Introduction to Resistor Circuits

(Tom Co 1/17/2008)

I. Voltage and Current

A. Definitions and Units

1. Current (I) = amount of electrical charge flowing per unit time

   Units: 1 ampere (A) = 1 coulomb/second

2. Voltage (V): potential energy per unit charge
   (also known as electromotive force, emf)

   Units: 1 volt (V) = 1 joule/coulomb

3. Power (P): power (energy per unit time)

   Units: 1 watt (W) = 1 joule/second

   $$P = V I$$

B. Conventions

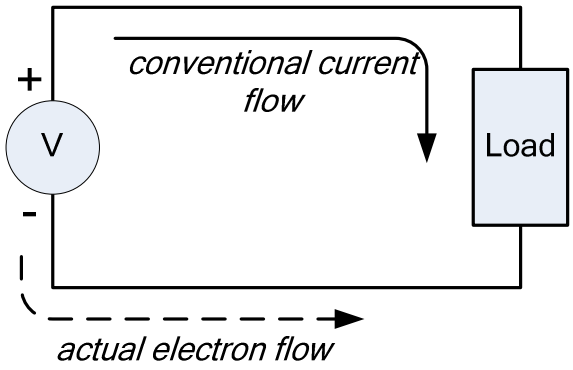

Figure 1. Electrical Loop

II. Resistors and Ohm's Law:

Resistance (R) = $\frac{V}{I}$

Units: 1 ohm (Ω) = 1 volt/ampere

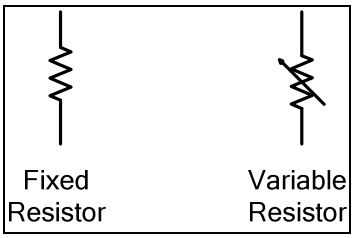

Figure 2. Symbols for Resistors.

Main application of resistors: to set the current flow in a wire for a given voltage.

An Analogy with Pipe Flow:

Voltage	Differential pressure (driving force)
Current	Volumetric flow rate
Resistance	Flow resistance

Main application of resistors: to set the current flow in a wire for a given voltage.

III. Kirchoff's Laws

a) Sum of voltage around a closed loop is zero
b) Sum of current at a junction is zero

Some conventions:

a) V_a = voltage at point a with respect to ground.
b) V_{ab} = $V_a - V_b$.
c) I_{ab} = current flow from point a to point b.
d) V_{ab} = $- V_{ba}$
e) I_{ab} = $- I_{ba}$
f) P_{ab} = $V_{ab} I_{ab}$

Loop A: $V_{ga} + V_{ab} + V_{bd} + V_{de} + V_{ef} + V_{fg} = 0$

Loop B: $V_{db} + V_{bc} + V_{ce} + V_{ed} = 0$

Junction a: $I_{ga} + I_{ba} = 0$

Junction b: $I_{ab} + I_{db} + I_{cb} = 0$

Junction c: $I_{bc} + I_{ec} = 0$

Junction d: $I_{bd} + I_{ed} = 0$

Junction e: $I_{de} + I_{ce} + I_{fe} = 0$

Junction f: $I_{ef} + I_{gf} = 0$

Junction g: $I_{ag} + I_{fg} = 0$

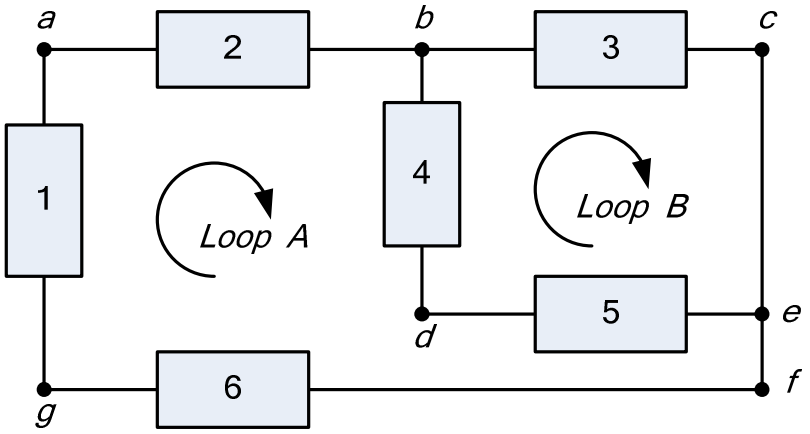

Figure 3.

Example 1:

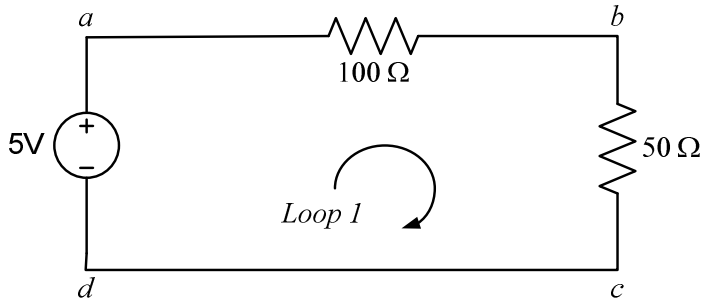

Figure 4.

Determine: V_{ab}, V_{bc} and P_{bc}

Solution:

Step 1. Write out Kirchoff's Law for Voltage and Current

$$\text{Loop } 1: \quad V_{da} + V_{ab} + V_{bc} + V_{cd} = 0$$
$$\text{Junction } a: \quad I_{da} + I_{ba} = 0$$
$$\text{Junction } b: \quad I_{ab} + I_{cb} = 0$$
$$\text{Junction } c: \quad I_{bc} + I_{cd} = 0$$
$$\text{Junction } d: \quad I_{ad} + I_{cd} = 0$$

Step 2. Substitute given voltage sources, implement Ohm's law and rewrite in terms of current with subscripts in set order (e.g. alphabetical in our case).

$$\text{Loop } 1: \quad -5 + 100I_{ab} + 50I_{bc} + 0I_{cd} = 0$$
$$\text{Junction } a: \quad -I_{ad} - I_{ab} = 0 \rightarrow -I_{ad} = I_{ab}$$
$$\text{Junction } b: \quad I_{ab} - I_{bc} = 0 \rightarrow I_{bc} = I_{ab}$$
$$\text{Junction } c: \quad I_{bc} - I_{cd} = 0 \rightarrow I_{cd} = I_{bc} = I_{ab}$$
$$\text{Junction } d: \quad I_{ad} - I_{cd} = 0 \rightarrow I_{ad} = I_{ab}$$

Step 3. Solve for all the currents I_{ij} and calculate the desired values in the circuit.

$$I_{ab} = \frac{1}{30}\,\text{amp} = I_{bc} = I_{cd} = I_{da}$$

$$\rightarrow \quad V_{ab} = \frac{10}{3}\,\text{volts}, \quad V_{bc} = \frac{5}{3}\,\text{volts}, \quad P_{bc} = \frac{5}{90}\,\text{watts}$$

Remarks:

1. Let a circuit consists of one voltage source V_1 and a set of resistors R_1, R_2,…, R_n. Then the voltage across resistor R_k is given by
$$V_{(R_k)} = \left(\frac{R_k}{R_1 + \cdots + R_n}\right) V_1$$
2. At points that connect only two wires: current flow in = current flow out. Thus, at point j,
$$I_{ij} = I_{jk}$$

Example 2:

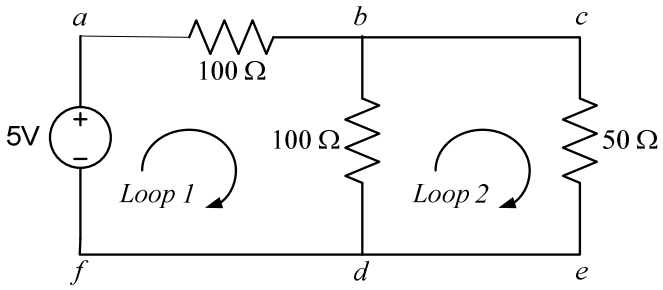

Figure 5.

Determine: V_{ab}, V_{bd}, V_{ce}, I_{bd}, I_{ce}, P_{ce}

Solution:

Step 1. Write out Kirchoff's Law for Voltage and Current

$$\text{Loop } 1: \quad V_{fa} + V_{ab} + V_{bd} + V_{df} = 0$$
$$\text{Loop } 2: \quad V_{bc} + V_{ce} + V_{ed} + V_{db} = 0$$
$$\text{Junction } b: \quad I_{ab} - I_{bd} - I_{bc} = 0$$
$$\text{Junction } d: \quad I_{bd} - I_{de} - I_{df} = 0$$
$$\text{Junction } a \text{ and } f: \quad I_{df} = I_{fa} = I_{ab}$$
$$\text{Junction } c \text{ and } e: \quad I_{bc} = I_{ce} = I_{ed}$$

Step 2. Substitute given voltage sources and implement Ohm's law

$$\text{Loop } 1: \quad -5 + 100I_{ab} + 100I_{bd} = 0$$
$$\text{Loop } 2: \quad 50I_{ce} - 100I_{bd} = 0$$
$$\text{Junction } b: \quad I_{ab} - I_{bd} - I_{ce} = 0$$
$$\text{Junction } d: \quad I_{bd} - I_{ce} - I_{ab} = 0$$

Step 3. Solve for all the currents I_{ij} and calculate the desired values in the circuit.

$$I_{bd} = \frac{1}{80}\text{amp}$$

$$I_{ce} = I_{bc} = I_{ed} = \frac{2}{80}\text{amp}$$

$$I_{ab} = I_{fa} = I_{df} = \frac{3}{80}\text{amp}$$

$$\rightarrow \quad V_{ab} = \frac{300}{80}\text{volts}; \quad V_{bd} = \frac{100}{80}\text{volts}; \quad V_{ce} = \frac{100}{80}\text{volts}$$

Consequence of Kirchoff's law:

a) Series

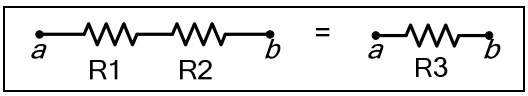

Figure 6.

$$\boxed{R3 = R1 + R2}$$

b) Parallel

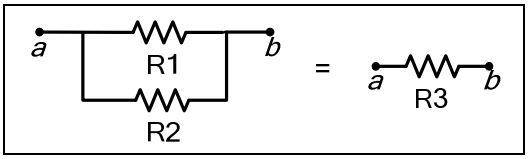

Figure 7.

$$\boxed{\frac{1}{R3} = \frac{1}{R1} + \frac{1}{R2}}$$

$$\boxed{\begin{aligned} \rightarrow \quad V_{(R_1)} = V_{(R_2)} &= \frac{R_1 R_2}{R_1 + R_2} I_{ab} \\ I_{(R_1)} &= \frac{R_2}{R_1 + R_2} I_{ab} \\ I_{(R_2)} &= \frac{R_1}{R_1 + R_2} I_{ab} \end{aligned}}$$

Example 2 (revisited) :

Using the parallel circuit formula above,

$$\boxed{V_{bd} = \frac{100 \cdot 50}{100 + 50} I_{ab}}$$

Substituting into Loop 1:

$$\boxed{\begin{aligned} &-5 + 100 I_{ab} + V_{bd} = 0 \\ &100 I_{ab} + \left(\frac{5000}{150}\right) I_{ab} = 5 \quad \rightarrow \quad I_{ab} = \frac{5}{100 + \frac{5000}{150}} = \frac{3}{80} \text{amp} \end{aligned}}$$

Drills:

1. Find V_{ab} and V_{bc} for the circuit in Figure 8.

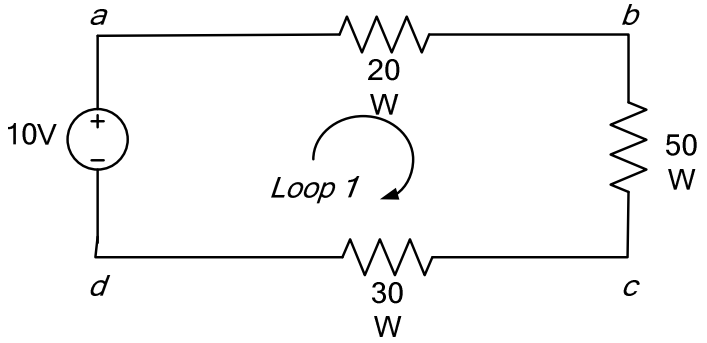

Figure 8.

Answer: V_{ab}=2 volts, V_{bc}=5 volts

2. Find V_{be} and V_{ef} for the circuit in Figure 9.

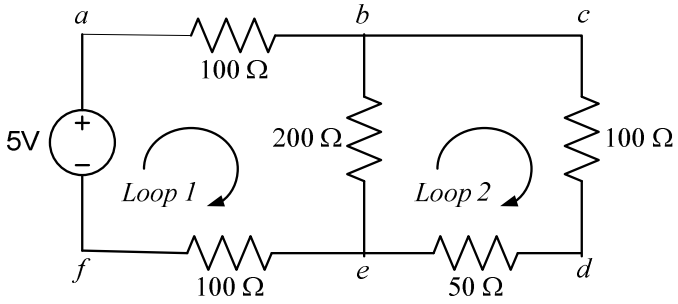

Figure 9.

Answer: V_{be}=1.5 volts, V_{ef}=1.75 volts