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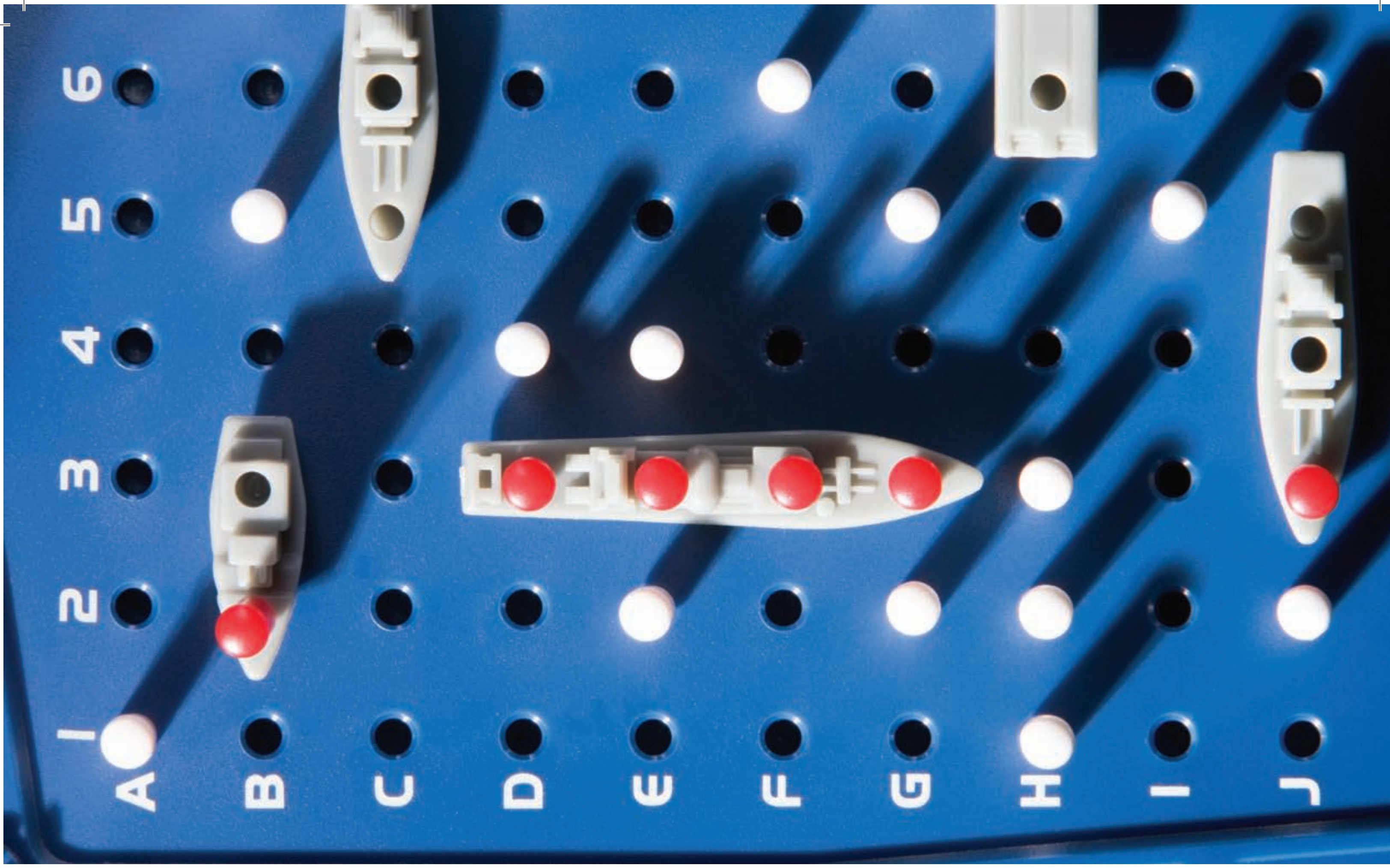

How to Beat Kindergartners at Battleship

Stephen J. Robinson

Every father must come to grips with the fact that his sons will eventually defeat him in virtually every head-to-head competition they embark upon, a prime example being on the basketball court when Junior's a teenager. For me, this looming defeat came far too suddenly and far too early when I was playing my five-year-old in the strategy game Battleship. I had erroneously assumed that I, having a head start of decades, would always have the upper hand in games of the mind, so this unexpected defeat was nearly too much to handle.

Now, a good father would be proud that his young boy had made such progress since his days in diapers. Unfortunately for my children, I am not a good father, and my competitive alter ego found little solace in this turn of events. The following is my attempt to use the power of statistics to gain the upper hand on any and all kindergartners who dare challenge me in Battleship.

Setup

Battleship is a globally popular board game, created around 100 years ago. The most typical setup consists of a 10 × 10 square grid on which a player places five one-dimensional warships either horizontally or vertically—an aircraft carrier occupying five spaces on the grid, a four-space battleship, a three-space destroyer, a three-space submarine, and a two-space patrol boat. His opponent does the same, and each player alternately tries to guess the location of his opponent's ships with colored pegs. When a guess is successful, it counts as a hit, and when all of the grid points of a specific ship have been hit, the opponent announces which ship was "sunk." The first player to successfully sink all five of his opponent's filled grid points (17 total grid points) wins.

There are several ways to vary games of Battleship: grid size and shape, ship size and shape, medium (e.g., pen-and-paper, plastic, electronic), and small rule modifications. For simplicity, the focus of this article will be on the predominant historical mode of play: a 10 × 10 square grid with the five one-dimensional ships detailed above.

Strategies

Each player places his ships in what he believes will be the configuration most difficult to detect. The biggest "rookie" mistake is placing two ships adjacent to each other on the grid. It is easy to see why in Figure 1.

	A	B	C	D	E
1					
2		X			
3		X	O	O	
4		X			
5		X			

Figure 1. A poor placement of two ships

The Xs represent a battleship, while the Os represent a patrol boat. Suppose one's opponent guesses B3 and thus gets a hit. On his next turns, he might then reasonably guess C3 and D3, at which point you inform him that he sunk your patrol boat. He then immediately realizes he has happened upon another of your ships covering B3 and proceeds to quickly sink it in the next few turns, instead of what you would like him to do: randomly guess and miss other points on the board. Because there is a significant amount of luck involved in Battleship, avoiding mistakes like this one is key.

Most players, even those with all of their baby teeth, innately know to add spaces between ships to increase their chances of winning. The first point to address here, then, is how spaces affect ship configurations. For the purposes of this article, space is defined in Figure 2.

4	3	2	2	3	4
3	2	1	1	2	3
2	1	X	X	1	2
3	2	1	1	2	3
4	3	2	2	3	4

Figure 2. The definition of "space" in this article. The ship is represented by Xs.

That is, if the ship in the figure is represented by Xs, and there is at least one space between it and other ships, no other ship can occupy the spaces marked by 1s. If there are at least two spaces between ships, no other ship can occupy the spaces marked by 1s or 2s, and so on.

Notice that, since ships cannot be arranged diagonally, having one space between ships removes the possibility of the "rookie mistake" mentioned above. Thus, the first point of strategy when placing your ships is to have at least one space between them. Considering several grid sizes, simulations show that this comes at the cost of reducing the number of possible ship configurations by about an order of magnitude for the larger grids, as seen in Table 1. (These numbers represent the configurations you would create and see on your game

Table 1: The Number of Possible Ways to Configure Ships on Various Grid Sizes with Various Numbers of Minimum Spaces Between Ships

grid dimensions	minimum space between ships: 0	1	2	3	4	5
5 × 5	8.1×10^{4}					
6 × 6	6.7×10^{6}	3.3×10^{3}				
7 × 7	1.2×10^{8}	1.9×10^{6}	4.0×10^{2}			
8 × 8	1.1×10^{9}	6.4×10^{6}	5.2×10^{5}			
9 × 9	6.8×10^{9}	8.1×10^{8}	3.0×10^{7}	1.6×10^{5}		
8 × 12	2.2×10^{10}	3.7×10^{9}	3.2×10^{8}	6.8×10^{6}	1.5×10^{4}	
10 × 10	3.0×10^{10}	5.8×10^{9}	5.2×10^{8}	1.5×10^{7}	2.2×10^{4}	
11 × 11	1.1×10^{11}	2.9×10^{10}	4.5×10^{9}	3.3×10^{8}	6.8×10^{6}	1.2×10^{4}

board, not what your opponent would "see" regarding your ships on his. For example, switching a submarine and destroyer looks different to you and counts as two configurations, while your opponent wouldn't notice the difference. However, changing ship directions—up vs. down and left vs. right—while a ship occupies the same grid points does not count differently.)

Probabilities

So let's assume we're playing on a 10 × 10 grid and I know you would do the smart thing and put spaces between your ships. (Of course, you could know that I would think that and not put spaces between your ships to throw me off, but we'll save that for the psychology journals.) Is there anything else I can do to maximize my chances of winning? The answer, as to not abruptly end this article, is fortunately yes.

Simulations of ship-by-ship placement reveal not only the number of ship configurations in a specific grid, but also how many of those configurations occupy a particular grid point. Then, finding probabilities is straightforward (see sidebar).

Consider Figure 3, which represents the probabilities of finding a ship at a given point in a 10 × 10, one-space minimum configuration. (All the possibilities in Table 1 have been examined in this way as well, but are omitted for brevity.) Notice the symmetry regarding 90˚ rotations and reflections about horizontal, vertical, and diagonal axes. This is a necessary condition of any rectangular Battleship grid.

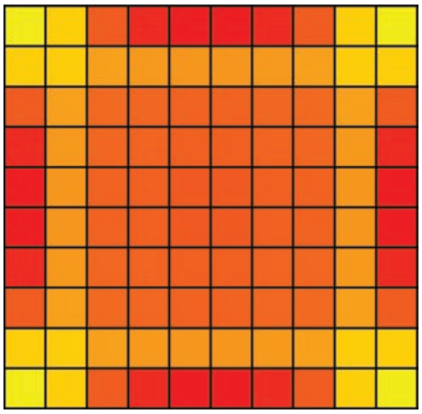

Figure 3. The probabilities of a ship being found at a specific grid point in a 10 × 10, one-space minimum configuration. Yellow represents an 11.0% chance of finding a ship there, red represents a 20.0% chance of finding a ship there, and the remaining shades of orange are linear fits in between. The average probability is 17/100 = 17%.

Figure 3 makes it clear that this knowledge of probability can greatly (i.e., by nearly a factor of two) increase one's chances of getting a hit on not just the first guess, but all guesses until the game is over. Still, a hit or miss at any grid point will alter the probabilities of the other grid points, especially those nearest the hit or miss. Therefore, the more guesses one makes, the less applicable Figure 3 becomes. Given the extraordinarily large number of possible ways to take, say, 40 shots, and the possibilities of each of those being a hit or miss, every scenario cannot be considered—we must develop general rules for how to guess.

Where to Guess if You Get a Hit

Suppose you get lucky and correctly guess one point on your opponent's grid. Most often, you will not know which ship you've hit (unless you and your opponent decide to play that way) or where to guess next with certainty. In those cases, the only hard and fast rule would be to guess at a grid point directly adjacent to the successful shot. Blindly, for a point off the edge of the grid, that leaves four choices—up, down, left, right—each with a 25% chance of success. But considering that there is no middle grid point on a 10 × 10 grid, any shot must introduce asymmetry.

For example, consider Figure 4, which shows a random shot with the assumption that a hit was made and the remaining probabilities that result. Naturally, the highest probability next-guesses (the redder squares) are directly adjacent to the hit.

Less obviously, Figure 4 recommends avoiding diagonally adjacent squares to the last hit (unless the last hit sunk the ship) because the one-space strategy will keep ships away from the high-probability squares. Simulations also show there is a small advantage to guessing adjacent squares toward the center of the grid, instead of away from the center, because it is harder to fit the bigger ships near the edges.

In Figure 4, the eight grid points around the hit have ship probabilities, starting from the upper left and moving clockwise, of 1.5%, 39.2%, 1.2%, 33.7%, 0.6%, 33.7%, 1.2%, and 39.2%.

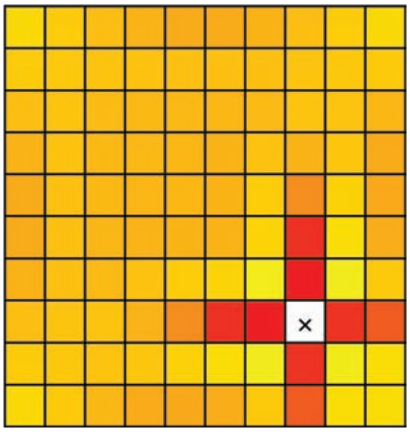

Figure 4. A hit with the first shot taken at point H8. Redder shades represent higher probabilities.

Where to Guess if You Miss

Clearly, the best-case scenario in Battleship is continually getting lucky and finding your opponent's ships, but the more likely scenario is missing early and often. How should you proceed in such a case? Figure 5 shows exactly how to proceed (i.e., to maximize the probability of the next shot being successful) under the assumption that you always miss if the probability is less than 100%.

59	20	48		1					21
	40		8	38	27	28	29	25	39
11		9			23				
	10	34	35	36	37	22		26	60
49			12				24	50	
3	33		51	13		30	52		2
	53	15		54	14	55		7	42
32		56	16		41		6		43
	17		57	19		5			44
58		18	61	62	4	45	46	47	31

Figure 5. The best possible way to proceed under the changing probabilities of continually missing your opponent if the probability of a hit is less than 100%. Shots 25, 27–29, 34–37, 39, 42–47, and 61–62 are all 100% certainties of successful shots; that is, they represent the ships' locations.

Notice, then, that if probabilities are taken into account, one will never need more than 62 shots to defeat his opponent. This is not obvious, as it appears that several spaces are still open to allow for ships, but the overall configuration would not fit in the remaining spaces with the earlier hits. Figure 5 was developed through simulation, but if one follows the numbers, the indication is that the best path ultimately leaves no room for an aircraft carrier, a good strategy to follow. Notice also that the shot probabilities are affected by previous shots. That is, if I miss at a certain location, the probability that an adjacent square has a ship should go down dramatically because ships use rows and columns of adjacent squares. This can be seen in Figure 6, which shows a miss at the spot where Figure 4 showed a hit. Here, the eight grid points around the miss have ship probabilities, starting from the upper left and moving clockwise, of 21.0%, 13.0%, 19.5%, 12.4%, 17.5%, 12.4%, 19.5%, and 13.0%.

The numbers in Table 1 were formulated by adding each ship in each possible space and configuration on the board based on the available positions allowed by already-placed ships. When five ships are successfully placed, that counts as one configuration. The probability of finding a ship at a given grid point (Figure 3) is then found by dividing the number of configurations with a ship at that point by the total number of possible configurations.

Figure 4 (6) was created in a similar fashion, but by only including (excluding) those configurations that have a ship covering grid point H8. Figure 5 (7) follows a path of high (low) probability by choosing to miss (i.e., simulate in the fashion of Figure 6) at the highest (lowest) probability points; thus, 62 (83) configurations had to be run, each relying on the results of the previous simulation. Figure 8 makes no assumptions about whether a shot will be a hit or miss; rather, it takes into account the probability of each happening using the best, worst, and random guessing strategies. A weighted average then gives the overall results after each shot.

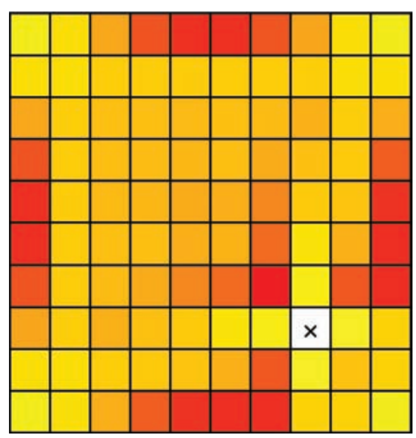

Figure 6. A miss with the first shot taken at point H8. Redder shades represent higher probabilities of ship placement.

However, since it is very likely that a successful shot will be made before the 25th shot (and thus, the exact pattern in Figure 5 will not be followed), the patterns that emerge from Figure 3 and Figure 6 determine the best types of shots that can be made. For example, follow the first 19 shots in Figure 5. After a loose spiral pattern in the first four shots (as suggested by Figure 3), the path quickly changes to adjacent diagonals (as suggested by Figure 6). This is clearly the most efficient way to eliminate possibilities, especially under the assumption that your opponent is placing spaces between his ships.

Comparing Figure 4 and Figure 6, we see opposite effects from hits and misses. That is, in addition to the probabilities that remain from Figure 3, the lowest probabilities are now in the same row or column as the miss, and the highest probabilities jump to diagonal spaces. Namely, if the conventional high-chance spaces to guess have already been chosen, try the spaces diagonal to the miss (the spiral pattern works best, but isolated misses again yield the highest probability next-guess toward the center).

How to Place Your Ships

If your opponent is a random guesser, there is no favorable way to arrange your ships. But if we assume he knows the tricks detailed above and is an intelligent player (or even worse, a computer with the ability to quickly calculate probabilities), we can help ourselves by considering Figure 7. This figure, like Figure 5, shows a possible pattern of guessing based on ship configuration probabilities and the assumption of only unsuccessful tries, but now taking the worst possible path instead of the best. That is, the lowest probabilities are the ones guessed until shots 84–100 (shown with ×) hit their targets.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
71	72	73	79	80	83	×	×	74	82
81	31	34	32	33	35	36	37	38	×
×	42	59	61	62	63	64	65	46	×
×	39	56	51	52	53	54	58	43	×
×	41	60	66	67	68	69	70	45	×
76	40	55	47	48	50	49	57	44	75
77	×	×	×	78	×	×	×	×	×

Figure 7. Assuming no hits are made, this shows the worst possible way to proceed and the remaining ships.

Looking at the results, this worst possible guessing pattern reveals ships along the sides, except the patrol boat. Since this figure shows the worst possible way to guess, it must be the most likely overall configuration, and thus, the worst possible configuration for your own ships (or any rotation or reflection thereof). That is, if your opponent is familiar with the probabilities discussed here, you have a very small chance of winning with such a configuration. (If you make no assumptions about putting spaces between ships, the worst possible configuration is piling all your ships in one corner—see the Appendix at *http://chance.amstat.org/category/supplemental*.)

Let's consider why Figure 7 is an especially awful configuration. First, notice that three ships have spaces in the highest probability squares on the grid—a definite configuration to avoid. Furthermore, four of the ships are along the edges. When an opponent successfully hits your ship along an edge, it leaves only three reasonable choices for his next shot, whereas normally there are four. You've increased his odds from roughly 25% to 33%. It's even worse in the corners, where the probability of a hit in the next shot is 50%. At first glance, requiring as much space between ships as possible (four spaces for a 10 × 10 grid) may seem like a good idea because it forces your opponent to guess all over the board. However, it has the unintended consequence of forcing your ships to the edges, which, as shown, is unadvisable.

Reality

In reality, things play out neither as wonderfully nor terribly as the scenarios presented thus far. For an accurate analysis, we must consult a binary tree that considers the probabilities of hitting and missing a particular grid point while taking into account the history of guesses so far. However, even if only 30 shots are needed to win (which assumes that more than half the shots are successful), the number of possible ways to proceed is enormous. For example, if the 30th shot were the final one, it would be a hit, but the other 29 shots with 16 hits could have happened in any pattern. Thus, there are $\binom{29}{16} = 6.8 \times 10^7$ ways this could happen. In addition, you can choose to take your first 30 shots on a 10 × 10 grid in ${}_{100}P_{30} = 7.8 \times 10^{57}$ ways (albeit most of them unintelligent). In either case, the computing power necessary to evaluate all of the possibilities—each hit or miss itself requires the evaluation of billions of ship configurations (see Table 1)—is currently out of reach.

Still, simulation is possible if we 1) use just a few guesses (six was chosen here to allow reasonable simulation times) and 2) use one of three extreme scenarios (best possible guessing using continually updated probabilities, random guessing, and worst possible guessing—for the sadists out there). Then, the effects of good guessing on winning Battleship can be seen clearly. Figure 8 details the average number of hits made after each shot up to six shots.

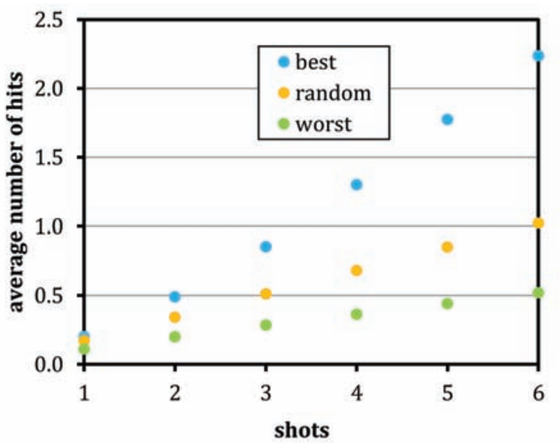

Figure 8. The average number of hits if a player plays using the best or worst probabilities to take the next shot, or simply plays by taking random shots.

Notice that, as most of us tend to play Battleship more randomly than we'd like to think, considering the probabilities yields a significant advantage over the usual modus operandi. Below is a summary of how to best place your ships and search for your opponent's, again assuming a 10 × 10, minimum one-space grid.

Summary

How to Configure Ships

1. Never let ships touch. This allows your opponent to stumble upon previously hidden ships.
2. Do not place ships along the edges; otherwise, it will be easier for your opponent to make his next move.

How to Shoot

1. Start with the midpoints on the sides of the grids. Those are the most likely places to find your opponent (unless he's read this article).
2. Create diagonal patterns that squeeze potential ships into smaller and smaller regions until you get a hit.
3. After a hit, the next shot should be adjacent to the hit, toward the center of the grid. Avoid diagonally adjacent squares.
4. After a miss, the next shot should be at a square diagonally adjacent to the miss, toward the center of the grid. Avoid nearby shots in the same row and column as the miss.
5. Unless you're playing a computer and have plenty of time to calculate probabilities, refer to Figure 3 for the most likely positions of your opponent. This is especially helpful when playing Battleship in "salvo" mode, in which each opponent takes more than one shot per turn.
6. When all else fails, tell your kindergartner you'll take him to McDonald's if he'll let you win.

Conclusion

If, unlike me, you're one of those good parents who lets his children win these types of games, take heart; you can use these tips to play at your absolute worst. Then, you can save the winning strategies for Battleship nights with your friends. C

Further Reading

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About the Author

Steve Robinson is an assistant professor of physics at Belmont University in Nashville, Tennessee. He earned his PhD in electrical engineering from the University of Illinois at Urbana-Champaign in 2007. His main lines of research are in condensed matter physics and nanoscale optics, but he enjoys games of chance and recently published an article on a prediction method for the NCAA men's basketball tournament. He is married with three children.