CM 3450
In-Class Drills
Sept. 3, 2008

Name: ____

1. **Word Equation Editor**. Use the key-stroke method to:

   a) Input the Navier Stokes equation

   $$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \nabla\cdot\mathbf{T} + \mathbf{f}$$

   b) Input the Cauchy stress tensor

   $$\sigma_{ij} = \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix}$$

   c) Repeat a) and b) and include equation numbers, i.e.

   $$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \nabla\cdot\mathbf{T} + \mathbf{f} \qquad (1)$$

   $$\sigma_{ij} = \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix} \qquad (2)$$

2. Implementing Successive Substitution.

Solve the pipeline problem a.

3. Implementing Data Tables

Solve the pipleline problem b and plot the results.

Calculations of Flow Rate in a Pipeline

(based on Cutlip and Shacham, 2008, pp. 110-118)

I. Working Equations

The mechanical energy balance is given by

$$-\frac{1}{2}v^2 + g\Delta z + \frac{g_c \Delta P}{\rho} + 2\frac{f_F L_v^2}{D} = 0 \tag{1}$$

where the Fanning friction factor is given by:

$$f_F = \begin{cases} \dfrac{16}{Re} & \text{if } Re < 2100 \\ \dfrac{1}{16}\left(\log\left[\dfrac{\frac{\epsilon}{D}}{3.7} - \dfrac{5.02}{Re}\log\left(\dfrac{\frac{\epsilon}{D}}{3.7} + \dfrac{14.5}{Re}\right)\right]\right)^{-2} & \text{if } Re > 2100 \end{cases} \tag{2}$$

and Reynold's number is given by

$$Re = \frac{\rho v D}{\mu} \tag{3}$$

We will use successive substitution to solve for the velocity v using the following rearrangement of equation (1):

$$v = \sqrt{\frac{g\Delta z + \frac{g_c \Delta P}{\rho}}{\frac{1}{2} - 2\frac{f_F L}{D}}} \tag{4}$$

In addition, we will use the following correlation for the density and viscosity of water:

$$\rho = 62.122 + 0.0122T - 1.54 \times 10^{-4}T^2 + 2.65 \times 10^{-7}T^3 - 2.24 \times 10^{-10}T^4 \tag{5}$$

$$\mu = \exp\left(-11.0318 + \frac{1057.51}{T + 214.624}\right) \tag{6}$$

where T is in $°F$, ρ is in $\frac{lb_m}{ft^3}$ and μ is in $\frac{lb_m}{ft \cdot s}$.

II. Additional Data:

ϵ (surface roughness)	0.00015 ft (for steel pipes)	
D (inside diameter)	4.026 in	4" Sch 40
	5.047 in	5" Sch 40
	6.065 in	6" Sch 40
	7.981 in	8" Sch 40

III. Problem Statement

a) Obtain the velocity in *ft/s* for: $T = 60°F, \Delta P = -150\ psi, \Delta z = 300\ ft, L = 1000\ ft$, and 8" diameter Sch 40 commercial steel pipe, using successive substitution.

b) Calculate the flow velocities in ft/s for L =500, 1000, …, 10,000 *ft* for different Sch 40 commercial steel pipes of nominal diameters: 4", 5", 6" and 8".

IV. Solution

a) Fill-in the spreadsheet such as the one in Figure 1.

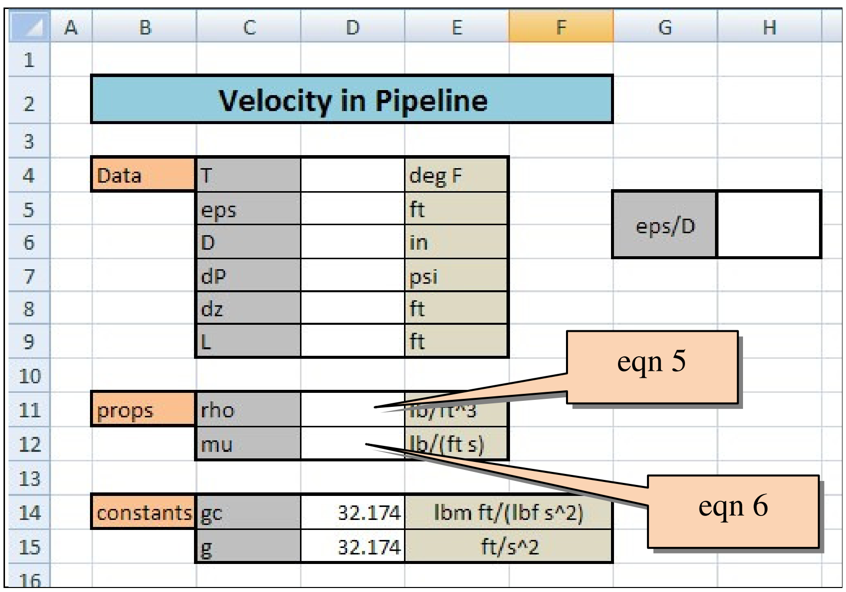

Figure 1. Sample setup

then build the columns for successive substitution as in Figure 2.

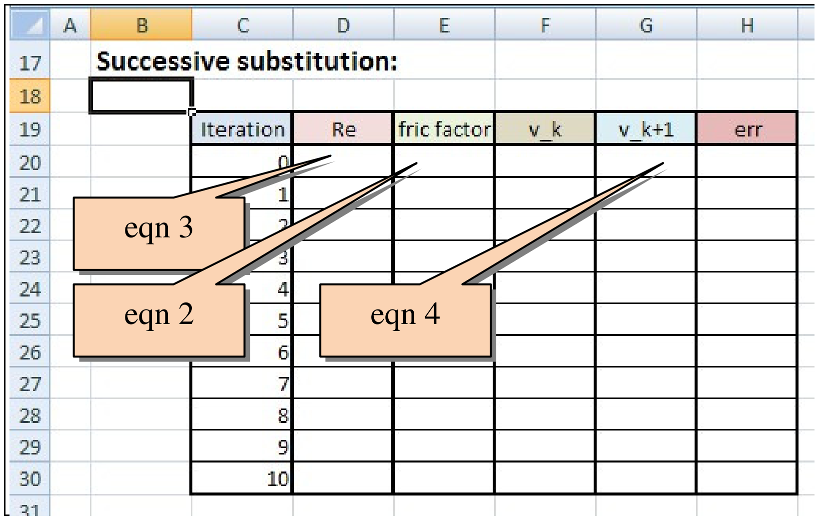

Figure 2. Successive substitution table.

b) Build a data table by referring to address of converged value for velocity but varying the value of *L* and *D*.