Introduction to Dynamic Simulation in Matlab via Transfer Functions

(tbco 3/2/2019)

Motivation:

- Transfer functions allow for modular block representation of the dynamic behavior of a process from input variables to the output variables
- They also allow for efficient modeling, analysis and design of complex networks of containing given blocks and blocks designed to achieve improved controlled response.

Initializing a Simulink run

- In a Matlab command window enter: **simulink**. (Alternatively, you may click the **[Simulink]** button in the "Home" tab menu.
- Create a blank model or open an existing *.mdl file. A simulink model window should appear.
- To view available blocks for a simulink model, select **[View]→[Library Browser]** to open another window that contains several menu choices.

Example 1. Simulating a first order process.

Consider the following first order ODE model

$$\tau \frac{dx}{dt} + x = K_p u \tag{1}$$

Assuming zero initial conditions, the transfer function from input u to output x is given by

$$x = \left(\frac{K_p}{\tau s + 1}\right) u \tag{2}$$

and the transfer function is given by

$$G_p(s) = \left(\frac{K_p}{\tau s + 1}\right) \tag{3}$$

> **Note:** the form given in (3), i.e. with the standard form in which the denominator has "1" as the coefficient of s^0, is technically referred to as a "first-order lag with a gain of K_p with time constant τ ", or if $K_p = 1$ it is called a "first-order lag with time constant τ".

Next, suppose we want to simulate the process in which the input u is given by the step function given by

$$u = \begin{cases} 2 & \text{if } t > 10 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Also, let the parameters of the process be set as $K_p = 4$ and $\tau = 7$.

Next, include the following blocks: (click the mouse on a empty spot in the window, then typing-and-select the block names as follows)

- **Step**
- **Transfer Fcn**
- **Scope**

Alternatively, you can drag blocks from the Library browser: a) **[Sources]→[Step],** b) **[Continuous]→[Transfer Fcn]**, c) **[Sinks]→[Scope]**.

Next, connect the **[Step]** block to the **[Transfer Fcn]** block, then connect the **[Transfer Fcn]** block to the **[Scope]** block.

Double-click on the **[Step]** block and the **[Transfer Fcn]** block to change their properties (be sure to click [OK] to accept changes):

- For the **[Step]** block, enter 10 for "*Step Time*" and 2 for "*Final Value*" to satisfy (4).
- For the **[Transfer Fcn]** block, enter [4] for "*Numerator Coefficients*" and [7,1] for the "*Denominator Coefficients*" to represent the transfer function (3) with $K_p = 4$ and $\tau = 7$.

Label the blocks (*click on existing labels to edit*) or lines (*double-click on connections then enter label*) as appropriate. The final model is shown in Figure 1.

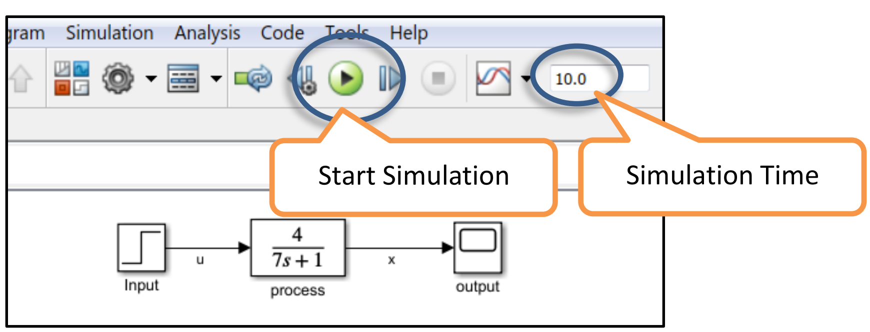

Figure 1. Simulink model for first order process given in example 1.

Change the "*Simulation Time*" (see Figure 1) from 10 to 50. Then click on "*Start Simulation*" button (see Figure 1).

Double-click on the **[Scope]** block (which we renamed "output" in Figure 1) to access the resulting plot of the simulation as shown in Figure 2.

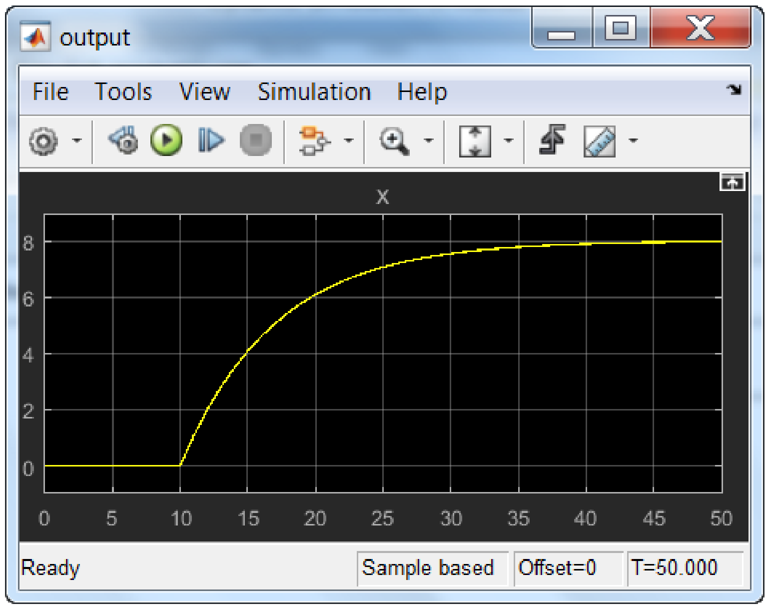

Figure 2. Plot of the simulation run.

Note the following:

- The process starts changing at the $t = 10$ which is when the input initiate a step change.
- The process gain is given by $K_p = (8-0)/(2-0) = 4$.
- The time when $x = 8 \cdot (0.632) \approx 5.08$ occurs at $t \approx 17$, which when subtracted from the $t_{step} = 10$, yields a time constant $\tau = 17 - 10 = 7$.

Example 2. Three-tank process (Open-loop, i.e. no feedback control)

Consider the three buffer-tanks in series shown in Figure 3.

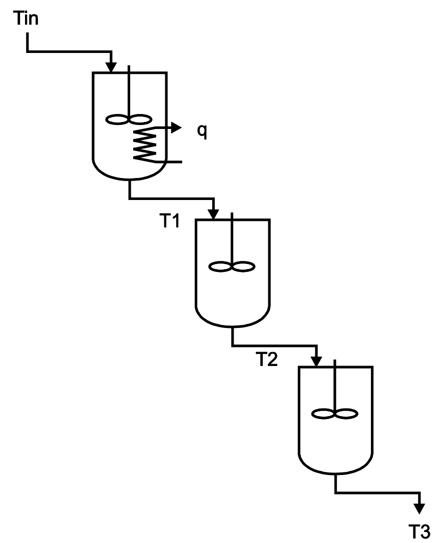

Figure 3. Three buffer tanks in series.

Assuming constant flow and tank liquid volume operation (together with the assumption of constant density and heat capacity of the liquid and well-mixed tanks), the model of the temperatures in each tank is given by

$$\begin{aligned}\tau_1 \frac{dT_1}{dt} + T_1 &= T_{in} + K_p q \\ \tau_2 \frac{dT_2}{dt} + T_2 &= T_1 \\ \tau_3 \frac{dT_3}{dt} + T_3 &= T_2\end{aligned} \tag{5}$$

Let the variables described in (5) already be deviation variables so that we can assume that all initial conditions to be zero. The transfer function representation of (5) is then given by

$$\begin{aligned}T_1 &= [G_1]T_{in} + [G_2]q \\ T_2 &= [G_3]T_1 \\ T_3 &= [G_4]T_2\end{aligned} \tag{6}$$

where G_i's are the corresponding transfer functions given by

$$\begin{aligned}G_1 &= \frac{1}{\tau_1 s + 1} \\ G_2 &= \frac{K_p}{\tau_1 s + 1} \\ G_3 &= \frac{1}{\tau_2 s + 1} \\ G_4 &= \frac{1}{\tau_3 s + 1}\end{aligned} \tag{7}$$

Let the parameters of the process be set as follows: $K_p = 5,\ \tau_1 = 7, \tau_2 = 6,\ \tau_3 = 4.$ Further, assume that q and T_{in} undergo step changes given by

$$q = \begin{cases} 2 & \text{if } t > 10 \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

$$T_{in} = \begin{cases} -4 & \text{if } t > 70 \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

Include blocks as shown in Figure 4 and change the settings to reflect the appropriate transfer functions and step functions. For the summing block, type "sum" (or drag block item **[Commonly used blocks]→[sum]**) and enter "|++" when requested. (Alternatively, double-click to change the "list of signs" property to "|++".) Connect the blocks as shown in Figure 4.

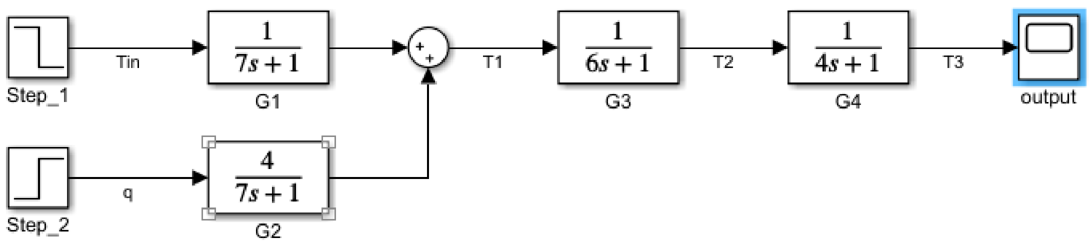

Figure 4. Transfer function network representation for the three-tank system.

Change the simulation time to 150 and run the simulation. The plot should resemble that shown in Figure 5.

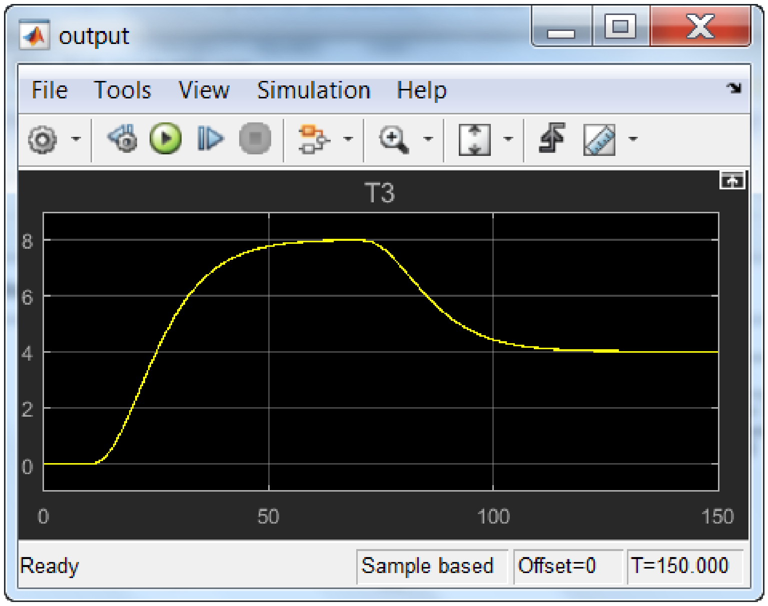

Figure 5. The result of the three-tank simulation.

Note the following:

- The process responds to the step increase in q the occurs at $t = 10$. The response is sluggish (slow initial change) compared to a first order process because T_3 undergoes a third order process response to the stimulus initiated in tank 1 by q.
- Another third process response is initiated at $t = 70$ due to the stimulus initiated in tank 1 by the decrease in T_{in}.

Example 3. Three-tank process with proportional control

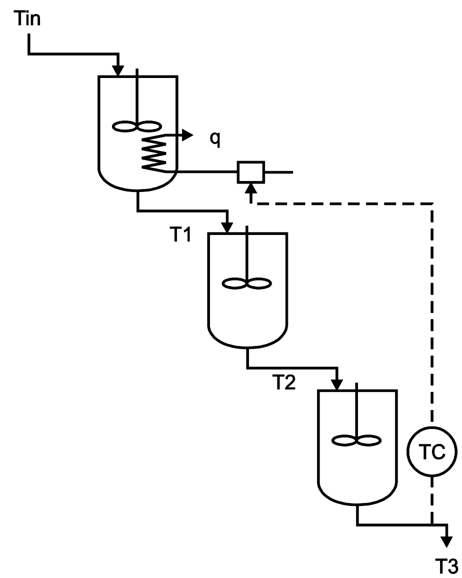

Figure 6. Three tank system with temperature control.

We extend example 2 to include a feedback control using proportional control as given by

$$q = K_c\left(T_3^{set} - T_3\right) \tag{10}$$

We will use the following step change in set point to observe the controlled response:

$$T_3^{set} = \begin{cases} 1 & \text{if } t > 10 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

The block diagram shown in Figure 4 will be modified by including two more blocks: a "**sum**" block (for "*list of signs*", enter "|+-" to change it to a difference) and a "**gain**" block (type "gain" or insert from library browser the element **[Commonly used blocks]→[Gain]**). Then connect and label as given in the block diagram shown in Figure 7.

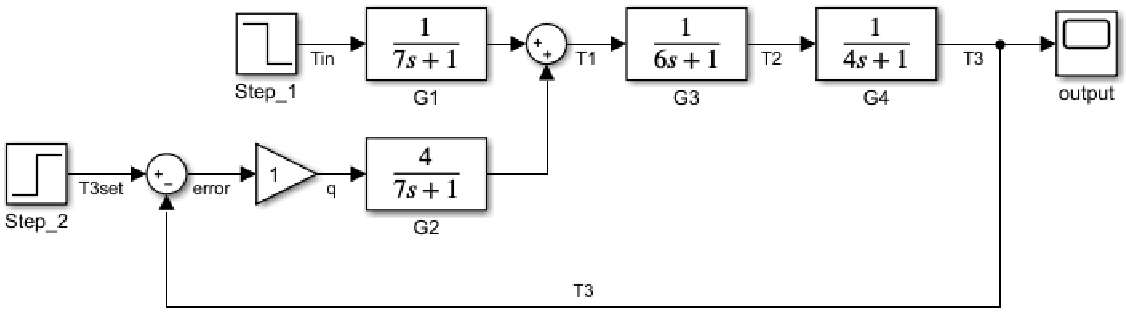

Figure 7. The block diagram for three-tank process with proportional control.

Using a proportional gain of $K_c = 1$, the performance is shown in Figure 8.

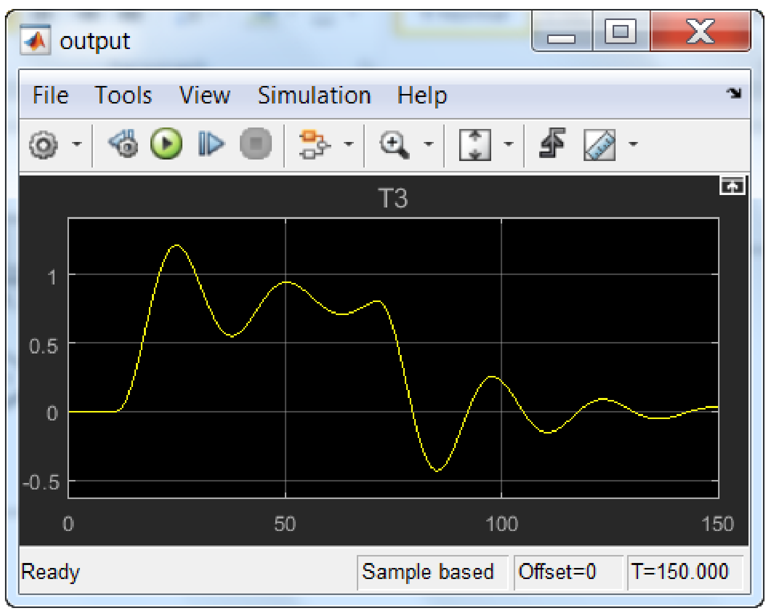

Figure 8. Performance of proporitonal control with $K_c = 1$ on three-tank process.

Note the following:

- Recall that the setpoint was stepped up to $T_3^{set} = 1$ at $t = 10$. The steady-state will be about 0.8, yielding an offset error of about 0.2. This shows the major limitation of using plain proportional control.
- The response is stable but oscillating. Decreasing K_c will reduce the oscillations but will result in worse steady state error offset.
- At $t = 70$, the effects of the step change in disturbance T_{in}has moved T_3 even further from the set-point showing that the compensation to reject disturbance effects was not satisfactory.

Example 4. Three-tank process with proportional-integral control

Now let us use a PI control for the three-tank system, described by the control law

$$q = K_c \left[e + \frac{1}{\tau_{int}} \int e \, dt \right] \tag{12}$$

where $e = T_3^{set} - T$ is the error signal.

The transfer function representation of the PI control becomes

$$q = (G_c)e \tag{13}$$

With G_c as the transfer function of the PI controller from e to manipulated variable q given by

$$G_c = K_c \cdot \left(1 + \frac{1}{\tau_{int}s}\right) = \frac{K_c \tau_{int} s + K_c}{\tau_{int} s} \tag{14}$$

Let us use the following PI tune values: $K_c = 0.2$ and $\tau_{int} = 10$. Note that we simply replaced the "Gain" block by the PI transfer function given in (14). (This is the advantage of design modularity within transfer function block diagram analysis - the other parts of the block diagram do not need to be modified when trying out different controllers.) The modified block diagram is shown in Figure 9.

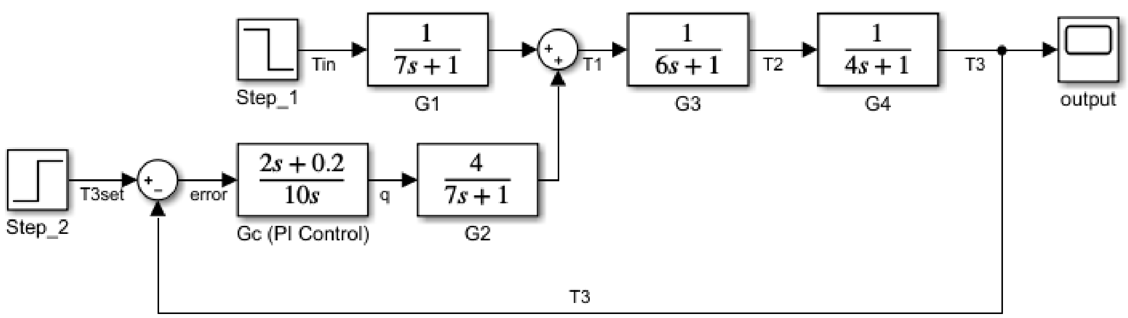

Figure 9. The block diagram for three-tank process with proportional-integral control.

Note: For the transfer function in G_c, the entry for the "denominator coefficients" should be "[10,0]".

The response of the PI control with : $K_c = 0.2$ and $\tau_{int} = 10$ is shown in Figure 10.

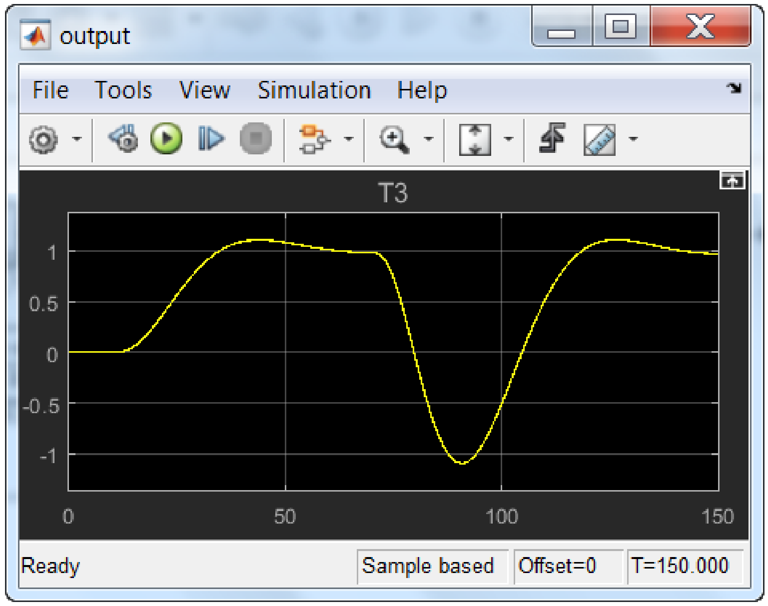

Figure 10. Performance of PI control with $K_c = 0.2$, $\tau_{int} = 10$, on three-tank process.

Note the following:

- The oscillations decreased for using $K_c = 0.2$, as compared to using $K_c = 1$ in example 3. There is still an overshoot but the percent overshoot is quite acceptable.
- The error eventually goes to zero as expected from the inclusion of an integral of error term.
- At $t = 70$, the step decrease in disturbance T_{in}initially moves T_3 down. However, the integral mode of the PI controller performed much better by successfully compensating for disturbance effects and moving T_3 back to the desired set-point.