CM 3310 Process Control

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Lecture 6

Application of ODE Analysis of Feedback Controlled Systems

- Recall the first order process where b is replaced by $K_p u$

$$\tau_p \frac{dy}{dt} + y = K_p u \quad y(0) = 0 \tag{1}$$

- Under step test, u, the controller output (CO) is independent of y, the process variable (PV)
- Under feedback control, the value of u will be coming from a "Control Algorithm"

Proportional Control

- The simplest control algorithm is the Proportional (P) control given by

$$u = K_c(y_{set} - y) = K_c e \tag{2}$$

where $e = (y_{set} - y)$ is also known as the "error signal".

- Qualitatively, this rule says

"when error is large, the controller output (u) should also be larger.
Likewise, if the error is small, less action is required."

- After substituting (2) into (1),

$$\tau_p \frac{dy}{dt} + y = K_p K_c(y_{set} - y)$$
$$\tau_p \frac{dy}{dt} + (1 + K_p K_c)y = K_p K_c(y_{set})$$
$$\left[\frac{\tau_p}{1 + K_p K_c}\right]\frac{dy}{dt} + y = \left[\frac{K_p K_c}{1 + K_p K_c}\right] y_{set}$$

Implications:

a) New time constant: $\hat{\tau} = \tau_p/(1 + K_pK_c)$

➔ With $K_pK_c > 0$, the "Closed loop Response" should be faster

b) New steady state: $y_{ss} = y_{set}[K_pK_c/(1 + K_pK_c)]$

Define "(steady state) offset error" as : $\Delta_{set} = x_{set} - \hat{x}_{ss}$

$$\Delta_{set} = \left(\frac{1}{1 + K_pK_c}\right) y_{set} \rightarrow (\text{for } y_{set} \neq 0, \Delta_{set} = 0 \text{ only when } K_pK_c \rightarrow \infty)$$

Note:

a) Large K_c can amplify noise in the measurements

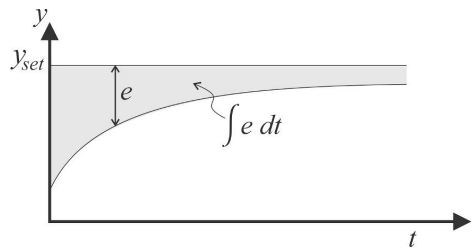
b) Large K_c can yield large controller output (CO) actions → valves and actuators will likely saturate back and forth fast and frequently.

Thus, proportional control is used when a level of steady state offset is tolerable, e.g. in level control of surge tanks.

Question: Will there also be steady state offset if proportional control is used for the general high order linear (self-regulating) processes?
What about for linear integrating processes?

Proportional Integral (PI) Control

- Recall that one of the main shortcomings of Proportional Control is that a steady state offset will likely be present.
- Additional push is needed to close the error ($e = y_{set} - y$) to zero.
- One effective choice is to calculate the extra push based on the area information generated by the gap.



→ ***PI control algorithm:***

$$u = K_c\left(e + \frac{1}{\tau_I}\int e\,dt\right) \tag{3}$$

where, τ_I is the integral time and K_c is control gain

Remarks:

1. The "integral time" τ_I is also known as the "reset time". And the units are often in seconds or minutes.
2. Since the integral time is in the denominator, increasing the magnitude of τ_I will reduce the influence of the integral of error term.
3. An alternative version would use the "integral rate", $\tau_R = 1/\tau_I$, which will have units of per second or per minute. Thus the PI control would become

$$u = K_c\left(e + \tau_R\int e\,dt\right) \tag{4}$$

4. Both versions in (3) and (4) has the control gain outside the parenthesis. These forms are then often referred to as "dependent PI form". The "independent PI form" is given by

$$u = K_c e + K_I\int e\,dt \tag{5}$$

where, $K_I = \text{integral gain} = K_c/\tau_I = K_c\tau_R$

(This means one needs to check the version that a control equipment is implementing)

Question: So, does this modification to the Proportional Control algorithm remove the steady state offset?

Substitute PI algorithm (3) in (1):

$$\tau_p \frac{dy}{dt} + y = K_p K_c \left([y_{set} - y] + \frac{1}{\tau_I} \int [y_{set} - y]\, dt \right) \quad (6)$$

Next, take a derivative d/dt on both sides to remove the integration operation,

$$\tau_p \frac{d^2 y}{dt^2} + \frac{dy}{dt} = K_p K_c \left(\left[\frac{dy_{set}}{dt} - \frac{dy}{dt} \right] + \frac{1}{\tau_I} [y_{set} - y] \right)$$

$$\tau_p \frac{d^2 y}{dt^2} + \left(1 + K_p K_c\right) \frac{dy}{dt} + \frac{K_p K_c}{\tau_I} y = \frac{K_p K_c}{\tau_I} y_{set} + K_p K_c \frac{dy_{set}}{dt}$$

Or

$$\left[\frac{\tau_p \tau_I}{K_p K_c} \right] \frac{d^2 y}{dt^2} + \left[\frac{\left(1 + K_p K_c\right) \tau_I}{K_p K_c} \right] \frac{dy}{dt} + y = y_{set} + \tau_I \frac{dy_{set}}{dt} \quad (7)$$

So, after using PI control, the steady state (i.e. set all time derivatives to zero) is:

$y_{ss} = y_{set}$ → no offset!

Caution: the process was initially a first-order lag, but it became a second order process with the introduction of the integral term of the PI

→ damping may be reduced too much and create large overshoots if K_c and τ_I are not chosen properly.

To analyze this, recast (7) into the second order form with τ_n and damping coefficient ζ:

$$\tau_n^2 \frac{d^2 y}{dt^2} + 2\zeta \tau_n \frac{dy}{dt} + y = y_{set} + \tau_I \frac{dy_{set}}{dt} \quad (8)$$

Thus, with $\tau_n^2 = \frac{\tau_p \tau_I}{K_p K_c}$ and $2\zeta \tau_n = \frac{(1+K_p K_c)\tau_I}{K_p K_c}$, we find

$$\tau_n = \sqrt{\frac{\tau_p \tau_I}{K_p K_c}} \quad \text{and} \quad \zeta = \left[\frac{1 + K_p K_c}{2\sqrt{K_p K_c}}\right]\left(\sqrt{\frac{\tau_I}{\tau_p}}\right)$$

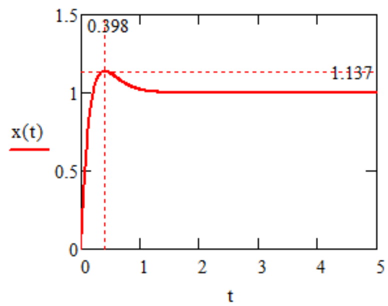
➔ a small τ_I can make $\zeta < 1$, i.e. "Underdamped"

Example:

Given an open-loop first-order process under manual control,

$$5\frac{dx}{dt} + x = 12\,u$$

Then using a PI controller with $K_c = 4$ and $\tau_{Int} = 0.35$ will yield a percent overshoot of $PO = 13.7\%$ as shown in the following plot:



Proportion-Integral-Derivative (PID) Control

- Recall that Proportional Controllers successfully moved process variables (PV) close to set points (SP), but not quite closing the gap (it tends to "sleep" before finishing the job)
- Then we added an integral of error term to close the gap and it worked, but in doing so, it might be possible that the chosen settings for integral time (aka reset time) may have introduce oscillations. Actually, high proportional gains can also increase oscillations
- Can we add another term to try to smooth out overshoots?

Proposed solution: add a derivative term

The "dependent ideal form" of the PID is now

$$u = K_c\left(e + \frac{1}{\tau_I}\int edt + \tau_{Der}\frac{de}{dt}\right) \tag{9}$$

where, τ_{Der} is another tuning parameter known as the "derivative time" (aka "rate time") in units of seconds or minutes.

Remarks:

1. Although the derivative term appears to be additive, it contributes by lessening the control action. For instance, let y_{set} be constant. Then

$$\frac{de}{dt} = \frac{d}{dt}(y_{set} - y) = -\frac{dy}{dt}$$

Hence, another available version of the ideal PID is given by

$$u = K_c\left(e + \frac{1}{\tau_I}\int edt - \tau_{Der}\frac{dy}{dt}\right) \tag{10}$$

2. The forms in (9) and (10) are tagged as "ideal" because a circuit hardware cannot really implement the algorithm exactly. Because the measurements often contain noise, the signals need to be filtered. We will defer the various forms for later lectures in which we could use transfer functions to describe the filters used.

Summary and Further Discussion:

1. P control is the simplest to use and tune. It is often implemented when a minimum levels of steady state error offsets are tolerated. (e.g. some tank level control)
2. PI control removes the steady state offset, but it could introduce underdamped behavior containing undesired oscillations and overshoots. However, some tuning rules have been found to reduce these problems. These are applied where processes with short time constants and yet steady state offsets need to be eliminated fast (e.g. flow control)
3. PID control can improve the response time by allowing higher proportional gain yet balanced/smoothened by the derivative term. These are applied where uncontrolled processes tend to be sluggish and yet needs to be controlled without steady state offset (e.g. temperature control of system with high heat capacities)

4. If deviation variable $\tilde{u}$ was used instead for the PID rule, we need to add the “bias” term u_o. Thus, with $\tilde{u} = u - u_o$, the ideal PID rule is often written as

$$u = u_o + K_c \left(e + \frac{1}{\tau_I} \int edt + \tau_{Der} \frac{de}{dt} \right) \quad (11)$$