Functions

Vocabulary: Think of a function as processor; if you feed it an acceptable input then it will return to you some output (for mathematical functions, both input and output will be numbers).

Fill in the blanks with the correct formal mathematical terms:

1. The set of allowable inputs is called the ____________ of the function.
2. The set of all possible outputs is called the ____________ of the function.

The rest of the questions on this page deal with the function f defined by

$$f(x) = \frac{x-3}{x-5}.$$

We'll also be looking at the graph, so have your calculator handy.

(Remember: the notation above means that when the input is $\clubsuit$, then the output will be $(\clubsuit - 3)/(\clubsuit - 5)$. "The point $(\heartsuit, \spadesuit)$ is on the graph" means that if $\heartsuit$ is the input then $\spadesuit$ is the output. In other words, the horizontal distance indicates the input and the vertical distance gives the resulting output.)

3. Determine symbolically whether or not $(3, 14)$ is on the graph. Check your answer with a graph on your calculator.
4. What is $f(4)$? (Find the exact answer symbolically and check it graphically.)
5. If $x = 4$, what is $f(x)$?
6. If $f(p) = 2$, what is p? (Check your answer both symbolically and graphically.)
7. What is the domain of f?

Challenge:

8. If $f(p) = q$, what is p? (Check your answer.)
9. Determine the range of f algebraically (based on your answer to the previous question) and check it graphically.

Increasing/decreasing functions and symmetry
What's wrong with each of the following statements?

10. "The function $f(x) = x^2$ is an increasing function. Looking at the right half of the graph, the function is obviously increasing (for $x > 0$) because it goes up. And when we look to the left of the origin the graph also goes up. So the function is increasing for all values of x."

11. "That $f(x) = \sqrt{x^2}$ sure is an odd function!"

12. "Every function is either even or odd."

Now we will examine the **postage rate function**.

The post office web page (http://www.usps.gov) gives the following information on rates for mailings weighing 13 ounces or less in July 2002:

Single-Piece Rates:	First ounce	\$0.37
	Each additional ounce (or fraction thereof)	\$0.23

Weight not over (oz.)	1	2	3	4	5	6	7	8	9	10	11	12	13
	\$0.37	0.60	0.83	1.06	1.29	1.52	1.75	1.98	2.21	2.44	2.67	2.90	3.13

Let's use $P(w)$ to represent the price (in dollars) to mail a parcel weighing w ounces.

13. Write a sentence which means the same thing as "$P(9.2) = 2.41$." (Don't use any symbols except numerals.)

14. Regarding price P as a function of weight w, which is the input and which is the output of this function?

15. If $w = 3.5$, what is $P(w)$?

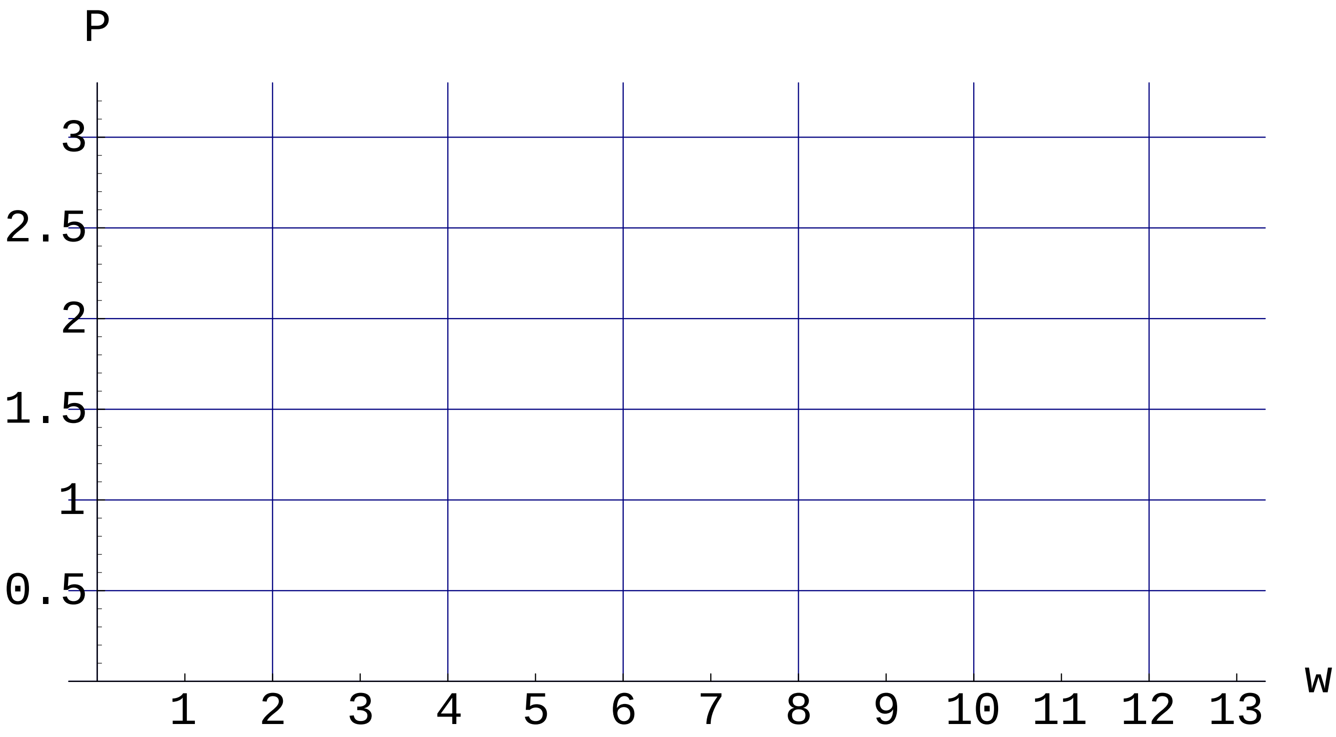
16. If $P(w) = 1.72$, what is w? (Think?)

17. Is P a function of w? How do you know?

18. What is the domain of $P(w)$?

19. What is the range of $P(w)$?

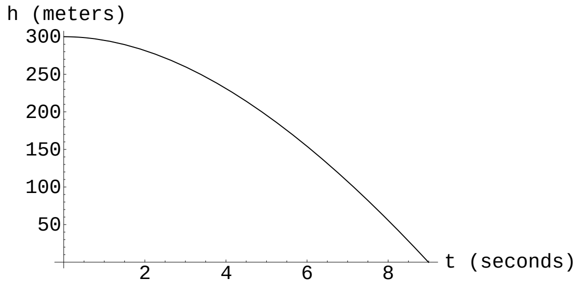
20. Draw the graph of the postage rate function $P(w)$ on the axes shown.

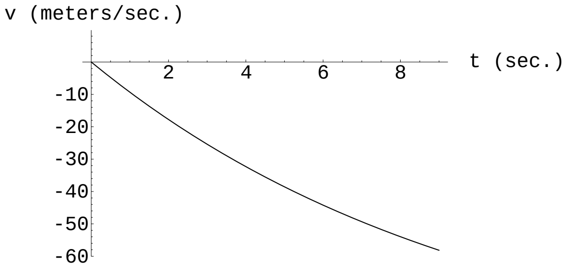


21. Try to write a piecewise-defined formula for the postal rate function. (It's rather boring, so you might just want to do it for $0 \leq w \leq 5$, say.)

Falling Bodies

Suppose you are dropping rocks off the edge of a 300 meter high cliff, and that the height h and velocity v of the rock are given by the following graphs.





22. What is the domain of the height function? ...of the velocity function?

23. What is the range of the height function? ...of the velocity function?

24. Is height an increasing function of time, a decreasing function of time, or neither?

25. If $t = 2$, what is $h(t)$? (approx.)

26. What is $v(2)$? (approx.)

27. Write a sentence which says the same thing as the statement "h(0)=300."

28. What is the speed of the rock when it is at a height of 250 meters? (approx.)

29. Find the average rate of change of the height function over the first two seconds. Include units. (Remember: the average rate of change of a function is change in output divided by change in input. Units for average rate of change are (units of output)/(units of input).)

30. What is the meaning of the number you found in the previous exercise?

31. Find the average rate of change of the velocity function over the first two seconds. Include units.

32. What is the meaning of the number you found in the previous exercise?

For Instructors:

SUGGESTIONS on how to INTRODUCE worksheet #4

Be sure you have mentioned the following topics and definitions:

- **Functions** as a process (feed it an **input** and it gives you an **output**)
- **Domain** and **range**
- Defining functions by **graphs**: (x, y) pairs, height above input-value gives corresponding output-value, how to see domain and range
- Defining functions by **tables**: is it only a representative set from a function defined on a larger domain or is it the whole function (with a discrete domain)?
- Defining functions by **formulas**: when is the formula valid?
- **Increasing/decreasing** (on an interval)
- **average rate of change**: change in output per change in input
- **Even and odd** functions (give at least one example of a function which is NEITHER!)