Trigonometric functions and their graphs

1. For each of the following observations about the unit circle, write an equation involving trig. functions which means the same thing. (The observations are not terribly precise, so you may have to think a little and draw a picture to guess at what the observer is thinking.)

   The first problem has been done as an example.

   (a) "Suppose you start at $(1,0)$ and move a certain distance around the circle. Whether you go clockwise or counter-clockwise, you end up at a point having the same x-coordinate."

   $$\cos(\theta) = \cos(-\theta)$$

   (b) "Suppose you start at $(1,0)$ and move a certain distance around the circle, looking at your y-coordinate. Depending on which way you go, the y-coordinate changes by a minus sign."

   (c) "Pick any point on the unit circle and make a right triangle by drawing a vertical line from that point to the x axis. The lengths of the sides of this triangle must satisfy the Pythagorean theorem."

   (d) "Pick any point (x,y) on the unit circle. If you go all the way around the circle, you end up at the same point (x,y) again." (two equations here: one for the x- and one for the y-coordinate)

   (e) "Pick any point (x,y) on the unit circle. The point directly opposite this point has the same x-coordinate, but with the opposite sign."

   (f) "Pick any point (x,y) on the unit circle. The point directly opposite this point has the same y-coordinate, but with the opposite sign."

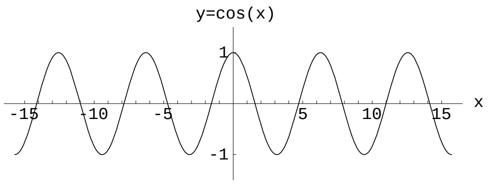
   (g) "Pick any point (x,y) on the unit circle and look at the point which is an angle of $\frac{\pi}{2}$ away from it. The x-coordinate of the first point is the same as the y coordinate of the second point."

   (h) "The x-coordinate of a point on the unit circle is never bigger than 1 or less than -1."

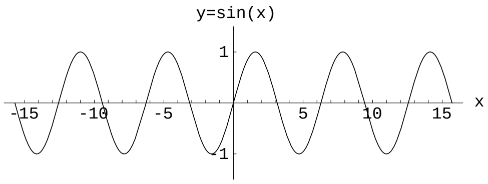
   (i) "The y-coordinate of a point on the unit circle is never bigger than 1 or less than -1."

2. For each of the observations/equations on the first page, what does this tell you about the **graph** of the functions $f(x) = \cos(x)$ and/or $f(x) = \sin(x)$?

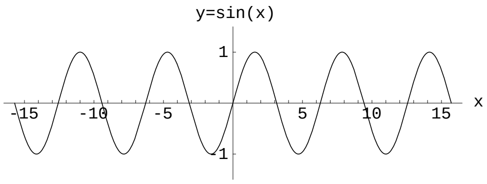
(a) $\cos(x) = \cos(-x)$



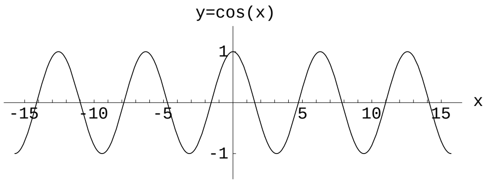
(b) $\sin(x) = -\sin(-x)$



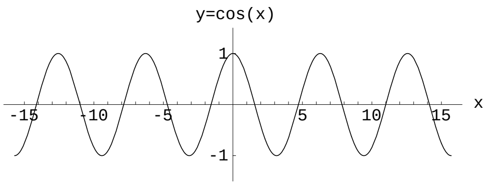
(d) $\sin(x) = \sin(x + 2\pi)$



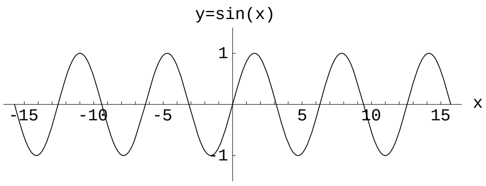
$\cos(x) = \cos(x + 2\pi)$



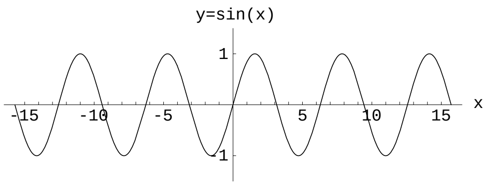
(e) $\cos(x + \pi) = -\cos(x)$



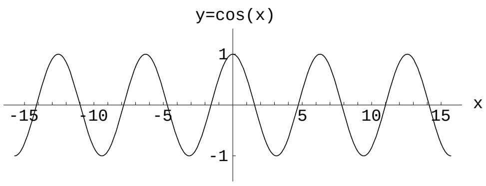
(f) $\sin(x + \pi) = -\sin(x)$



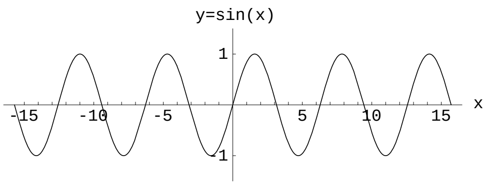
(g) $\cos(x) = \sin(x+\pi/2)$ or $\cos(x-\pi/2) = \sin(x)$



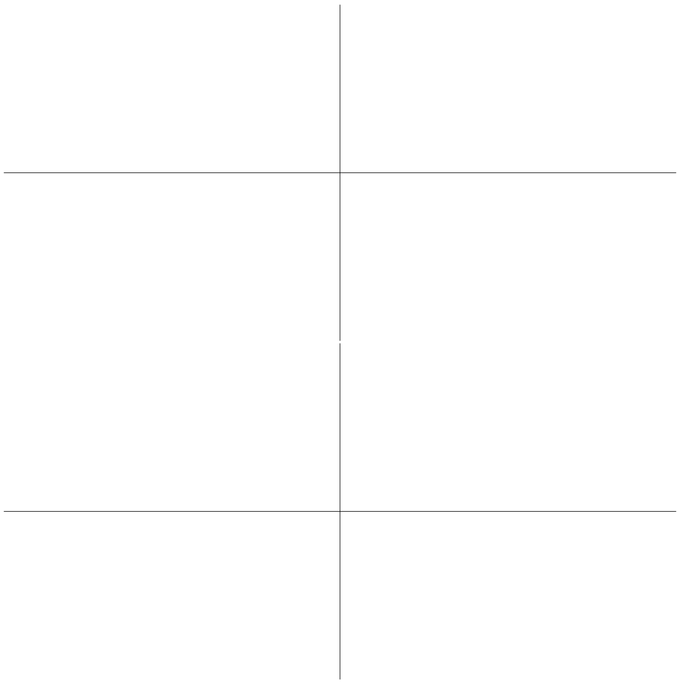
(h) $-1 \leq \cos(x) \leq 1$ or $|\cos(x)| \leq 1$



(i) $-1 \leq \sin(x) \leq 1$ or $|\sin(x)| \leq 1$



3. Use your calculator to show the graphs of $y = \sin(4x)$ and $y = \sin(4x + 4)$ together. Meanwhile, have another member of your team show the graphs of $y = \sin(4x)$ and $y = \sin(4x + 1)$ together. Sketch the graphs here:



How are they related? Which graph represents a left shift of $y = \sin(4x)$ by one unit? How could you tell this from the formula?

Skip the following challenge problem and come back to it if you have time later.

4. **Biorhythms.** In the (somewhat questionable[1]) theory of biorhythms, a sine function of the form

$$P = 50\sin(\omega t) + 50$$

is used to measure the percent P of a person's potential at time t, where t is measured in days and $t = 0$ is the person's birthday and ω is a constant which depends on the type of potential being considered. Three characteristics are commonly measured:

Physical potential: period of 23 days
Emotional potential: period of 28 days
Intellectual potential: period of 33 days

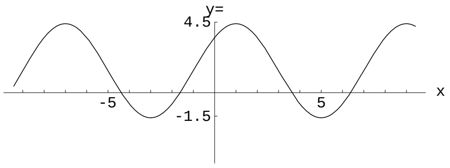
(a) Find ω for all three functions.

(b) Graph all three functions.

(c) Is there a time t when all three characteristics have 100% potential? If so, when is it? (How old will the person be when this happens?)

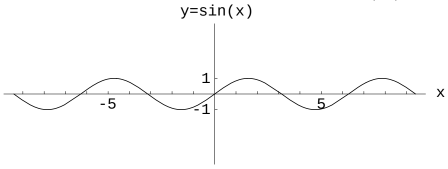
[1]Some say "bogus"

5. In order to find the equation for the function graphed below, we will use transformations to stretch and shift the graph of $y = \sin(x)$ until it looks right.

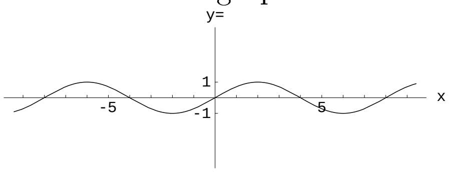


This function has peaks at $x = -7$, $x = 1$, $x = 9$, ..., and its maximum and minimum values are 4.5 and -1.5. What is the period of this function? What is its amplitude?

(a) First we deal with the horizontal stretch. The graph of $y = \sin(x)$:



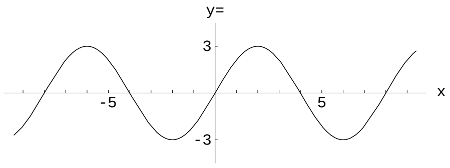
has a period of ____________ and we need to have a period of ____________. By what factor must we stretch $\sin(x)$ horizontally to get the right period? ______
Write the formula for the stretched graph shown in the picture below.



function:____________

Check your formula by graphing it on your calculator.

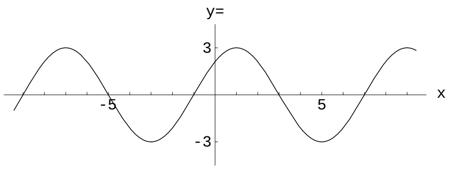
(b) Now we stretch the graph vertically to get the correct amplitude. Use your function above and transform it to get the function with the proper vertical stretch.



function:____________

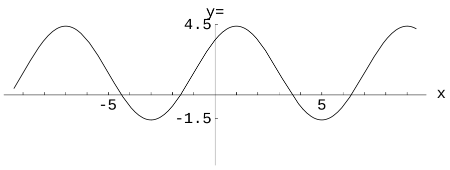
Again, check your formula by graphing it on your calculator.

(c) Now shift your graph one unit to the left so that the peak occurs at $x = 1$, as shown.



function:____________

(d) Finally, shift the graph up so that the function has a range of $-1.5 \leq y \leq 4.5$.



function:____________

6. If you have more time, sketch the graph of a shifted sinusoidal curve (with period and amplitude clearly marked) and challenge another team to find a formula for the function you have graphed. (If no other group is up to your challenge, have each member of the group make a challenge graph for the group member sitting to their right.)

For Instructors:

Suggestions on how to introduce worksheet #12

For the second page, the students should be pointing to the graphs and drawing on them to describe to each other what property of the graph is a result of the equality/inequality. Do the first one for them as an example: pick some positive x value and note the y-value on the graph; then look at "$-x$" and show that the graph has the same y-value there.

Be sure students are familiar with

- The unit-circle definition of sine and cosine
- The meaning of "period" and "amplitude"
- Basic transformations of equations: horizontal and vertical shifts and stretches
- How to view multiple graphs on their calculator