CM3310 Spring 2008

(Dr. Tom Co, 2/10/2008)

Lecture 9. PID Tuning and Introduction to Laplace Transforms

1. PID Tuning Methods

a.) Cohen-Coon Method (Open-loop Test)

Step 1: Perform a step test to obtain the parameters of a FOPTD (first order plus time delay) model

i. Make sure the process is at an initial steady state
ii. Introduce a step change in the manipulated variable
iii. Wait until the process settles at a new steady state

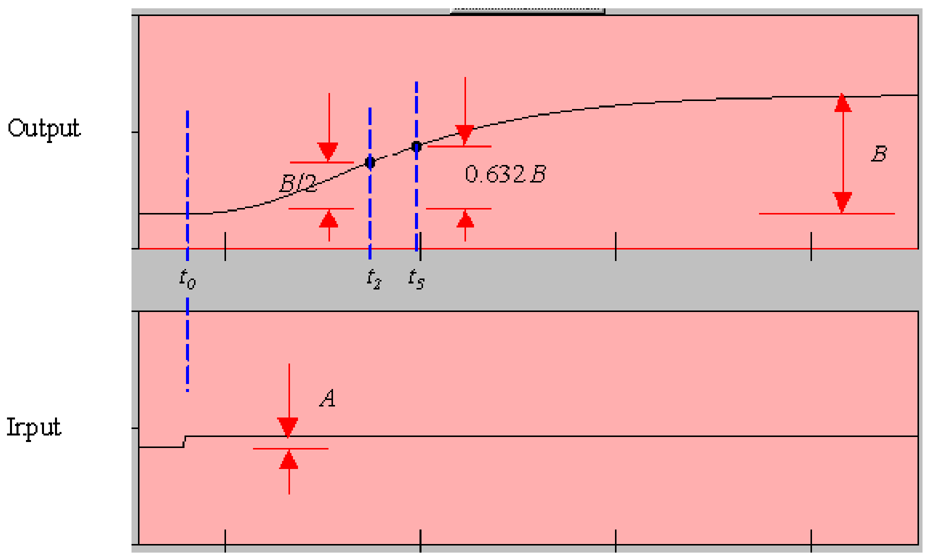


Figure 1. Step Test for Cohen-Coon Tuning.

Step 2: Calculate process parameters: $t_1, \tau, \tau_{del}, K, r$ as follows

$$t_1 = \frac{t_2 - (\ln(2))t_3}{1 - \ln(2)}$$

$$\tau = t_3 - t_1$$

$$\tau_{del} = t_1 - t_0$$

$$K = \frac{B}{A}$$

$$r = \frac{\tau_{del}}{\tau}$$

Step 3: Using the process parameters, use the prescribed values given by Cohen and Coon.

Table 1. Cohen-Coon Tuning Rules

	K_c	τ_{Int}	τ_{Der}
P	$\frac{1}{rK}\left(1+\frac{r}{3}\right)$		
PI	$\frac{1}{rK}\left(0.9+\frac{r}{12}\right)$	$\tau_{del}\frac{30+3r}{9+20r}$	
PID	$\frac{1}{rK}\left(\frac{4}{3}+\frac{r}{4}\right)$	$\tau_{del}\frac{32+6r}{13+8r}$	$\tau_{del}\frac{4}{11+2r}$

b.) Ziegler-Nichols Method (Closed-loop P-ControlTest)

Step 1: Determine the sign of process gain (e.g. open loop test as in Cohen-Coon).
Step 2: Implement a proportional control and introducing a new set-point.
Step 3: Increase proportional gain until sustained periodic oscillation.
Step 4: Record ultimate gain and ultimate period: K_u and P_u.
Step 5: Evaluate control parameters as prescribed by Ziegler and Nichols

Table 2. Ziegler Nichols Tuning Rules

	K_c	τ_{Int}	τ_{Der}
P	$\frac{K_u}{2}$		
PI	$\frac{K_u}{2.2}$	$\frac{P_u}{1.2}$	
PID	$\frac{K_u}{1.7}$	$\frac{P_u}{2}$	$\frac{P_u}{8}$

c.) Tyreus-Luyben Method (Closed-loop P-Control test)

Step 1-4: Same as steps 1 to 4 of Ziegler-Nichols method above

Step 5: Evaluate control parameters as prescribed by Tyreus and Luyben

Table 2. Tyreus-Luyben Tuning Rules for PI and PID

	K_c	τ_{Int}	τ_{Der}
PI	$\frac{K_u}{3.2}$	$2.2P_u$	
PID	$\frac{K_u}{2.2}$	$2.2P_u$	$\frac{P_u}{6.3}$

d.) Autotune Method (Closed-loop On-Off test)

Step 1: Let process settle to a steady state

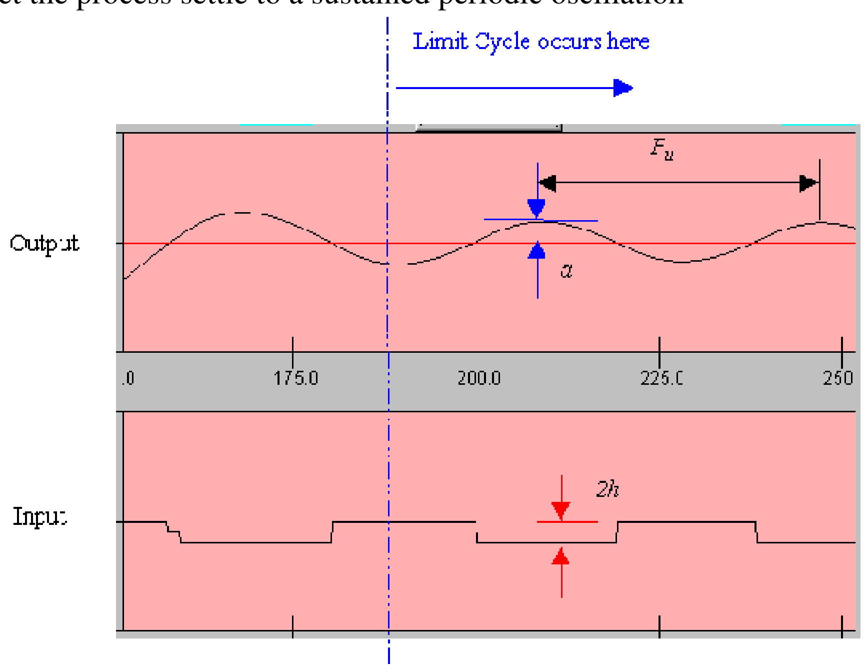
Step 2: Move the setpoint to the current steady state

Step 3: Implement an on-off (relay) controller

$$\text{If process gain is positive, } u = \begin{cases} u_0 + h & if \;\; e \geq 0 \\ u_0 - h & if \;\; e < 0 \end{cases}$$

$$\text{If process gain is negative, } u = \begin{cases} u_0 - h & if \;\; e \geq 0 \\ u_0 + h & if \;\; e < 0 \end{cases}$$

Step 4: Let the process settle to a sustained periodic oscillation



Step 5: Evaluate ultimate gain using autotune formulas (P_u can be obtain from the plots)

$$K_u = \frac{4}{\pi}\frac{h}{a}$$

Step 6: Use either Ziegler-Nichols or Tyreus-Luyben prescribed tunings

II. Introduction to Laplace Transforms

A. Motivation
1. Can convert linear differential equations into algebraic equations
2. Allow for modular design and analysis via transfer function blocks

B. Definition/Procedure

Given: a function in time, $f(t)$

Step 1: Multiply $f(t)$ by another function e^{-st}.

Step 2: Integrate the product with respect to t from $t = 0$ to $t = \infty$:

$$L[f(t)] = \int_0^\infty f(t)e^{-st}dt = \hat{f}(s)$$

Remarks:
1. s is known as the Laplace transform variable which is a complex variable constrained to have positive real parts.
2. Integration by parts are often implemented during evaluation of the integral.

C. Table of Laplace Transform (see pages 89-90 for a larger table)

	$f(t)$	$L[f(t)]$ or $\hat{f}(s)$
Step function	$S(t) = \begin{cases} 0 & if\ t \le 0 \\ 1 & if\ t > 0 \end{cases}$	$\frac{1}{s}$
Exponential	e^{-at}	$\frac{1}{s+a}$
Sine	$\sin(at)$	$\frac{a}{s^2+a^2}$
Cosine	$\cos(at)$	$\frac{s}{s^2+a^2}$
Power	t^n	$\frac{n!}{s^{n+1}}$
Delta Impulse	$\delta(t) = \frac{dS(t)}{dt}$	1

D. Properties of Laplace Transform

Linearity	$L[\alpha f(t) + \beta g(t)] = \alpha L[f(t)] + \beta L[g(t)]$
1st Shifting Theorem	$L[e^{-at} f(t)] = L[f(t)]\|_{s\to s+a}$
2nd Shifting Theorem (delayed functions)	$L[f(t-a)] = e^{-as} L[f(t)]$ (Note: valid only if $f(t<0) = 0$)
Transform of Derivatives	$L\left[\frac{d^n f}{dt^n}\right] = s^n L[f(t)] - \sum_{k=0}^{n-1} s^{n-k-1} \left.\frac{d^k f}{dt^k}\right\|_{t=0}$
Transform of Integral	$L\left[\int_0^t f\, d\tau\right] = \frac{1}{s} L[f(t)]$
Final Value Theorem	$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s\, L[f]$

E. Drills

1. Take the Laplace transforms of the following functions:
   a) $f(t) = 10e^{-2t} + 5\sin(3t)$
   b) $f(t) = e^{-2.5t}\cos(2t)$
   c) $f(t) = \cos(2.5t - 3)$ (Hint: use formula for cosine of sums.)
   d) $f(t) = e^{-5t} t^3$
   e) $f(t) = \begin{cases} 5 & if\ \ t \le 4 \\ -2 & if\ \ t > 4 \end{cases}$
   (Hint: rewrite the function first as a sum of delayed step functions)
2. Determine the inverse Laplace transforms of the following using the tables above:
   a) $\hat{f}(s) = \frac{4}{s} + \frac{2}{s+3} - \frac{1}{s^2+4}$
   b) $\hat{f}(s) = \frac{12}{(s+2)^2+25}$
   c) $\hat{f}(s) = \frac{2}{s^2+2s+2}$ (Hint: convert the denominator to a form $(s+a)^2 + b^2$)
3. Use the theorem of transforms of derivatives to obtain the Laplace transform of

$$\frac{d^2}{dt^2}(e^{-2t})$$

Answers:

1. a) $\frac{10}{s+2}+\frac{15}{s^2+9}$
   b) $\frac{s+2.5}{(s+2.5)^2+4}$
   c) $\frac{s\cos(3)+2.5\sin(3)}{s^2+2.5^2}$
   d) $\frac{6}{(s+5)^4}$
   e) $-\frac{2}{s}+\frac{7}{s}e^{-4s}$

2. a) $4S(t)+2e^{-3t}-\frac{1}{2}\sin(2t)$
   b) $\frac{12}{5}e^{-2t}\sin(5t)$
   c) $2e^{-t}\sin(t)$

3. $s^2\frac{1}{s+2}-\left(s+(-2)\right)=\frac{4}{s+2}$