**Solving System of Equations: Nonlinear Case**

(Dr. Tom Co, 10/5/08)

## Introduction:

A problem contains a set of nonlinear equations with $n$ unknowns and $n$ equations

$$f_1(x_1, \dots, x_n) = 0$$
$$\vdots$$
$$f_n(x_1, \dots, x_n) = 0$$

Often, a numerical approach is needed.

## MathCad procedure:

1. Set up the problem by defining all the variables and the nonlinear functions. In doing so, one needs to also fix an initial value for the unknown, i.e. this approach is like roots() function in that we need initial guesses.
2. Insert a "solve" block. This block is marked by the keywords: ***Given*** and the ***Find***( ) function. Inside the solve block, write the equations (be careful to use **[ctrl =]** when writing these equations).
3. Units are allowed. Make sure the initial guesses have the appropriate units.

## Example:

We want to solve for $x$, $y$ and $z$ that satisfies the following three equations:

$$\left(\frac{(x-2)}{2}\right)^2 + (y-2)^2 = 25 - (z+1)^2$$
$$(x+y)^2 - z + 3 = 0$$
$$2x + 3y - z = -2$$

1. Define variables and functions:

$$f_{left}(x,y,z) := \left[\frac{(x-2)}{2}\right]^2 + (y-2)^2$$

$$f_{right}(x,y,z) := 25 - (z+1)^2$$

$$g(x,y,z) := (x+y)^2 - z + 3$$

$$h(x,y,z) := 2x + 3y - z$$

$$x := 1 \qquad y := 1 \qquad z := 1$$

2. Enter the solve block:

Given

$$f_{left}(x,y,z) = f_{right}(x,y,z)$$

$$g(x,y,z) = 0$$

$$h(x,y,z) = -2$$

$$b := find(x,y,z) \qquad b = \begin{pmatrix} 0.687 \\ 0.063 \\ 3.562 \end{pmatrix}$$

3. Extract the unknowns:

$$x := b_0 \qquad y := b_1 \qquad z := b_2$$

$$x = 0.687 \qquad y = 0.063 \qquad z = 3.562$$

**Remarks:**

1. Units are allowed. However, one needs to use the following format:

$$\begin{pmatrix} A \\ B \\ \vdots \end{pmatrix} := Find(A, B, \ldots)$$

2. Parameters and constraints can be included.