MA5630

Numerical Optimization

Final Project

Draft Report

Title:

Economic Load Dispatch and Optimal Power Flow in Power System

Team Members:

Zagros Shahooei

Guna R. Bharati

Barzin Moridian

Contents

1. Economic Load Dispatch

Electrical energy cannot be stored; it is generated from natural sources and delivered to the demands. A transmission system is used for delivery of electrical energy to the load points. In brief, an interconnected power system consists of three parts: 1. Generators, which produce the electrical energy; 2- Transmission lines, which transmits the produced energy to demands; 3- Loads, which consume the energy.

Since it is not possible to store electrical energy, the net energy generation in the system must be equal to the total system load and power losses. The main objective of power system is to supply the load continuously and as economic as possible. Planning the power generated by each generation unit and the system analysis is done in different steps from weeks until minutes before real time.

Economic (optimal) Load Dispatch (ELD) is the process of allocating generation among different generating units; in such a way that the overall cost of generation is minimized. In ELD problem we do not consider the power losses in transmission lines; so the total power generation must be equal to the total load. ELD is allocating loads to generation units with minimum cost while meeting the constraints. It is formulated as an optimization problem of minimizing the total costs of generation units. The total cost of generation includes fuel costs, costs of labor, supplies, maintenance. This cost depends on the amount of real power produced by the generator. Generation cost is considered as a quadratic function.

$C_{gi} = a_i {P_{gi}}^2 + b_i P_{gi} + d_i$ (1)

Total costs: $\mathrm{C} = \sum_{i=1}^{n} C_{gi}$ (2)

The objective function is to minimize the overall cost of power generation subject to the constraints.

$Minimize \sum_{i=1}^{n} C_{gi}$ (3)

Optimization constraints are as follows:

- Equality constraints: Energy balance equation. The total power generation must be equal to the demand.

  $P_D = \sum_{i=1}^{n} P_{gi}$ (4)

- Inequality Constraints: Generators' power output constraints>
  $P_{gi(\min)} \leq P_{gi} \leq P_{gi(\max)}$ (5)

2. DC Optimal Power Flow

ELD is the simplest planning method and it is used for long-term planning purposes. Most of the system constraints are not considered in ELD. The optimal power flow (OPF) problem seeks to control generation/consumption to optimize certain objectives such as minimizing the generation cost or power loss in the network.

Each load or generation point of power system is called a "bus" and different buses are connected together with transmission lines. Indeed the transmission lines have resistance and reactance which cause power loss. Considering all the system parameters, the optimization constraints are non-linear equations.

To clarify different power system parameters, a simple 3 bus system is shown in figure 1. Two types of power exist in power system, Active power and Reactive power. Active power relates to the resistive loads like electric heaters, lamps, and etc. Reactive lodes are related to motors and rotational loads. Transmission line parameters include resistance and inductance. Transmission line resistance results in active power loss and inductance result in reactive power loss.

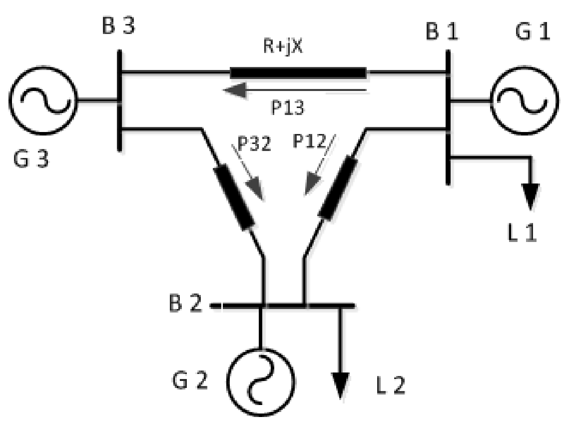

Figure 1. 3 bus power system

The voltages of each points (bus) in power system is a sinusoidal wave form with a frequency of 60 Hz. This means the voltage at each bus has an amplitude and a phase angle. The magnitude change of the voltages of different buses is because of transmission line resistance and having different phase angles is a result of transmission line inductance.

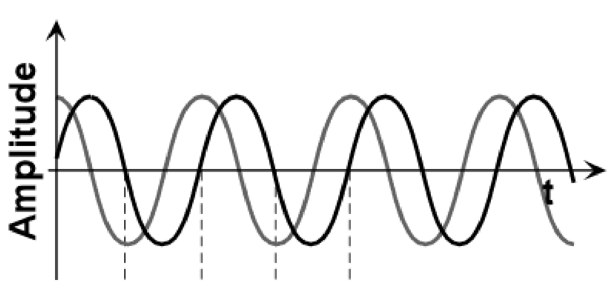

Figure 2. Sinusoidal voltage waveform

Nonlinear AC Optimal Power Flow (OPF) problems are approximated by linearized DC OPF problems to obtain real power solutions. In DC-OPF, we ignore the line resistances and reactive power flow in the system. Since the transmission line resistances are considered to be zero, all the voltage magnitudes throughout power system are equal to the nominal voltage of the system. The voltages are only different in phase angles. The objective function and constraints of DC-OPF are as follows:

The objective function is to minimize the overall cost of power generation subject to the constraints.

$$Minimize \ \sum_{i=1}^{n} C_{gi} \qquad (6)$$

Optimization constraints are as follows:

- Equality constraints:
  - Energy balance equations. For each bus i in the system:

$$P_{gi} - P_{Di} = \sum_{j=1}^{n} B_{ij}(\delta_i - \delta_j) \qquad (7)$$

$$B_{ij} = \frac{1}{X_{ij}} \qquad (8)$$

  - Voltage magnitude; for each bus i in the system:

$$|V_i| = 1 \qquad (9)$$

- Inequality Constraints:
  - Generators' power output constraints

$$P_{gi(\min)} \leq P_{gi} \leq P_{gi(\max)} \qquad (10)$$

  - Phase angle constraints:

$$\delta_{i(\min)} \leq \delta_i \leq \delta_{i(\max)} \qquad (11)$$

3. AC Optimal Power Flow

The ACOPF is at the heart of power system operation; it is being done by system operator and is solved form every day for day-ahead markets, every hour, and even every 5 minutes. With advances in computing power and solution algorithms, we can model more of the constraints and remove unnecessary approximations that were previously required to find a solution in reasonable time. Optimal power flow is sometimes referred to as security-constrained economic dispatch. As described before, simpler version of OPF, known as DCOPF, assumes all voltage magnitudes are fixed; indeed, DCOPF is a linearized form of a full alternating current network (ACOPF).

There are four quantities at each bus: voltage magnitude (V), voltage angle (θ), real power (P), and reactive power (Q). In a power flow solution, buses are classified into three bus types: PQ, PV and slack. PQ buses generally correspond to loads and PV buses to generators. Generator buses are called PV buses because power and voltage magnitude are fixed; load buses are known as PQ buses because real and reactive power are fixed. Slack or reference buses have a fixed voltage magnitude and voltage angle.

Table 1. Different power system buses

Bus Type	**Fixed Quantities**	**Variable Quantities**	**Physical model**
PV	Real power (P) Voltage Magnitude (|V|)	Reactive Power Voltage angle	Generator
PQ	Real Power (P) Reactive Power (Q)	Voltage Magnitude Voltage angle	Load
Slack	Voltage Magnitude (|V|) Voltage angle (δ)	Real power Reactive Power	An arbitrary generator

Again the objective function is to minimize the overall cost of power generation needed to supply the demands and power loss in the system; and subject to the constraints.

$$Minimize \sum_{i=1}^{n} C_{gi} \qquad (12)$$

Optimization constraints are as follows:

- Equality constraints:
    - Energy balance equations. For each bus i in the system:

$$P_{gi} - P_{Di} = V_i \sum_{j=1}^{n} V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \qquad (13)$$

$$Q_{gi} - Q_{Di} = V_i \sum_{j=1}^{n} V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij}) \qquad (14)$$

- Inequality Constraints:
    - Generators' active power output constraints

$$P_{gi(\min)} \leq P_{gi} \leq P_{gi(\max)} \qquad (15)$$

    - Generators' reactive power output constraints

$$Q_{gi(\min)} \leq Q_{gi} \leq Q_{gi(\max)} \qquad (16)$$

    - Voltage constraints:

$$\delta_{i(\min)} \leq \delta_i \leq \delta_{i(\max)} \qquad (17)$$

    - Phase angle constraints:

$$\delta_{i(\min)} \leq \delta_i \leq \delta_{i(\max)} \qquad (18)$$

In the above equations Y is admittance matrix which indicates the transmission line parameters. Y Matrix or Ybus is an $n \times n$ matrix describing a power system with n buses. It represents the nodal admittance of the buses in a power system. In realistic systems which contain thousands of buses, each bus in a real power system is usually connected to a few other buses through the transmission lines. The Y Matrix is also one of the data requirements needed to formulate a power flow study.

$$Y = \begin{bmatrix} Y_{11} & \cdots & Y_{1n} \\ \vdots & \ddots & \vdots \\ Y_{n1} & \cdots & Y_{nn} \end{bmatrix} \qquad (19)$$

$$Y_{ij} = \begin{cases} y_{ii} + \sum_{i \neq j} y_{ij} & if\ i = j \\ -y_{ij} & if\ i \neq j \end{cases} \qquad (20)$$

y_{ij} is admittance parameter of the transmission line between bus i and bus j. It is a complex number and considering line resistance (R) and reactance (X) which were mentioned before:

$$y_{ij} = \frac{1}{R_{ij}} + j\frac{1}{X_{ij}} = |y_{ij}|\angle\theta_{ij} \qquad (21)$$

4. Test systems and Results

Different test systems with different sizes are being tested and the optimization problem is coded in GAMS, Mathematica, and MATLAB.

4.1. Simple 3 Bus Test System

This test system is shown in figure 3. The cost functions associated with each of the generators are also presented. Table 2 shows the power output limits of the generators.

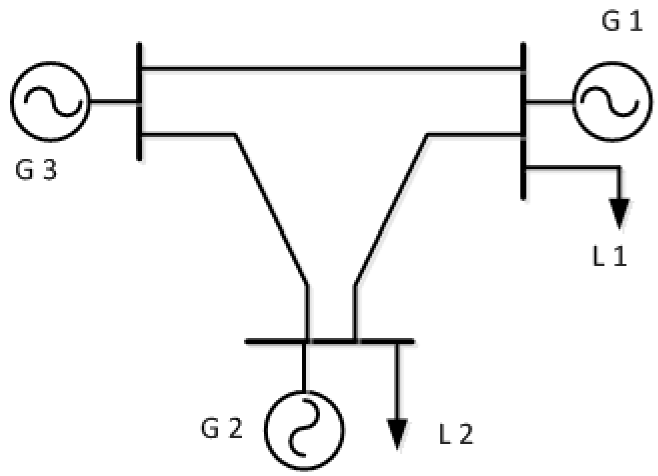

Figure 3. 3 bus test system

$C_{g1} = 0.00128{P_{gi}}^2 + 6.48P_{gi} + 459$

$C_{g2} = 0.00194{P_{gi}}^2 + 7.85P_{gi} + 310$

$C_{g3} = 0.00482{P_{gi}}^2 + 7.97P_{gi} + 78$

$Total\ demand = P_{D1} + P_{D2} = 850\ MW$

Table 2. generator power output constraints

i	$P_{min\,i}$ (MW)	$P_{max\,i}$ (MW)
1	150	600
2	100	400
3	50	200

4.1.1. Introduction to GAMS :

General Algebraic Modeling System (GAMS) is a high-level modeling system for mathematical programming and optimization. It is specifically designed for modeling linear, nonlinear and mixed integer optimization problems. The system is especially useful with large, complex problems. GAMS is available for use on personal computers, workstations, mainframes and supercomputers[1].

GAMS is able to formulate optimization problems with different solvers for different types of problem classes. That means switching from one model type to another can be done with a minimum of effort. You can even use the same data, variables, and equations in different types of models at the same time [1].

GAMS supports the following basic model types:

Solver	Description
LP	Linear Programming
MIP	Mixed-Integer Programming
NLP	Non-Linear Programming
MCP	Mixed Complementarity Problems
MPEC	Mathematical Programs with Equilibrium Constraints
CNS	Constrained Nonlinear Systems
DNLP	Non-Linear Programming with Discontinuous Derivatives
MINLP	Mixed-Integer Non-Linear Programming
QCP	Quadratic Constrained Programs
MIQCP	Mixed Integer Quadratic Constrained Programs

Without a valid GAMS license the system will operate as a free demo system with following limits:

- Number of constraints and variables: 300
- Number of nonzero elements: 2000 (of which 1000 nonlinear)
- Number of discrete variables: 50

Problem identification in GAMS is very simple and straight forward. It is developed by defining the variables, constants, parameters, and equations. Then the desired solver is selected and as a result all the optimization variable values are detected. In this step using some screen shots from GAMS, different steps of solving ELD optimization problem are presented:

- Defining cost coefficients, total power demand; and different variables including generators power output, objective function.

```
set i /1*3/;
alias (i,j)
table coef(i,j)
         1        2       3
1      0.00128   6.48    459
2      0.00194   7.85    310
3      0.00482   7.97    78

* Total Load
scalar PD /850/


positive variables PG(i);

variables obj;

equations Pbal, Objfn;
```

- In the next step the inequality constraints are presented.

```
* Limits for generators
parameters Pmin(i)
            /1   150
             2   100
             3   50/

parameters Pmax(i)
            /1   600
             2   400
             3   200/;

PG.up(i)=Pmax(i);
PG.lo(i)=Pmin(i);
```

- Finally, the objective function and power balance equations are defined. Since we have a quadretic objective function, we use NLP solver and then run the simulation.

```
* Objective Function
Objfn.. obj=e=sum(i,coef(i,'1')*PG(i)*PG(i)+coef(i,'2')*PG(i)+coef(i,'3'));
Pbal.. sum(i,PG(i))=e=PD;

model ELD /all/

* Choosing Non-linear Programming Solver
solve ELD minimizing obj using NLP
option decimals=4;

* Display the results
display obj.l;
display PG.l;
display Pbal.m;
```

Same problems were tested in MATLAB and Mathematica platforms,

The results of ELD optimization for 3 bus system are presented in table 3.

Table 3. ELD results for 3 bus test system

Generators(i)	**P_{gi}(MW)**		
	MATLAB	**Mathematica**	**GAMS**
1	600	600	600
2	187.13	187.13	187.13
3	62.86	62.8698	62.87
Total Cost	7252.83	7252.83	7252.83
Execution Time	0.09s	0.08s	0.04s
No of Iteration	13	13	5

4.2. 6 Bus Test System

Figure 4 shows the 6 bus test system and we will develop different optimization problems including ELD, DC-OPF and AC-OPF.

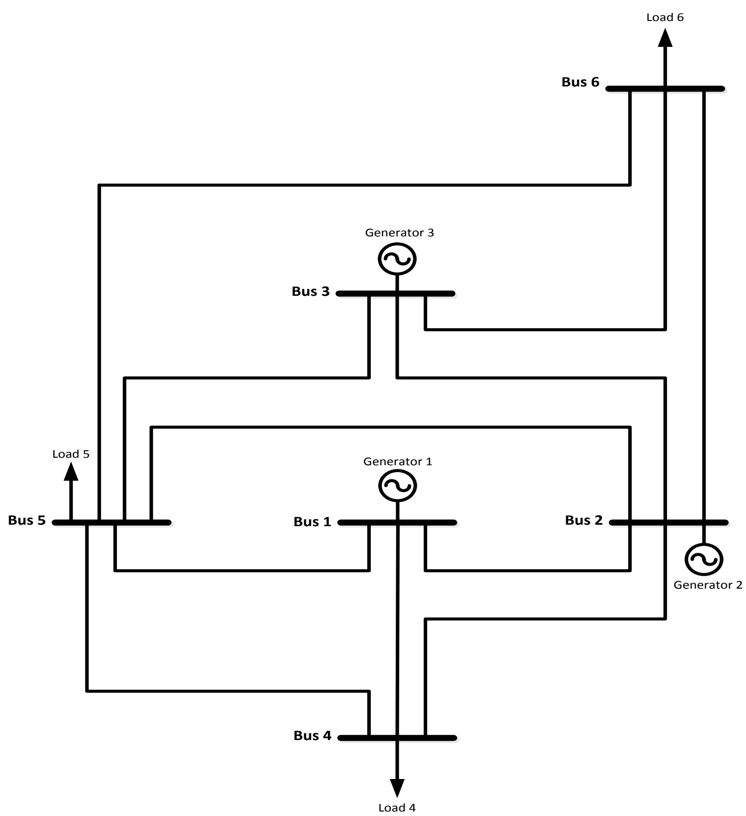

Figure 4. 6 bus test system

4.2.1. Economic Load Dispatch

ELD problem is similar to the previous test system. Since we have 3 generators in this system, our optimization problem includes 3 variables which are generator power outputs. Table 4 shows the optimized value of the variables and also the objective function which is the total cost.

Table 4. ELD results for 6 bus system

	P_{gi}(MW)		
Generators(i)	**MATLAB**	**Mathematica**	**GAMS**
1	50.2	50.036	50
2	129.8	129.963	130
3	100	100	100
Total Cost	1471.9	1471.9	1471.9
Execution Time	0.442s	0.077s	0.473s
No of Iteration	13	13	5

4.2.2. DC-OPF

In DC-OPF, other than generator power outputs, the voltage phase angles (δ_i) are also variables (except for slack bus). So we will have total 8 variables.

Different steps of Solving DC-OPF in GAMS are as follows:

- Defining cost coefficients, line parameters, load demands, and etc.

```
set i /1*6/;
alias (i,j)
set k /1*3/;

table coef(i,k)
          1        2       3
1      0.0015      8     300
2      0.0005      8     450
3      0.001     7.5     700
4         0        0       0
5         0        0       0
6         0        0       0

table B(i,j)
          1         2         3         4         5         6
1      -11.748      4         0       4.706     3.102       0
2         4      -23.195    3.846     8.001       3       1.454
3         0       3.846   -16.567      0        3.175     9.6157
4      4.706      8.001      0      -14.633       2         0
5      3.102        3     3.175        2      -14.137       3
6         0       1.454   9.6157       0          3       -17

parameters PD(i)
       /1  0
        2  0
        3  0
        4  0.9
        5  1
        6  0.9/

scalar pu /100/;
```

- Defining different variables and up and down limits for generator power outputs.

```
parameters Pmin(i)
        /1  0.5
         2  0.5
         3  0.3
         4   0
         5   0
         6   0/

parameters Pmax(i)
        /1  1.5
         2   2
         3   1
         4   0
         5   0
         6   0/



variables PG(i);
variables obj;
variables del(i);

equations Pinj(i), Objfn;


PG.lo(i)=Pmin(i)  ;
PG.up(i)=Pmax(i);
PG.fx('4')=0;
PG.fx('5')=0;
PG.fx('6')=0;

del.fx('2')=0;
```

- Then the objective function and equality constraints are defined as below:

Objective Function: $Minimize\ \sum_{i=1}^{n} C_{gi}$

Equality Constraint: $P_{gi} - P_{Di} = \sum_{j=1}^{n} B_{ij}(\delta_i - \delta_j)$

```
Objfn..   obj=e=sum(i,coef(i,'1')*PG(i)*PG(i)+coef(i,'2')*PG(i)+coef(i,'3'));
Pinj(i).. PG(i)-PD(i)=e=(sum(j,B(i,j)*(del(i)-del(j))));


model ELD /all/

solve ELD minimizing obj using NLP
option decimals=4;
display obj.l;
display PG.l;
display Pinj.m;

parameters deldeg(i);
deldeg(i)=180*del.l(i)/pi;
display deldeg;
```

The results of optimization are displayed and they are shown in table 4

Table 4: 6 BUS DC-OPF Results

Generators(i)	**P_{gi}(MW)**			**δ_i**		
	MATLAB	**Mathematica**	**GAMS**	**MATLAB**	**Mathematic**	**GAMS**
1	131.12	50.6	50	0.074	-0.01474	-0.0199
2	50	129.399	130	0	0	0
3	10	100	100	0.0133	0.001209	-0.0197
4				-0.0473	-0.079335	-0.0827
5				-0.0704	-0.098641	0.1115
6				-0.0578	-0.06937	0.1011
Total Cost	1471.99	1471.9	1471.9			
Execution Time	0.1655s	0.087s	0.564s			
No of Iteration	9	11	5			

4.2.3. AC-OPF

In AC-OPF besides the generators active power output and the voltage phase angles, the load bus voltage magnitudes and generator reactive power outputs are also optimization variables; which means we totally have 14 variables. The objective function and also the constraints are non-linear equations.

Different steps of defining variables and parameters are similar to DC-OPF; however, all the transmission line parameter details (Ybus matrix) and load reactive power consumptions need to be defined as well. Then the objective function and active and reactive power balance equations are defined as below:

Objective Function:

$$Minimize \sum_{i=1}^{n} C_{gi}$$

Equality Constraints:

$$P_{gi} - P_{Di} = V_i \sum_{j=1}^{n} V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij})$$

$$Q_{gi} - Q_{Di} = V_i \sum_{j=1}^{n} V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij})$$

```
Costf..   cost=e=sum(M,coef(M,'1')*Pgen(M)*Pgen(M)+coef(M,'2')*Pgen(M)+coef(M,'3'));
Pbal(N).. Pgen(N)-Pload(N)=e=sum(M,(voltage(N)*voltage(M)*Y(N,M)*cos(theta(N,M)+delta(M)-delta(N))));
Qbal(N).. Qgen(N)-Qload(N)=e=-sum(M,(voltage(N)*voltage(M)*Y(N,M)*sin(theta(N,M)+delta(M)-delta(N))));
```

Table 5 shows the optimization results with the orange background color.

Generators(i)	**P_{gi}(MW)**			**δ_i**		
	MATLAB	**Mathematica**	**GAMS**	**MATLAB**	**Mathematic**	**GAMS(^{0})**
1	76.89	76.89	76.891	0.0177	0.01765	1.011
2	112.44	112.44	112.445	0	0	0
3	100	100	100	0.0034	0.003427	0.197
4				-0.0415	-0.041454	-2.375
5				-0.0606	-0.006064	-3.474
6				-0.0496	-0.049625	-2.844

Generators(i)	**Q_{gi}(MVAr)**			**V_i**		
	MATLAB	**Mathematica**	**GAMS**	**MATLAB**	**Mathematic**	**GAMS**
1	35.04	35.04	35.044	1.05	1.05	1.05
2	69.97	69.99	69.968	1.05	1.05	1.05
3	59.82	59.818	59.819	1.05	1.05	1.05
4				0.9855	0.9855	0.98554
5				0.9683	0.96826	0.96827
6				0.9924	0.99241	0.99241
Total Cost	1472.65	1472.65	172.649			
Execution Time	0.1263s	0.16s	0.697			
No of Iteration	11	11	18			

Table 5: Results of 6 BUS AC-OPF

4.3. 14 BUS Test System:

An IEEE standard 14 bus system was taken as the cost optimization test system. Below is the electrical diagram and data used for the optimization.

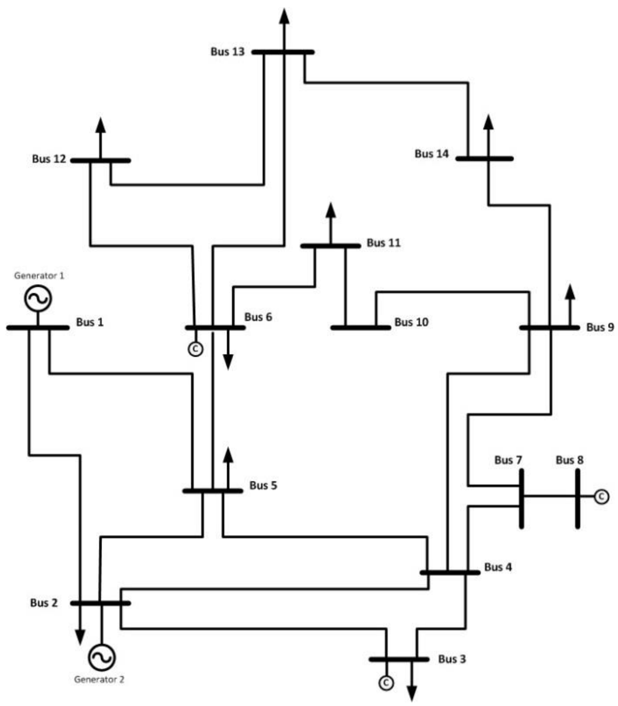

Figure: 14 Bus System

Bus No.	Pd (pu)	Qd (pu)	Pg(max)	Pg(min)	Qg(max)	Qg(min)
1	0	0	6	0.1	10	-10
2	0.217	0.127	3	0.1	0.5	-0.4
3	0.942	0.19			0.4	0
4	0.478	0				
5	0.076	0.016				
6	0.612	0.075			0.24	-0.06
7	0	0				
8	0.5	0				
9	0.2850	0.166			0.24	-0.06
10	0.09	0.058				
11	0.035	0.018				
12	0.061	0.016				
13	0.135	0.058				
14	0.149	0.05				

Table: 14 bus data (IEEE)

Initially Impedance matrix was calculated form the bus system data and was subjected to the equality constraints. Voltage and delta constraints were taken same for this system. For the specific constraints where voltage can vary between -5 to 5%, MATLAB and GAMS were not able to give feasible solution. In this case Mathematica was able to give a feasible solution as shown below.

Bus No.	P_{gi}			δ_i		
	MATLAB	Mathematica	GAMS	MATLAB	Mathematica	GAMS
1		0.505			0.003	
2		2.1723			0	
3					-0.144	
4					-0.1093	
5					-0.0896	
6					-0.1935	
7					-0.1683	
8					-0.16834	
9					-0.2	
10					-0.2044	
11					-0.2014	
12					-0.2097	
13					-0.21068	
14					-0.22436	

Bus No.	Q_{gi}			V_i		
	MATLAB	Mathematica	GAMS	MATLAB	Mathematica	GAMS
1		-0.0907			1.05	
2		0.01585			1.05	
3		0.25582			1.0177	
4					1.01759	
5					1.0228	
6		0.218877			1.014	
7					1.0155	
8		0.2055			1.05	
9					0.994	
10					0.99	
11					0.9982	
12					0.99801	
13					0.99267	
14					0.9745	
Total Cost		771.424				
Execution time		0.459s				
Iteration		22				

Voltage constraints were changed such that voltage in each bus can vary from -10 to 10%. In this case MATLAB and GAMS were able to find a feasible solution as shown below.

Bus No.	**P_{gi}**			**δ_i**		
	MATLAB	**Mathematica**	**GAMS**	**MATLAB**	**Mathematica**	**GAMS**
1	0.464	0.505	0.50746	0	0.003	0.0033
2	2.2561	2.1723	2.17982	0	0	0
3				-0.1657	-0.144	-0.16048
4				-0.1349	-0.1093	-0.12148
5				-0.111	-0.0896	-0.09954
6				-0.2114	-0.1935	-0.2143
7				-0.1979	-0.1683	-0.18656
8				-0.1979	-0.16834	-0.18656
9				-0.231	-0.2	-0.22153
10				-0.2402	-0.2044	-0.22636
11				-0.2333	-0.2014	-0.22303
12				-0.2363	-0.2097	-0.23214
13				-0.2576	-0.21068	-0.2332
14				-0.2808	-0.22436	-0.24834

Bus No.	**Q_{gi}**			**V_i**		
	MATLAB	**Mathematica**	**GAMS**	**MATLAB**	**Mathematica**	**GAMS**
1	0.0475	-0.0907	-0.0827	1	1.05	1
2	-0.1386	0.01585	0.01931	1	1.05	1
3	0.4	0.25582	0.28808	1.0188	1.0177	0.9685
4				1.0256	1.01759	0.96737
5				1.0208	1.0228	0.97247
6	0.24	0.218877	0.24	1.0028	1.014	0.96773
7				1.0086	1.0155	0.96988
8	0.24	0.2055	0.24	0.9648	1.05	1.01167
9				1.0286	0.994	0.94676
10				1.0346	0.99	0.94224
11				1.0247	0.9982	0.95103
12				1.0172	0.99801	0.95081
13				1.0386	0.99267	0.94518
14				1.0597	0.9745	0.92594
Total Cost	771.763	771.424	771.501			
Execution time	1.055s	0.459s	0.68s			
Iteration	16	22	24			

5. Conclusion

GAMS is a dedicated software for the optimization problem, its result are very good compared to MATLAB and Mathematica for small number of variables, however with increase in variables MATLAB and Mathematica are comparable to the performance of GAMS. Both MATLAB and Mathematica were using interior point method to find the optimal solution, though the environment of MATLAB and Mathematica are similar, MATLAB was comparably good because of the fact that Mathematica couldn't give solution when initial conditions were not close to certain value. In one case Mathematica out bids both GAMS and MATLAB i.e 14 bus system with voltage constraints within -5 to 5%. In this case Mathematica was the only way to get solution. As a conclusion we can say all of the platform were successful and were best depending on the test they were subjected to. Finally, the concept of optimization was clearly observed in different test system and platforms. As a future enhancement we can use this optimization problem in higher bus system which is very practical.