## War

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War is a popular card game among children in the United States. You simply need a standard 52 card deck of playing cards to play. War is most often played as a 2 player game but the rules can easily change to allow for more players. The deck is split as evenly as possible between the players. In the case of 2 players each gets 26 cards. The players take the card off the top of their deck and place it on the table. The highest valued card wins. In the game of war Aces are considered the highest. In the event of a tie (war) each player plays 2 face down cards and one face up card. The highest valued card of the second round wins all the cards on the table. If there is another tie the process is repeated until a winner is established. If a player is unable to play the required number of cards for a war they play as many as possible and treat their last card as the face up card. If this results in a tie then the player who has no cards in their hand loses.

We were interested in how long does the average game last? How many wars on average are there per game? How many double wars and so on. To do this we created a simulation using the program Wolfram Mathematica 8 to collect various statistics about the game. We created the deck by assigning all the cards a numerical value 2 through 14 in ascending order starting with 2 and ending with ace. We then made 4 copies of this list because there is no reason to differentiate between suits in this game and had our standard 52 card deck. We used a random sample function to simulate shuffling. We found that shuffling is crucial to playing the game or the game could end up in an infinite loop of the same wars being repeated. We used the take, drop, and join commands to simulate the movement of the cards during gameplay. We had to translate the rules of the game into logic statements. For example our game ended when either Player 1 or Player 2's hands were empty and the war pile was empty.

Our simulation gave us some interesting results to analyze. We ran the game 3,000,000 times and found the following statistics.

| Win percentage P1 | $49.968 \%$ |
| :--- | :--- |
| Win percentage P2 | $50.032 \%$ |
| Avg. Number of Turns | 342.108 |
| Avg. Number of Wars | 18.768 |
| Most Turns | 4071 |
| Least Turns | 15 |
| Most Wars | 219 |
| Most Double Wars | 19 |
| Most Triple Wars | 5 |
| Most Quadruple Wars | 2 |
| Probability of A War | .0533 |
| Probability of Ending On A War | .18933 |

The distribution of turns per game and wars per game was a non-normal distribution.


From the distribution of turns per game we found the probabilities of a game lasting a certain number of turns or fewer.

| $\mathbf{n}$ | $\mathbf{z}$ | $\mathbf{p}$ |
| :--- | ---: | ---: |
| $\mathrm{n}<100$ | -0.875 | 0.1908 |
| $\mathrm{n}<200$ | -0.513 | 0.304 |
| $\mathrm{n}<300$ | -0.152 | 0.4396 |
| $\mathrm{n}<400$ | 0.209 | 0.5828 |
| $\mathrm{n}<500$ | 0.57 | 0.7157 |
| $\mathrm{n}<600$ | 0.932 | 0.8243 |
| $\mathrm{n}<700$ | 1.293 | 0.902 |
| $\mathrm{n}<800$ | 1.654 | 0.95 |
| $\mathrm{n}<900$ | 2.015 | 0.978 |
| $\mathrm{n}<1000$ | 2.377 | 0.9913 |

We compared our data to the data found on the Wikipedia page for War (card game). Our simulations were a little different in that our "war" consisted of two face down cards under a face up card and theirs consisted of three face down cards. Also, we ran our data 3 million times, whereas the Wikipedia page had only 1 million. Wikipedia's average number of turns was 253.314 which was 89 fewer turns than ours. They had an average number of wars of 15.096 which was 4 fewer than ours. Their highest number of turns was 1,113 fewer than ours at 2,958 and their least number of turns was 3 fewer at 12. Their highest number of wars was 64 fewer than ours at 155 , their highest number of double wars was 7 fewer at 12 , our highest number of triple and quadruple wars were the same, and their highest number of quintuple wars were one greater than ours at 2

After we compared our data to Wikipedia's table we decided test out some variations to the game. We made it easy to implement the code by commenting out all the variations except the one we wanted running. We looked at what would happen if we played with 2 full decks, if we played with only one face down card in a war, and if we used 4 face down cards in a war. The results were somewhat
what we expected. Using two full decks made the game last about 3.5 times longer than the standard game. The distribution of turns per game was much wider, but still unusual.


| $\mathbf{n}$ | $\mathbf{z}$ | $\mathbf{p}$ |
| :--- | ---: | ---: |
| $n<400$ | -0.849 | 0.1979 |
| $n<600$ | -0.66 | 0.2546 |
| $n<800$ | -0.42 | 0.3192 |
| $n<1000$ | -0.281 | 0.3894 |
| $n<1200$ | -0.091 | 0.4637 |
| $n<1400$ | 0.099 | 0.5394 |
| $n<1600$ | 0.288 | 0.6133 |
| $n<1800$ | 0.478 | 0.6837 |
| $n<2000$ | 0.668 | 0.7479 |
| $n<2200$ | 0.857 | 0.8043 |
| $n<2400$ | 1.048 | 0.8527 |
| $n<2600$ | 1.236 | 0.8918 |
| $n<2800$ | 1.426 | 0.9231 |
| $n<3000$ | 1.616 | 0.947 |
| $n<3200$ | 1.805 | 0.9645 |
| $n<3400$ | 1.995 | 0.977 |

The number of face down cards seemed to have a more predictable effect. For every additional face down card in a war approximately 200 turns were shaved off the average game length. The distribution of turns per game did get slimmer as we predicted.

1-face down card:


| $\boldsymbol{n}$ | $\boldsymbol{z}$ | $\mathbf{p}$ |
| :--- | ---: | ---: |
| $\mathrm{n}<100$ | -0.962 | 0.168 |
| $\mathrm{n}<200$ | -0.7 | 0.242 |
| $\mathrm{n}<300$ | -0.437 | 0.3311 |
| $\mathrm{n}<400$ | -0.175 | 0.4305 |
| $\mathrm{n}<500$ | 0.087 | 0.5347 |
| $\mathrm{n}<600$ | 0.35 | 0.6368 |
| $\mathrm{n}<700$ | 0.612 | 0.7297 |
| $\mathrm{n}<800$ | 0.875 | 0.8092 |
| $\mathrm{n}<900$ | 1.137 | 0.8722 |
| $\mathrm{n}<1000$ | 1.4 | 0.9192 |
| $\mathrm{n}<1100$ | 1.662 | 0.9517 |
| $\mathrm{n}<1200$ | 1.925 | 0.9729 |

4-face down cards:


| $\boldsymbol{n}$ | $\boldsymbol{z}$ | $\boldsymbol{p}$ |
| :---: | ---: | ---: |
| $\mathrm{n}<100$ | -0.593 | 0.2766 |
| $\mathrm{n}<200$ | 0.082 | 0.5327 |
| $\mathrm{n}<300$ | 0.757 | 0.7755 |
| $\mathrm{n}<400$ | 1.437 | 0.9239 |
| $\mathrm{n}<500$ | 2.108 | 0.9825 |
| $\mathrm{n}<600$ | 2.783 | 0.9973 |

Future exploration of the game war might include, but not limited to, finding the probability of entering in an infinite loop if the shuffling function is removed, the effects that bringing in more players would have, what using more decks would do, and the amount of time the average game lasts.

## References

Giridharagopal, R. (2012, January 14). Here comes the science. Retrieved from http://www.rajgiri.net/index.php?page=6

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