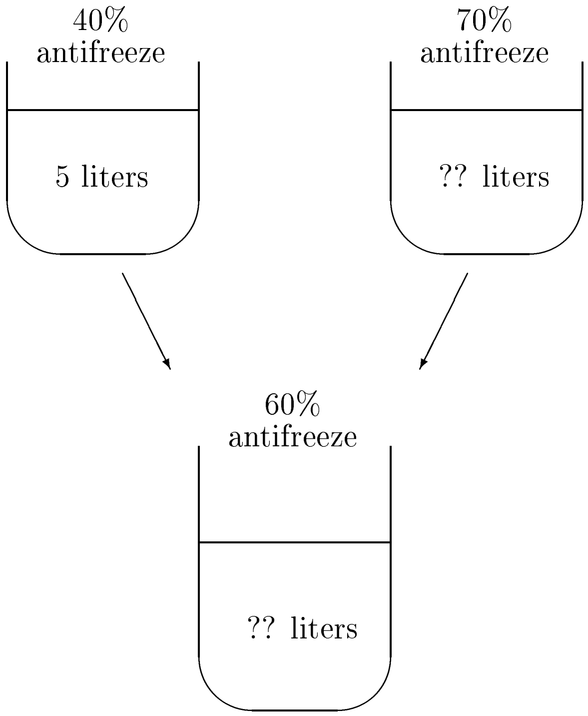
Setting Up Equations: Applications

For Instructors:

SUGGESTIONS on how to INTRODUCE worksheet #2

The Problem: Mixing Water and Antifreeze. How much of a 70% antifreeze solution must be added to 5 liters of a 40% antifreeze solution in order to obtain a 60% antifreeze mixture?



Read the problem to the students and sketch the diagram on the board. Try to get them to tell you how to label it.

Tell the students that you want to talk your way backwards through the problem before you have them work it forwards. If they know where they're supposed to be going, it'll be easier for them to get there.

Prompt the students to think about what they know, and what they need to know:

- What is the ultimate goal? (a final mixture that is 60% antifreeze)
- What information do you need about the final mixture in order to compute its percentage of antifreeze? (the volume of the mixture and the amount of antifreeze in the mixture)
- What information do you need in order to find the total volume of the mixture? (the volume of the 40% antifreeze and that of the 70% antifreeze)
- What information do you need in order to find the total amount of antifreeze in the final mixture? (the amount of antifreeze in each of the 40% and 70% solutions)

Explain that the first question on the worksheet will walk them through this problem. Then hand out the worksheets and have them work in their groups, discussing answers and asking questions as they work.

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Note: For this and all future worksheets, skip any portions marked “challenge” or “if you have time” and come back to them only if you’ve completed the rest of the worksheet.

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1. The following questions will guide you through the steps to solve the following problem.

   Mixing Water and Antifreeze. How much of a 70% antifreeze solution must be added to 5 liters of a 40% antifreeze solution in order to obtain a 60% antifreeze mixture?

   (a) The **unknown** quantity in this problem is the amount of 70% antifreeze needed, so give this unknown a “name” (i.e., choose a variable to represent this quantity). What are the units for this variable?

   $$\underline{\hspace{1.5cm}} = \text{(amount of 70\% solution), measured in } \underline{\hspace{3cm}}.$$

   (b) After mixing the 5 liters of the 40% solution with the unknown quantity of the 70% solution, what is the **total** amount of solution you will have? Include units.

   (c) How much antifreeze is in the 5 liters of the 40% solution?

   (d) How much antifreeze is in the unknown amount of the 70% solution?

   (e) What is the **total** amount of antifreeze in the mixture?

   (f) Use your answers above to write an expression for the **percentage** of antifreeze in the mixture.

   (g) Write an equation which says “The percentage of antifreeze in the mixture is equal to 60.”

   (h) Solve this equation (by hand) for the unknown.
   Does your solution make sense? How can you check your solution?

2. Now work through the previous problem, but using a 50% solution in place of the 70% solution.
   (a) STOP: Before you do any computations, think about whether the question makes any sense.
   (b) GO: Go through the same algebraic steps you did in the previous problem, with the 50% solution. What answer do you get? Does it make sense? (Challenge: Can you figure out a meaning for this answer?)

3. A square pond is enclosed by a deck that is three feet wide. The fence surrounding the deck is 100 feet long. Sketch a picture, labeling all known lengths. For any unknown quantities, represent each by a variable and label them in your sketch.

   Do you have enough information to determine the dimensions of the pond? If so, find the dimensions (is there only one answer?). If not, what additional information would be needed in order to determine the dimensions? (If you need any more information, invent it to solve the problem.)

   Would your answer/computations be different if the pond was a rectangle but not necessarily square?

4. The manager of a theater wants to know how many of its patrons are adults and how many are children. During a typical week, the theater sells 5200 tickets and has total receipts of \$20,335. The adult admission is \$4.75 and the children's admission is \$2.50.

   Answer the following questions to determine how many of the patrons in a typical week are children.

   (a) Choose variables to represent the two unknowns in this problem.

   (b) Write an equation (in terms of the unknowns) which is equivalent to the statement that "the theater sells 5200 tickets."

   Rearrange this equation so that it gives you a formula for the number of children in terms of the number of adults.

   (c) Write an expression (in terms of the unknowns) which represents the dollars received from the sale of tickets to adults.

   (d) Write an expression (in terms of the unknowns) which represents the dollars received from the sale of tickets to children.

   (e) Write an equation equivalent to the statement that "The total receipts are \$20,335."

   (f) Solve your two equations to find the number of children who go to the theater in a typical week. (Check your answer by computing the number of adults and then computing the total revenue.) Are there more children or adults who go to this theater?

IF YOU HAVE MORE TIME:

5. **Using Two Pumps.** A large (5-horsepower) pump can empty a pool in 5 hours, while a smaller (3-horsepower) pump can empty the same pool in 8 hours. The pumps are used together to begin emptying this pool. After two hours, the smaller pump breaks down. How much longer must the larger pump run before the pool is emptied?

   (Hint: Decide on variables to represent any unknowns, then start trying to find relationships between the variables. Make sure that everyone in your group knows precisely what each variable represents!)

   Check: Is your answer logical? Should it take more or less than 5 hours (total) to empty the pool?