Accuracy of FEAST

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Constructing a real symmetric matrix using rotations

```
Clear[SymMatWithEvals]
SymMatWithEvals[evals_List]:= Module[
{n=Length[evals],Q,A},
(* Form a random rotation *)
Q=QRDecomposition[RandomVariate[NormalDistribution[0,1],{n,n}]][[1]]
(* Similarity transforms the input eigenvalues to be unrecognizable
*)
A=Q.DiagonalMatrix[evals].Transpose[Q];
0.5 (A + Transpose[A]) (*fixes floating point asymmetry*)
]
```

Sample of making a symmetric matrix

```
evals={1,25,50,400,1000}
A=SymMatWithEvals[evals];
MatrixForm[A]
Eigenvalues[A]
```

$$\{1,25,50,400,1000\}$$

$$\begin{pmatrix} 75.1925 & 132.076 & 19.3242 & 89.9505 & -119.583 \\ 132.076 & 824.58 & 261.923 & 2.72812 & -212.403 \\ 19.3242 & 261.923 & 148.499 & -47.7745 & -15.6489 \\ 89.9505 & 2.72812 & -47.7745 & 227.988 & -159.199 \\ -119.583 & -212.403 & -15.6489 & -159.199 & 199.74 \end{pmatrix}$$

$$\{1000., 400., 50., 25., 1.\}$$

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Looking for 1 eigenvalue

Eigenvalues	λ_{min}	λ_{max}	M	Output
1,25,50,400,1000	0	5	1	{1.,0.}
1,25,50,400,1000	20	30	1	{25.,0.}
1,25,50,400,1000	45	55	1	{50.,Indeterminate}
1,25,50,400,1000	350	500	1	{400.,123.586}
1,25,50,400,1000	900	1200	1	{1000.,365.714}

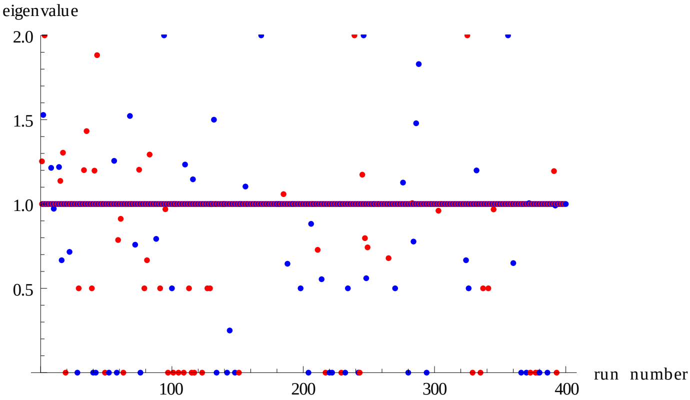
Looking for 2 eigenvalues

Eigenvalues	λ_{min}	λ_{max}	M	Output
1,25 ,50,400,1000	-2	30	2	{50.0013, 25.,1. }
1, 25,50 ,400,1000	20	75	2	{ 50.,25. ,1.}
1,25, 50,400 ,1000	40	500	2	{ 399.9489416370155‌4 , 49.99974501865308 , 19.06911267847751}
1,25,50, 400,1000	350	1200	2	{ 1000.,400. ,1.81701}

Finding multiple eigenvalues

Eigenvalues	λ_{min}	λ_{max}	M	Output
1,25,50,400,1000	0	60	3	{50., 25., 24.6902, 1., 0.}
1,25,50,400,1000	20	450	3	{1006.24, 400., 50., 25., 1.}
1,25,50,400,1000	40	1500	3	{1000., 400., 50., 25., 1.}
1,25,50,400,1000	0	500	4	{997.753, 400., 50., 25., 1.00061, 1.}
1,25,50,400,1000	20	1200	4	{1000., 400., 50., 40.0445, 25., 1.}
1,25,50,400,1000	0	1200	5	{1000., 400., 50., 49.9727, 25., 22.3177, 1., Indeterminate}

Finding the same eigenvalue multiple times



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Looking for eigenvalues near the interval

Eigenvalues	λ_{min}	λ_{max}	M	Desired λ	Output
{1, 25, 50, 400, 1000, 20, 45, 500}	0	1	1	1	{1.,0.5}
{1, 25, 50, 400, 1000, 20, 45, 500}	0	.9	1	1	{1.,0.}
{1, 25, 50, 400, 1000, 20, 45, 500}	-1	0	1	1	{23.6169,1.}
{1, 25, 50, 400, 1000, 20, 45, 500}	55	60	1	50	{50.,44.9802}

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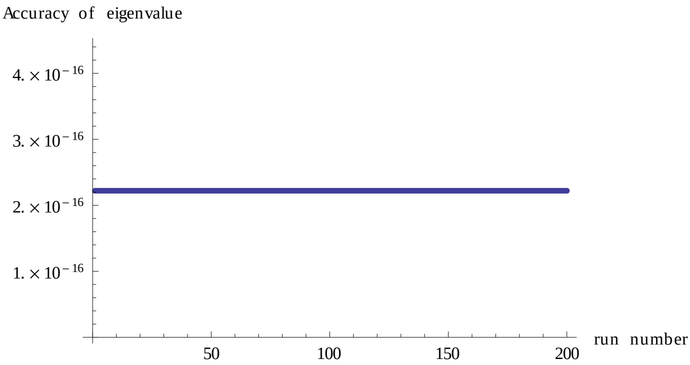
Testing accuracy of finding 1 eigenvalue multiple times



Testing accuracy with multiple residuals

Correct λ	Algorithm λ	$\frac{\lVert Ax_i - \lambda_i Bx_i \rVert_1}{\lVert Ax_i \rVert_1}$	$\frac{\lVert \lambda_{actual} - \lambda_{calculated} \rVert_2}{\lVert \lambda_{actual} \rVert_2}$
400	399.94894	0.011915	0.00012765
50	49.99975	0.0021485	5.09963×10^{-6}
50	50.	4.04765×10^{-10}	7.10543×10^{-16}
25	25.	4.13696×10^{-8}	8.5123×10^{-14}
1	1.	1.68769×10^{-7}	1.9762×10^{-14}

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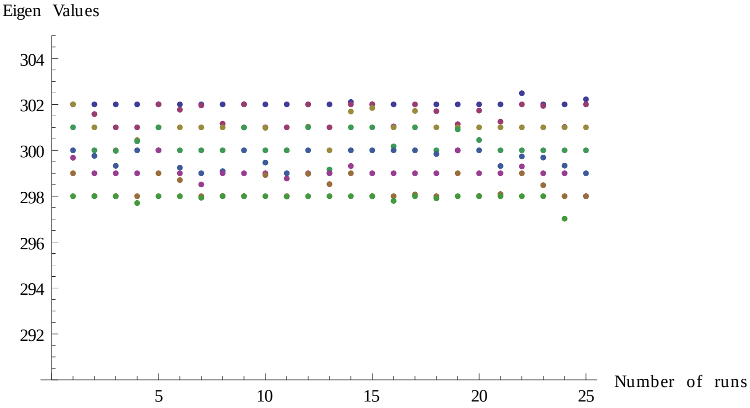
Accuracy of clustered eigenvalues

Correct eigenvalues: $\{1, 298, 299, 300, 301, 302, 600\}$

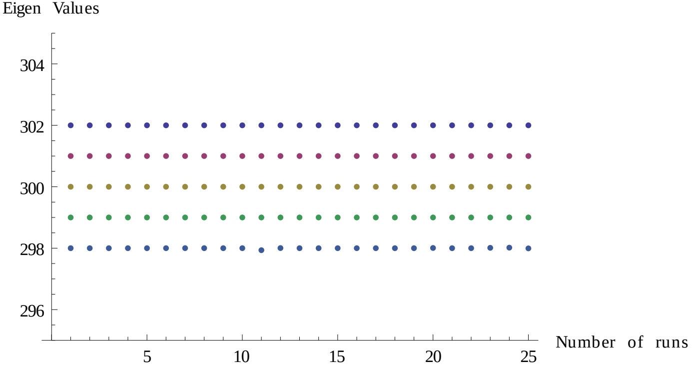
Range: $[290, 305]$



Accuracy of clustered eigenvalues

Correct eigenvalues: {1,298,299,300,301,302,600}

Range: [300,305]

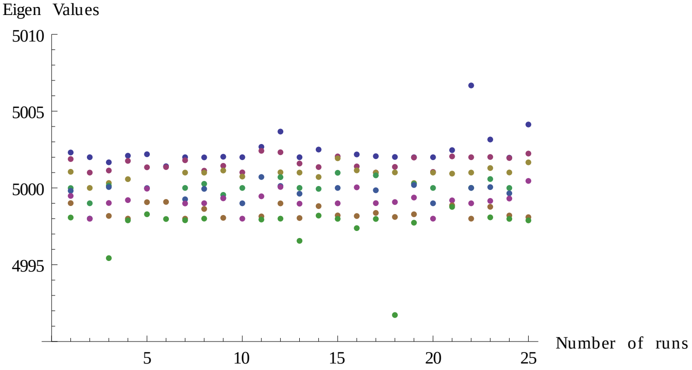


Algorithm gives complex eigenvalues

30 clustered eigenvalues

$M = 30$

Range: [4900,5050]



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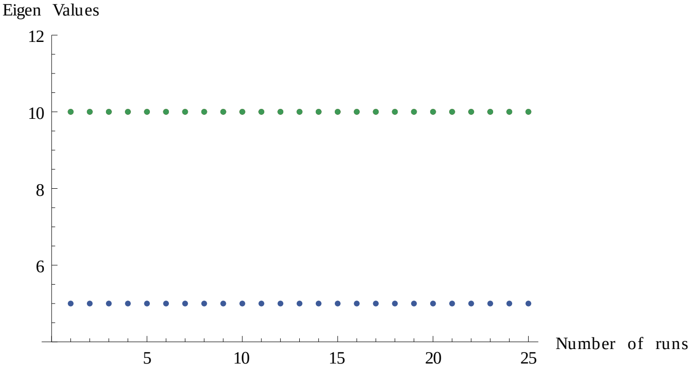
Repeated eigenvalues

Correct eigenvalues: {1, 5, 10, 10, 10, 15, 20, 25}

Range: [4,12]



Repeated eigenvalues

Correct λ	Calculated λ	Difference
5	5.000000000000304	$3.037570195374428 \times 10^{-13}$
10	10.000000000000004	$3.552713678800050 \times 10^{-15}$
10	10.000000000000012	$1.243449787580175 \times 10^{-14}$
10	10.000000000000357	$3.570477247194503 \times 10^{-13}$

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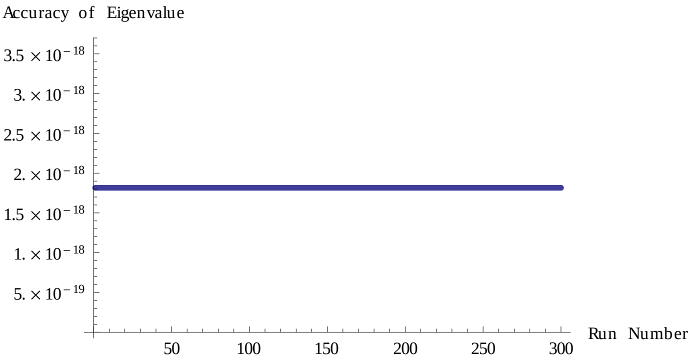
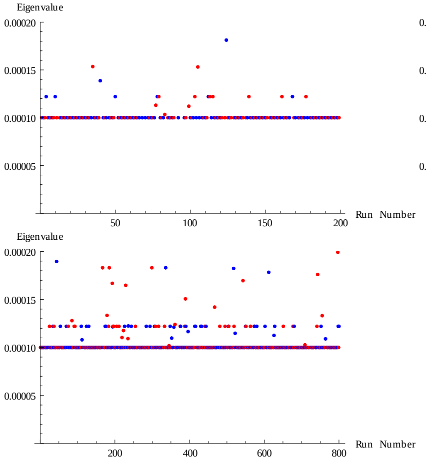
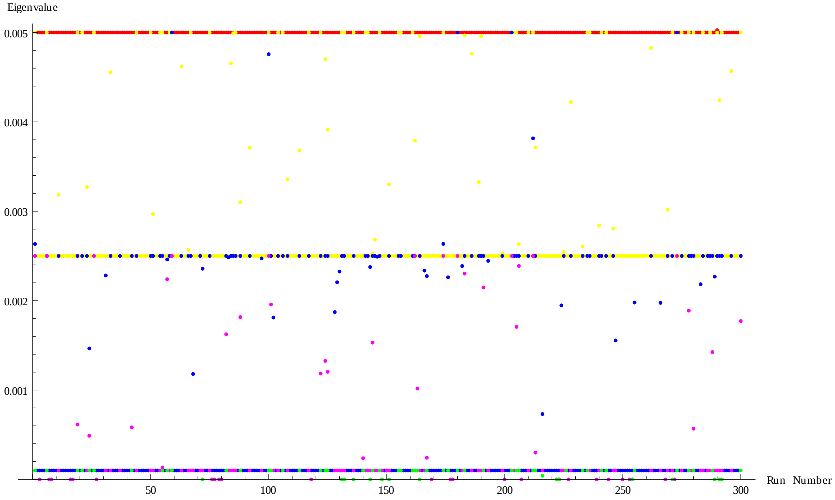
Accuracy of finding a small eigenvalue



Ghost eigenvalue



Finding 3 eigenvalues



Accuracy of finding 3 small eigenvalues

Correct λ	Algorithm λ	$\frac{\|Ax_i - \lambda_i Bx_i\|_1}{\|Ax_i\|_1}$	$\frac{\|\lambda_{actual} - \lambda_{calculated}\|_2}{\|\lambda_{actual}\|_2}$
0.0050	0.005	1.97687×10^{-11}	1.73472×10^{-16}
0.0025	0.0025	1.87104×10^{-9}	1.21431×10^{-15}
0.0001	0.0001	7.91298×10^{-9}	1.07065×10^{-14}

Questions?