Inverse Trigonometric Functions

Find the angles given by the following expressions (write exact answers – i.e., in terms of π). Split the work up between team members and use TI-89 calculators to get it done fast. Have each person start at the top or bottom of a column and work to the middle. The main idea is to get lots of examples so you can see patterns. Be sure your calculators are set to work in radians!

$\cos^{-1}\left(\cos(\frac{\pi}{4})\right) =$ $\quad$ $\sin^{-1}\left(\sin(\frac{\pi}{4})\right) =$

$\cos^{-1}\left(\cos(-\frac{\pi}{4})\right) =$ $\quad$ $\sin^{-1}\left(\sin(-\frac{\pi}{4})\right) =$

$\cos^{-1}\left(\cos(\frac{\pi}{3})\right) =$ $\quad$ $\sin^{-1}\left(\sin(\frac{\pi}{3})\right) =$

$\cos^{-1}\left(\cos(-\frac{\pi}{3})\right) =$ $\quad$ $\sin^{-1}\left(\sin(-\frac{\pi}{3})\right) =$

$\cos^{-1}\left(\cos(\frac{\pi}{2})\right) =$ $\quad$ $\sin^{-1}\left(\sin(\frac{\pi}{2})\right) =$

$\cos^{-1}\left(\cos(-\frac{\pi}{2})\right) =$ $\quad$ $\sin^{-1}\left(\sin(-\frac{\pi}{2})\right) =$

$\cos^{-1}\left(\cos(\frac{3\pi}{4})\right) =$ $\quad$ $\sin^{-1}\left(\sin(\frac{3\pi}{4})\right) =$

$\cos^{-1}\left(\cos(-\frac{3\pi}{4})\right) =$ $\quad$ $\sin^{-1}\left(\sin(-\frac{3\pi}{4})\right) =$

$\cos^{-1}\left(\cos(\frac{5\pi}{6})\right) =$ $\quad$ $\sin^{-1}\left(\sin(\frac{5\pi}{6})\right) =$

$\cos^{-1}\left(\cos(-\frac{5\pi}{6})\right) =$ $\quad$ $\sin^{-1}\left(\sin(-\frac{5\pi}{6})\right) =$

$\cos^{-1}(\cos(\pi)) =$ $\quad$ $\sin^{-1}(\sin(\pi)) =$

$\cos^{-1}(\cos(-\pi)) =$ $\quad$ $\sin^{-1}(\sin(-\pi)) =$

$\cos^{-1}\left(\cos(\frac{7\pi}{6})\right) =$ $\quad$ $\sin^{-1}\left(\sin(\frac{7\pi}{6})\right) =$

$\cos^{-1}\left(\cos(-\frac{7\pi}{6})\right) =$ $\quad$ $\sin^{-1}\left(\sin(-\frac{7\pi}{6})\right) =$

$\cos^{-1}\left(\cos(\frac{5\pi}{4})\right) =$ $\quad$ $\sin^{-1}\left(\sin(\frac{5\pi}{4})\right) =$

$\cos^{-1}\left(\cos(-\frac{5\pi}{4})\right) =$ $\quad$ $\sin^{-1}\left(\sin(-\frac{5\pi}{4})\right) =$

1. Based on your examples from the previous page, for which values of θ is it true that $\cos^{-1}(\cos(\theta)) = \theta$? Why?

Check with a few more examples (try some with $\theta > 2\pi$ or $\theta < -2\pi$).

2. For which values of θ is it true that $\sin^{-1}(\sin(\theta)) = \theta$? Why?

   Check with a few more examples (try some with $\theta > 2\pi$ or $\theta < -2\pi$).

3. Is it always true that $\cos\left(\cos^{-1}(\theta)\right) = \theta$? (Note the change in order of cos and $\cos^{-1}$. This makes a big difference!) Explain.

4. Is it always true that $\sin\left(\sin^{-1}(\theta)\right) = \theta$? Explain.

5. Use your calculator to compute each of the following.
   (a) $\cos^{-1}(\cos 2) =$
   (b) $\cos^{-1}(\cos 2.0) =$
   (c) $\cos(\cos^{-1} 2) =$
   (d) $\cos(\cos^{-1} 2.0) =$
   (e) $\sin^{-1}(\sin 2) =$
   (f) $\sin^{-1}(\sin 2.0) =$
   (g) $\sin(\sin^{-1} 2) =$
   (h) $\sin(\sin^{-1} 2.0) =$

   Note any weirdness. (CHECK: Do $\cos^{-1}(2)$ and $\sin^{-1}(2)$ make sense when you're using real numbers? How does your calculator display complex numbers, like $\sqrt{-1.0}$?)

6. Find the EXACT value of $\sin^{-1}(\sin(2))$ by drawing triangles in the unit circle. (Hint: In which quadrant is the angle of 2 radians?)

7. Sketch a right triangle where one of the acute angles, "θ," has a sine of $\frac{1}{4}$. Label the angle θ and the lengths of all three sides of the triangle. (Note: There are many answers; each one is a scaled and/or flipped version of the others, but the three angles involved are the same regardless of scale.)

8. Use your triangle to determine the value of $\cos(\theta)$. Notice that $\theta = \sin^{-1}(\frac{1}{4})$, so you have actually computed $\cos\left(\sin^{-1}(\frac{1}{4})\right)$. Check your answer with your calculator.

9. Now we'll do the same problem algebraically (rather than geometrically):
Suppose we have an acute angle θ and that $\sin(\theta) = \frac{1}{4}$. Use the fact that $\sin^2(\theta) + \cos^2(\theta) = 1$ to find $\cos(\theta)$.

(Does this agree with your answer to the previous problem?)

10. Almost nobody uses the inverse secant function. (In fact, most calculators don't even compute inverse secants. Does yours?) If $\sec(\theta) = x$, what is $\cos(\theta)$?

By interpreting "$\sec^{-1}(x)$" as "the angle whose secant is x," rewrite $\sec^{-1}(x)$ in terms of the inverse cosine function.

$\sec^{-1}(x) =$ ____________

(Check your answer with $x = 2$.)

11. Draw a right triangle where one of the acute angles, "θ," has a cosine of $\frac{2}{3}$. Use this to evaluate $\sin\left(\cos^{-1}(\frac{2}{3})\right)$. Check your answer with your calculator.

12. Evaluate $\sin\left(\tan^{-1}(\frac{3}{2})\right)$, by drawing a right triangle. Check your answer with your calculator.

13. Use your calculator to get decimal approximations for $\tan^{-1}(999.9)$, $\tan^{-1}(9999.9)$, $\tan^{-1}(99999.9)$, and $\tan^{-1}(999999.9)$ and the inverse tangent of some other really big number. Explain why these numbers are awfully close to $\frac{\pi}{2}$.

x	$\tan^{-1}(x)$
999.9	
9999.9	
99999.9	
999999.9	

14. **Challenge.** To find the EXACT value of $\phi = \sin^{-1}\left(\cos(\frac{5}{3})\right)$, decide which quadrant the angle $\frac{5}{3}$ radians is in and draw similar triangles in the unit circle. (Check your answer with your calculator.)

For Instructors:

Suggestions on how to introduce worksheet #14

Be sure students are familiar with

- the definitions of the inverse trig. functions ($\cos^{-1}(x)$, $\sin^{-1}(x)$, and $\tan^{-1}(x)$), including their domains and ranges.
- unit circle definitions of sine and cosine, and their connections to the inverse trig. functions.
- right triangle geometry (how lengths of sides are related to the sine, cosine, and tangent of an acute angle)