = D + L + U$ where $D_{i,j} =$ If$[i == j, a_{i,j}, 0]$, $L_{i,j} =$ If$[i > j, a_{i,j}, 0]$, and $U_{i,j} =$ If$[i < j, a_{i,j}, 0]$. Comment on the connection between this exercise and solving the equation $A.\vec{x} = \vec{b}$.