Right triangle geometry

Xena is going to a tennis match and plans to sit at center-court so she can see both players equally well. Unfortunately, she has a stiff neck and her head can only turn through an angle of $\frac{2\pi}{3}$ side-to-side. Our goal is to determine how far back she must sit so she can see everything.

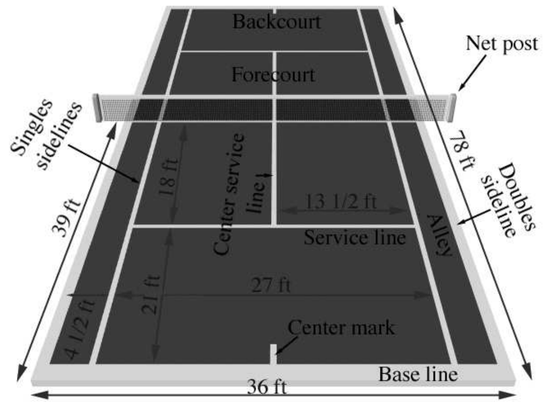
First draw a diagram of the geometry as seen from above (the view from a blimp). As shown below, a tennis court is 78 feet long and 36 feet wide. Assume that (during the game) the ball will always be in the region between the doubles sidelines and less than 10 feet beyond either base line. We'll call the distance from Xena to the nearest sideline "x". Draw Xena's head in your diagram and label the distance x.

Where would the ball be when Xena has to turn her head the most? Draw in the right triangle with one vertex at this point and one vertex at the location of Xena's head. Label the angle (in this right triangle) near Xena's head "θ".

What is the restriction on θ (due to Xena's stiff neck)?

Write an equation for $\sin(\theta)$ (or $\cos(\theta)$ or $\tan(\theta)$) in terms of x. Then use your knowledge of $\sin(\frac{\pi}{3})$ (or $\cos(\frac{\pi}{3})$ or $\tan(\frac{\pi}{3})$) to find x.

How many feet from the sideline should Xena be seated?



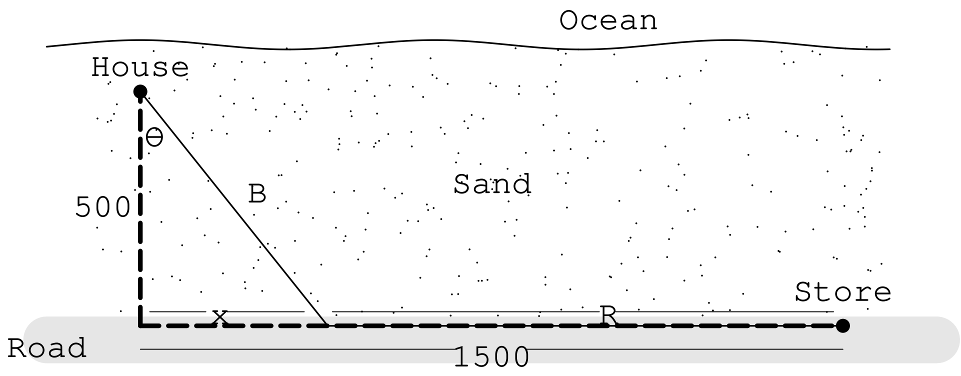
(image from http://muttley.ucdavis.edu/Curriculums/Tennis/Images/court_a.gif)

Suppose you want to walk from your beach house to the store down the street. You usually head straight across the sand directly towards the road (which is 500 feet away), turn left, and travel another 1500 feet down the road to the store (dashed lines in diagram).

It occurs to you one day that you could take off at a different angle from your house (solid lines in the diagram) and reduce the distance you have to walk. Assume that you can walk at a speed of 100 feet per minute along the beach and at a speed of 300 feet per minute along the road.

Our goal is to find the time T (in minutes) it takes to get to the store as a function of the angle θ.
Our steps will be

- Write the total trip time in terms of B and R, the distances walked along the beach and road, respectively.
- Rewrite R in terms of x (the side of the triangle in the diagram), so that the trip time is expressed in terms of the sides of the triangle.
- Rewrite the sides of the triangle (B and x) in terms of θ using the formulas for $\sin(\theta)$ and $\cos(\theta)$.



First we will check out the length of time it takes to walk to the store when you head straight across the sand.

1. If you walk along the dashed path, how much time do you spent walking on the beach? How much time is spend walking along the road? Add these two together to get the total trip time for the dashed path.

2. Suppose you walk ONLY on the beach, in a straight line from the house to the store. How far do you walk? How long does it take you to walk this distance?

Now consider the trip where θ varies:

3. If "B" is the distance (in feet) that you walk across the beach, how long does it take you to get to the road (in terms of B)?

4. If "R" is the distance (in feet) that you walk on the road, how long does this part of the trip take (in terms of R)?

5. Add your two times to write a formula for the total trip time T in terms of B and R.

6. Use the diagram to find a formula for R in terms of x.

7. Use this formula for R to write the total trip time T in terms of B and x.

Now we're ready to use the triangle:

8. Notice that the triangle in the picture is a right triangle. Write the formulas for $\sin(\theta)$ and $\cos(\theta)$ in terms of x and B (the lengths of the legs of the right triangle).

9. The expression for $\cos(\theta)$ should involve B. Solve this for B in terms of θ.

10. Now use the expression for $\sin(\theta)$ to find x in terms of B and θ.

11. Then use your result from the previous problem to "get rid of the B" and have an expression for x in terms of θ alone.

12. Plug in your formulas for B and x to find $T(\theta)$, the total trip time in terms of θ.

13. Plug in $\theta = 0$ to your formula to check it. What should $T(0)$ be?

14. Suppose you walk directly from your house to the store. What is the corresponding value of θ? Does your formula apply for values of θ which are larger than this value? What happens if you try?

15. Plot your formula for $T(\theta)$ on your graphing calculator and determine the minimum trip time.

16. Check that the minimum trip time is in fact less than the time required to walk directly to the store and the time to walk along the dashed path.

17. Suppose we had chosen to label the triangle differently: Label the angle at the lower right corner ϕ. What would your answers be in terms of ϕ?

18. Compare your formula to the one given on page 406 (problem 107). Explain the differences.

If you have more time:

19. Suppose you know that some angle θ is an acute angle (not necessarily the same "θ" as in the last problem). What can you say about $\sin(\theta)$? $\ldots\sec(\theta)$?

20. Consider an isosceles triangle (a triangle with two sides of equal length). If the length of the equal sides is a and the angle opposite these sides is θ, find a formula for the area A of the triangle as a function of a and θ. Pick a value for a and use your graphing calculator to determine the angle θ which maximizes the area $A(\theta)$.

21. If θ is an acute angle with $\sin(\theta) = 0.4$, find the exact value of

    (a) $\csc(\theta)$

    (b) $\cos(\theta)$

    (c) $\tan(\theta)$

    (d) $\cos^2(\theta)$

    (e) $\cos(\theta - \pi/2)$

22. If θ is an obtuse angle with $\sin(\theta) = 0.4$, find the exact value of

    (a) $\csc(\theta)$

    (b) $\cos(\theta)$

    (c) $\tan(\theta)$

    (d) $\cos^2(\theta)$

23. **Challenge:** Can you find the exact value of $\sin(1^o) + \sin(2^o) + \sin(3^o) + \sin(4^o) + \ldots + \sin(359^o)$? (Hint: Compare $\sin(1^o)$ to $\sin(359^o)$ by drawing triangles.)

For Instructors:

Suggestions on how to introduce worksheet #11

Be sure students are familiar with

- Pythagorean theorem
- How to compute $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$ from the lengths of sides of a right triangle with an acute angle θ.
- Trig. function values for angles that are multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$.

If students are clueless on the last problem, suggest they look at Table 6 on page 410 in the text.