CALCULUS with Piecewise Linear Functions

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Derivative = Slope

The "derivative of f at x_0" is the **slope** of the graph of $y = f(x)$ at $x = x_0$. It is usually denoted

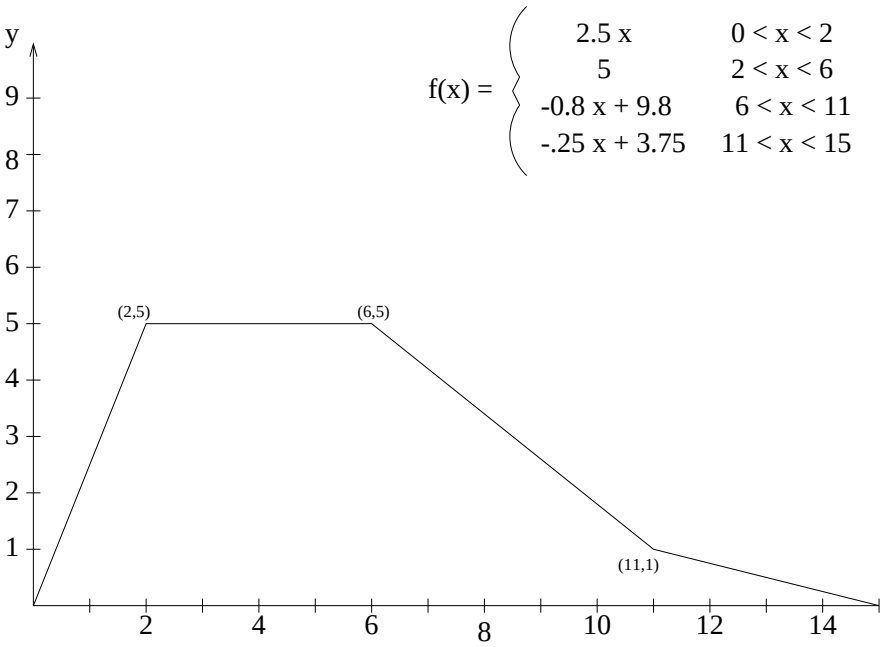
$$f'(x_0) \quad \text{or} \quad \frac{df}{dx}(x_0).$$

Integral = Area

The "integral of f from a to b" is the **area** under the graph of $y = f(x)$ and above the x-axis between $x = a$ on the left and $x = b$ on the right. It is usually denoted

$$\int_a^b f(x)dx.$$

Example



For the function $f(x)$ above, we have

$f'(1) = \frac{5}{2}$	and	$\int_0^2 f(x)dx = 5$
$f'(1.683) = \frac{5}{2}$		$\int_1^2 f(x)dx = 3.75$
$f'(5) = 0$		$\int_2^5 f(x)dx = 15$
$f'(6)$ is undefined		$\int_1^5 f(x)dx = 18.75.$

How are the last three integrals (areas) related? Why?

Worksheet

Below is the graph of a function $f(x)$. What is the value of

$$f'(1/2) \ ?$$

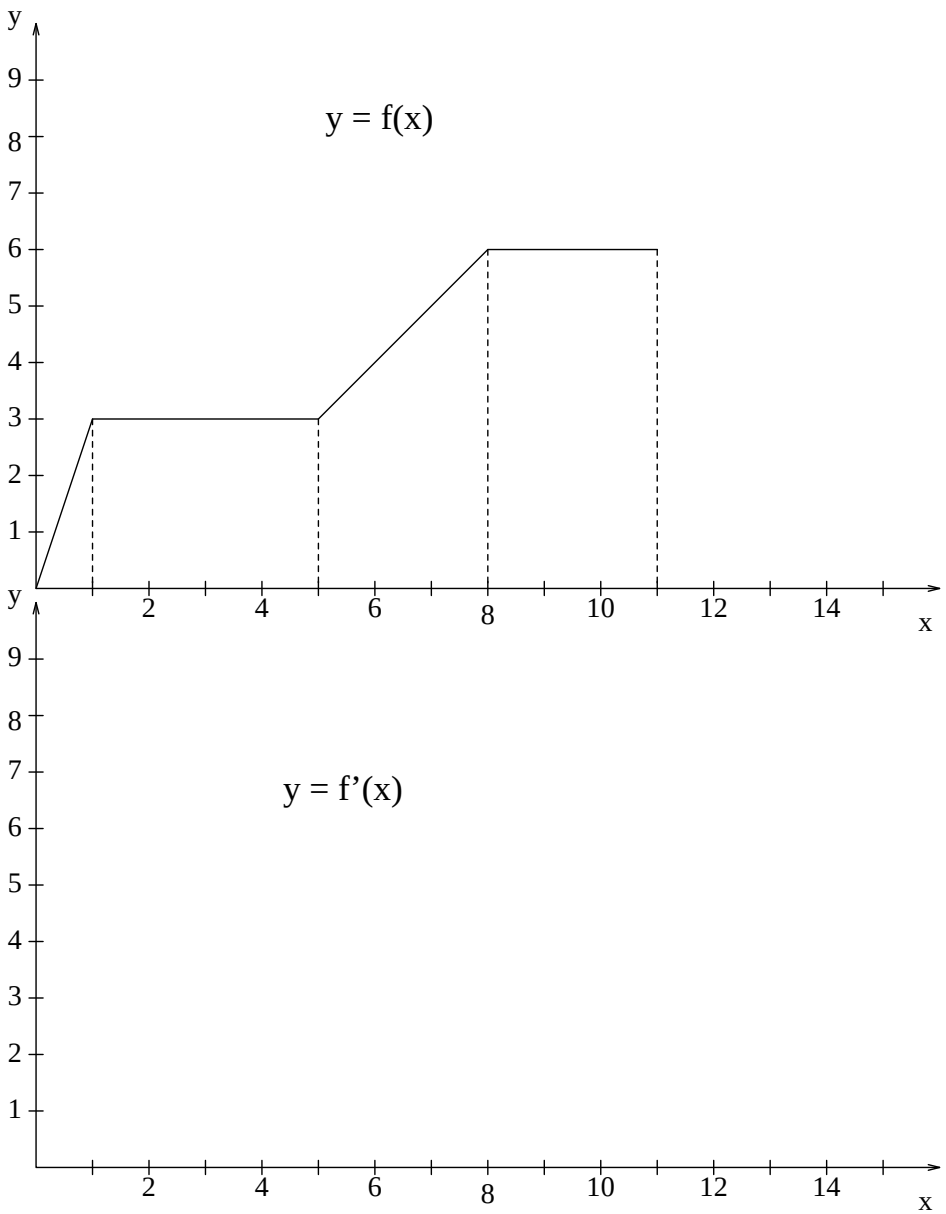
$$f'(2) \ ?$$

$$f'(4) \ ?$$

$$f'(6) \ ?$$

$$f'(10) \ ?$$

On the second set of axes below, sketch the graph of $f'(x)$.



After you have drawn the graph of $f'(x)$, calculate the integrals below.
Note: since it is "$f'(x)$" in the integral, you should use the graph of $f'(x)$ (not $f(x)$) to compute the areas.

$$\int_0^1 f'(x)dx =$$

$$\int_0^2 f'(x)dx =$$

$$\int_0^3 f'(x)dx =$$

$$\int_0^4 f'(x)dx =$$

$$\int_0^5 f'(x)dx =$$

$$\int_0^6 f'(x)dx =$$

$$\int_0^7 f'(x)dx =$$

$$\int_0^8 f'(x)dx =$$

$$\int_0^9 f'(x)dx =$$

$$\int_0^{10} f'(x)dx =$$

$$\int_0^{11} f'(x)dx =$$

Do you notice anything interesting here? (HINT: Compare the numbers above to the original function f evaluated at $x = 1$, $x = 2$, $x = 3$, $\ldots$, $x = 11$.) Write a formula describing what you notice about the integral

$$\int_0^b f'(x)dx.$$

IF YOU HAVE MORE TIME:

This formula will hold only when the original function goes through the origin (i.e., when $f(0) = 0$). How would the answers on this worksheet change if the whole graph of f were shifted up two units, so $f(0) = 2$? How would you alter the formula above?