310 Final
Name:_____

Show ALL your work and explain what you are doing!

1	**10**	
2	**10**	
3	**10**	
4	**10**	
5	**10**	
6	**10**	
7	**10**	
8	**10**	
9	**10**	
10	**10**	
Total	**100**	

1) [/10 pts] Solve $x\,y' - 4\,y = x^4$, $y(1) = 3$

```
DSolve[ {
  x y'[x] - 4 y[x] == x^4,
  y[1] == 3
    }, y[x], x]
```

$$\{\{y(x) \to \log(x)\, x^4 + 3\, x^4\}\}$$

2) [/10 pts] Find the general solution of

$$x\,y' + 4\,y = y^4\, x^4$$

```
DSolve[ {
 x y'[x] + 4 y[x] == y[x]^4 x^4
   }, y[x], x]
```

$$\left\{\left\{y(x)\to -\frac{2\,c_1^{8/3}}{\sqrt[3]{x^{12}-3\,x^4\,c_1^8}}\right\},\left\{y(x)\to \frac{2\,\sqrt[3]{-1}\,c_1^{8/3}}{\sqrt[3]{x^{12}-3\,x^4\,c_1^8}}\right\},\left\{y(x)\to -\frac{2\,(-1)^{2/3}\,c_1^{8/3}}{\sqrt[3]{x^{12}-3\,x^4\,c_1^8}}\right\}\right\}$$

3) [/10 pts] Find the general solution of

x^2 y' − x y + $x^2 + y^2 = 0$

```
DSolve[
 x^2 y'[x] - x y[x] + x^2 + y[x]^2 == 0,
 y[x], x]
```

$$\{\{y(x)\to -x\tan(c_1+\log(x))\}\}$$

4) [/10 pts] Find the General Solution of

$y^{(4)}$ + 6 $y^{(3)}$ + 8 y'' − 6 y' − 9 y = 36 − 150 e^{2x}

```
DSolve[
 y''''[x] + 6 y'''[x] + 8 y''[x] - 6 y'[x] - 9 y[x] == 36 - 150 E^(2 x),
 y[x], x] // Simplify
```

$$\{\{y(x)\to e^{-3x}c_1-2e^{2x}+e^{-3x}x\,c_2+e^{-x}c_3+e^{x}c_4-4\}\}$$

```
Expand[
 (r + 3)^2 (r - 1) (r + 1) ]
```

$$r^4+6r^3+8r^2-6r-9$$

5) [/10 pts] Find the Solution of

y'' + 4 y' + 13 y = 870 sin(2 x)

y(0) = 0

y'(0) = 0

```
9860 Sin[ 2 x]
```

```
DSolve[ {
 y''[x] + 4 y'[x] + 13 y[x] == 870 Sin[ 2 x],
 y[0] == 0,
 y'[0] == 0},
 y[x], x] // Simplify
```

$$\{\{y(x) \to -48\cos(2x) + 48\, e^{-2x}\cos(3x) + 54\sin(2x) - 4\, e^{-2x}\sin(3x)\}\}$$

6) [/10 pts] Find the general solution of

$y'' - 4y' + 4y = 18(1 + x)\, e^{2x}$