CM3310 Spring 2008

(Dr. Tom Co, 4/2/2008)

Lecture 20. Bode Plots For PID Tuning

1. **Recall**

$$LM(H) = 20\log\left(|H(i\omega)|\right)$$
$$\phi = \tan^{-1}\frac{Im\big(H(i\omega)\big)}{Re\big(H(i\omega)\big)}$$

Thus, the critical point (-1,0) of the Nyquist plot of $H(s)$ occurs when

$$LM(H) = 20\log(|-1|) = 0 \text{ dB}$$
$$\phi = \tan^{-1}\left(\frac{0}{-1}\right) = -180°$$

2. **Crossover frequencies: (see page 231-232)**

   a) The crossover frequency, ω_{co}, is the frequency where phase shift, ϕ, is equal to $-180°$.
   b) The phase-margin frequency, ω_{pm}, is the frequency where the amplitude ratio is 1, or when log modulus is equal to 0 dB.

3. **Bode Stability Criterion (based on Nyquist Criterion)**

   If the log modulus of $H(s)$ at $\omega = \omega_{\text{co}}$ is less than 0 dB, then the feedback system is stable.

4. Stability Margins

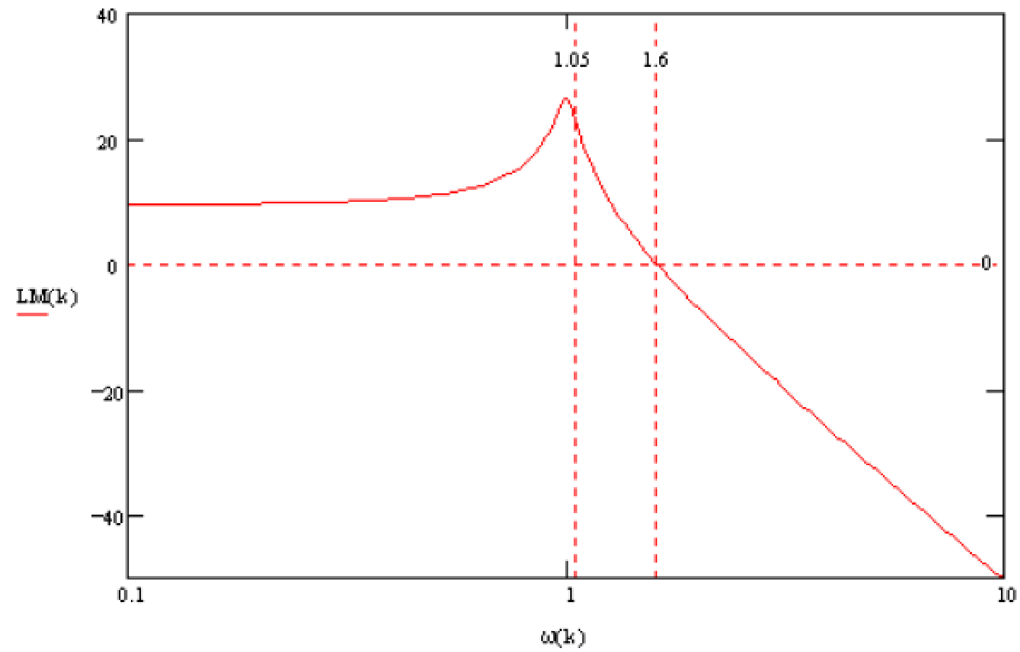
   a) Gain Margin
      - Determine the log modulus corresponding at $\omega = \omega_{\text{co}}$, i.e. LM_{co}.
      - The gain margin can then be evaluated as

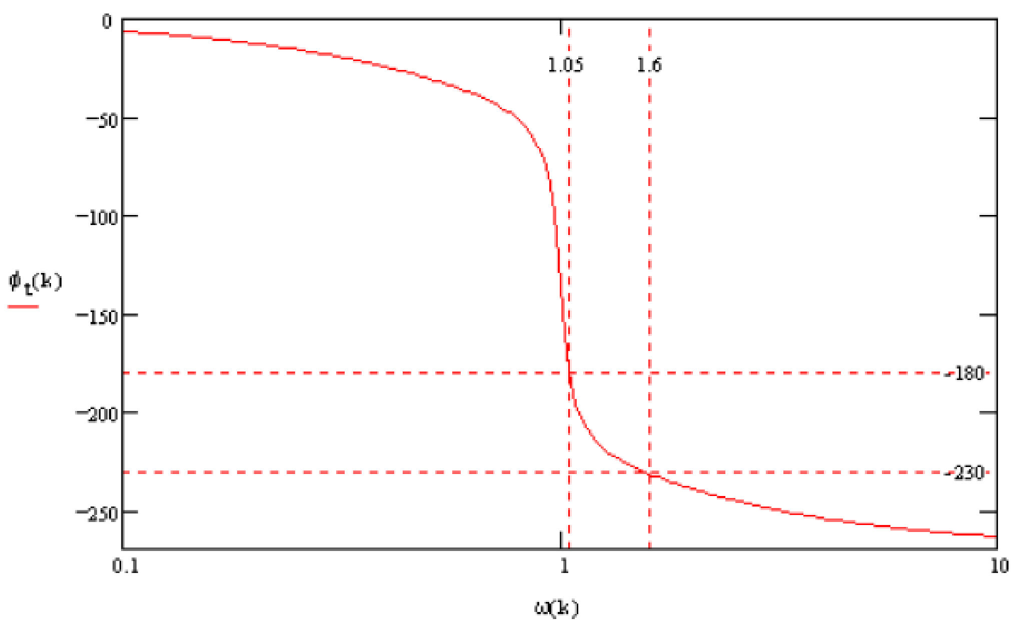
$$GM = 10^{\left(-\frac{LM_{\text{co}}}{20}\right)}$$

   b) Phase Margin

      - Simply measure the number of degrees above $-180°$ for the phase shift ϕ at $\omega = \omega_{\text{pm}}$.

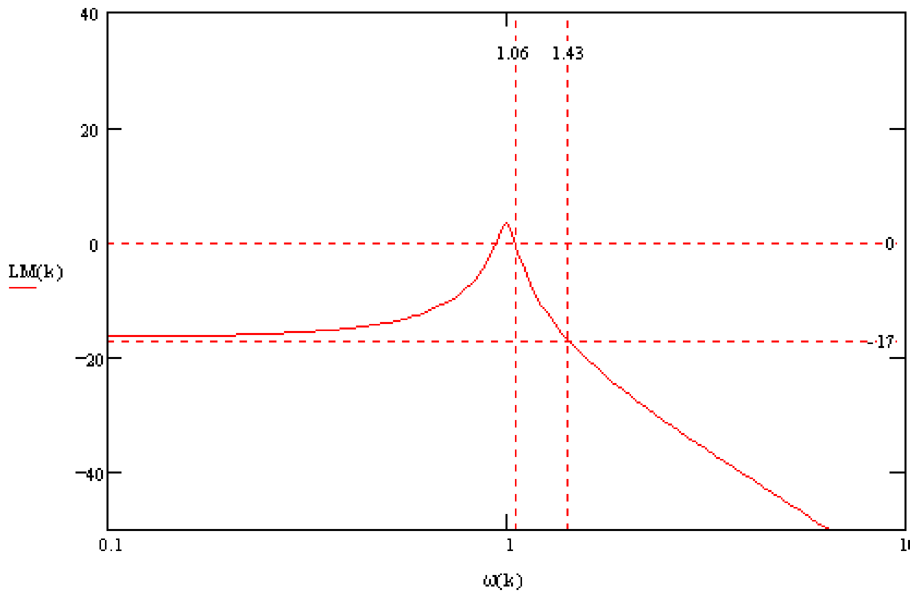
Example 1:

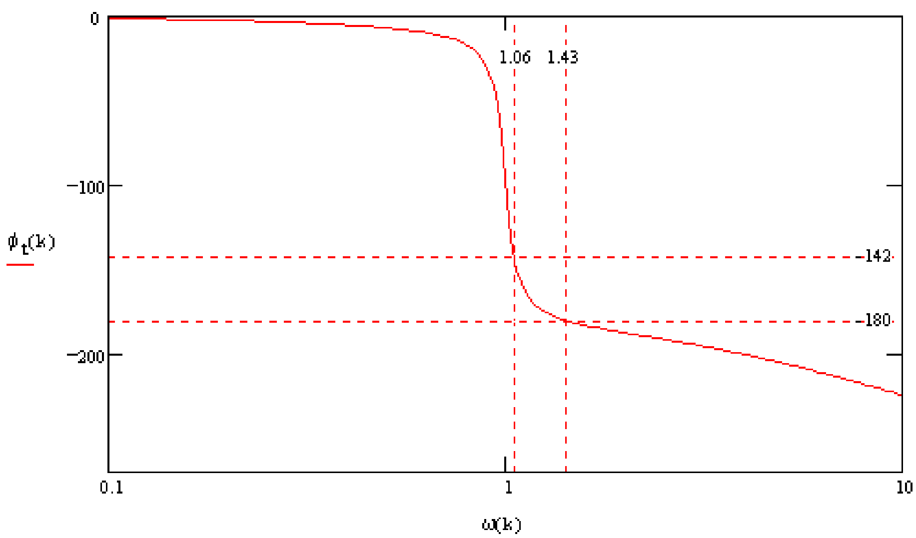




The crossover frequency is at $\omega_{co} = 1.05$ rad/sec. At this frequency the log modulus is above 0 dB. Thus the feedback process will be unstable. The phase-margin frequency is $\omega_{pm} = 1.6$ rad/sec.

Example 2:





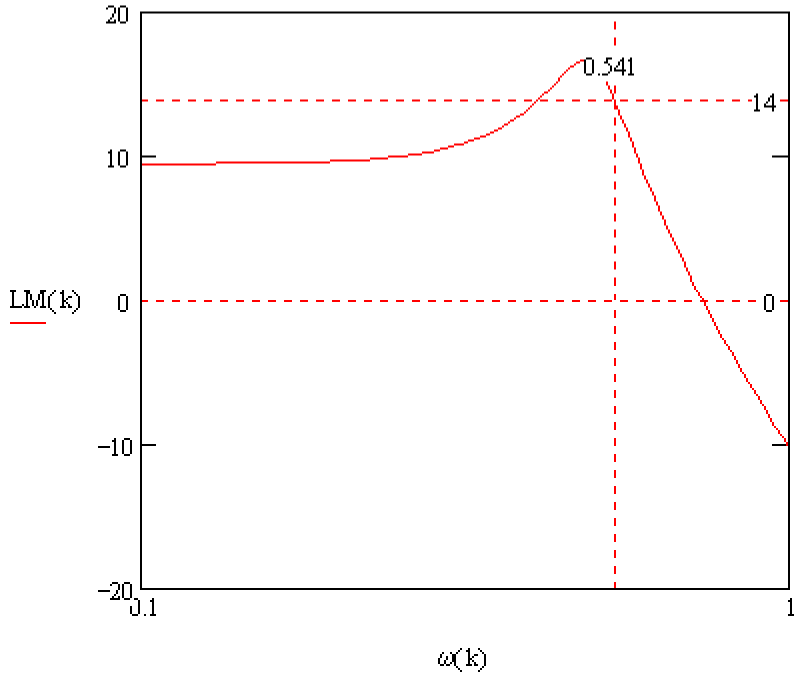
The phase crossover frequency is at 1.43 rad/sec, while the gain crossover frequency is at 1.06 rad/sec. The system is closed loop stable with the following stability margins:

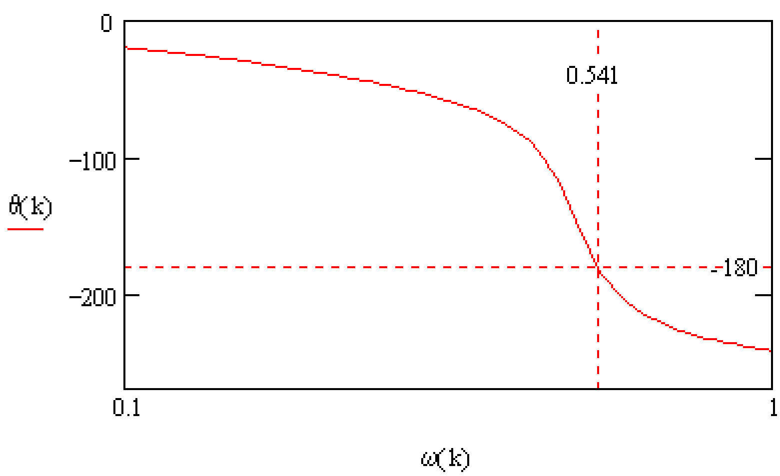
$$PM = 38° \qquad \text{and} \qquad GM = 10^{\frac{17}{20}} = 7.08$$

Bode Reshaping via PID Control (Ziegler-Nichols Design):

Suppose the process is described by the following transfer function:

$$G_p(s) := \frac{3}{\left[4\cdot\left(s^2\right) + 0.5\cdot s + 1\right]\cdot(3\cdot s + 1)}$$





Using Ziegler-Nichols method, we find that the ultimate gain is given by

Ultimate gain: $K_u = 10^{-14/20} = 0.2$

Ultimate period: Pu = 2π/0.541 = 11.6 sec

Based on the Ziegler-Nichols tuning of parameters, the resulting PID is given by

$$G_c(s) := K_c \cdot \left(\frac{\tau_I \cdot s + 1}{\tau_I \cdot s} \right) \cdot \left(\frac{\tau_d \cdot s + 1}{\alpha \cdot \tau_d \cdot s + 1} \right)$$

where $K_c = 0.118$, $\tau_I = 5.82$ and $\tau_d = 1.454$, with $\alpha = 0.01$. The Bode plots of this PID is given in Figure 2.

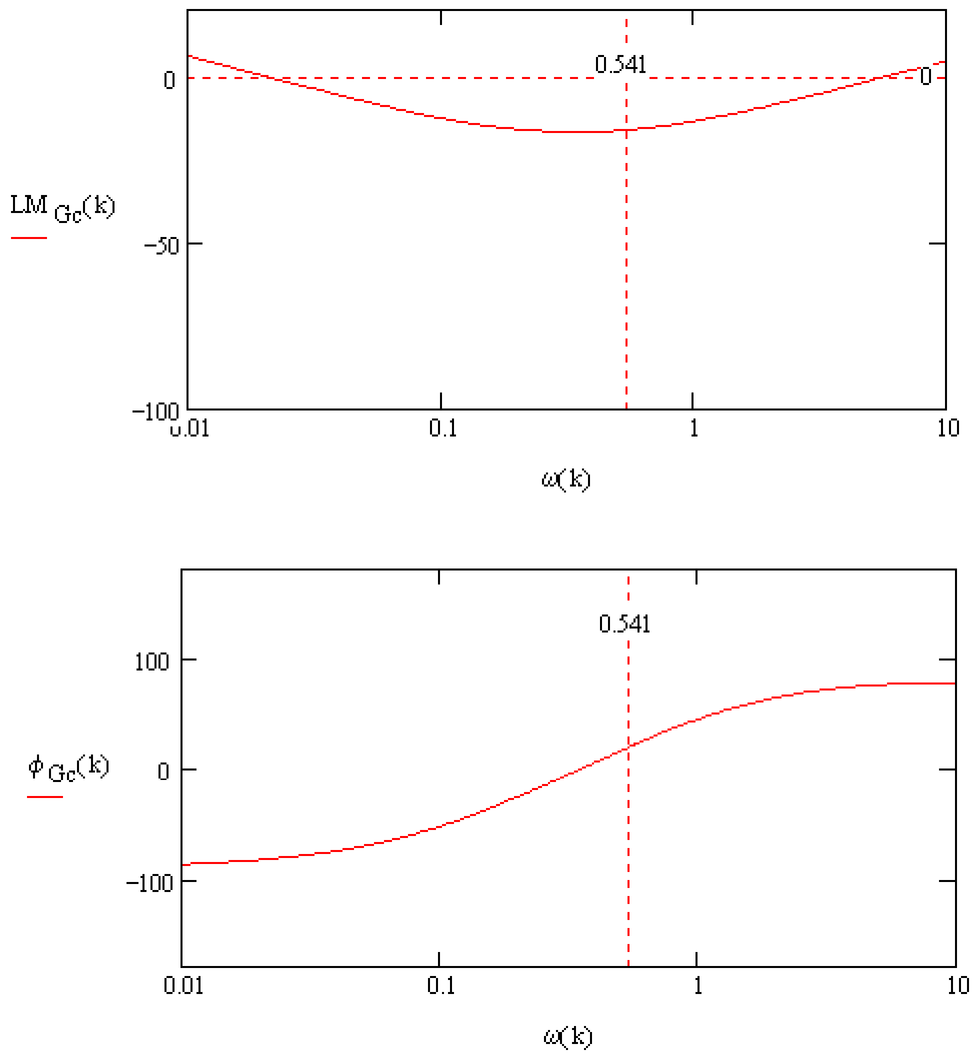


Figure 2.

Upon connecting G_c in series with G_p, the resulting Bode plots of G_cG_p are shown in Figure 3.

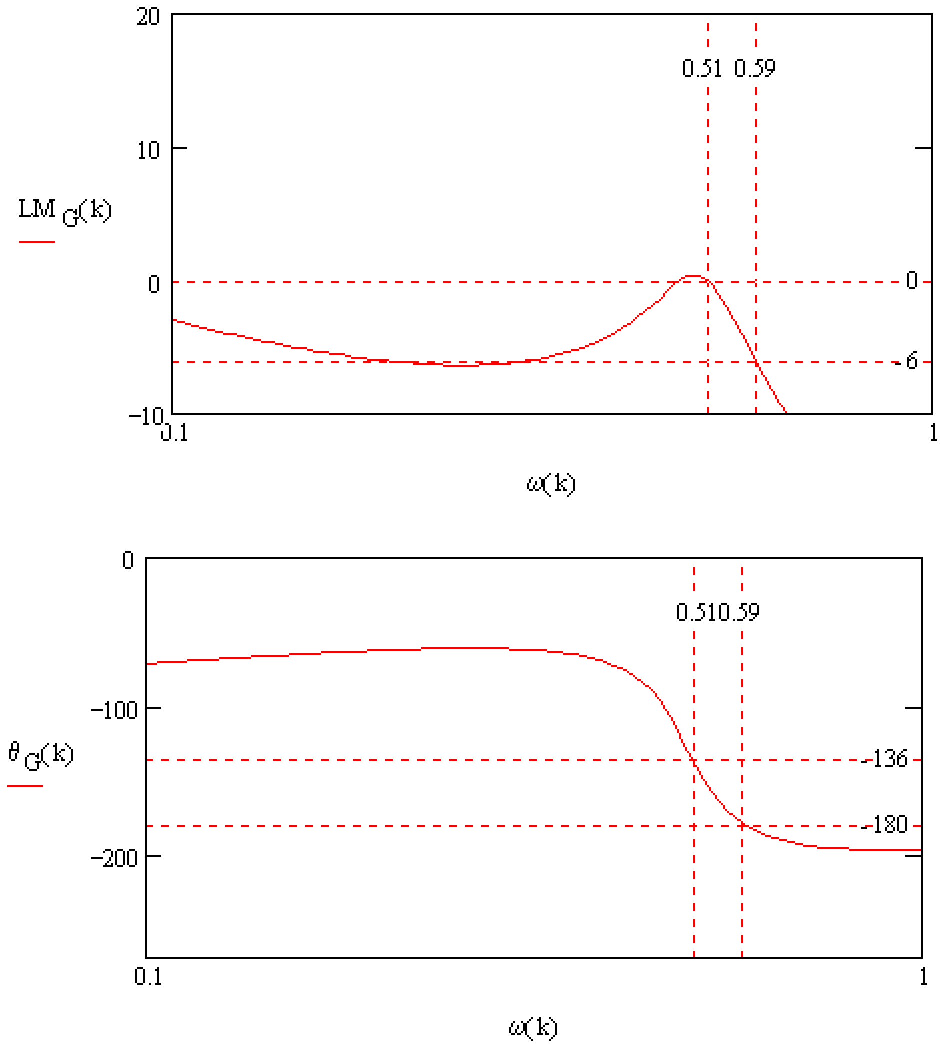


Figure 3.

Note that the result of implementing Ziegler-Nichols yields a gain margin=2 and phase margin approximately 45^o.