NAME: _____

FINAL EXAM
MA3160, Spring '06

Please **show work** or give reasoning for **every** answer. (No credit will be given for correct answers without an indication of how you arrived at your conclusion.)

If you obtain an answer or part of an answer with your **calculator**, please indicate what you punched into your calculator and what the output was.

If you use a **formula**, please write down the formula that you are using.

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1. Does the sphere described by $(x-1)^2+(y-2)^2+(z-3)^2=6$ intersect the y-z plane?

   If so, find an equation for the circle (or point) of intersection.
   If not, show that there is no intersection.

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2. Suppose a person's weight W (measured in pounds) depends on his average daily food intake x (measured in calories) and his average daily salt intake y (measured in milligrams):

$$W = W(x,y).$$

   (a) What are the units for $\dfrac{\partial W}{\partial x}$?

   (b) Given that $W(2500,1500)=222$, $W_x(2500,1500)=.08$, $W_y(2500,1500)=.004$, estimate the value of $W(2500,1000)$.

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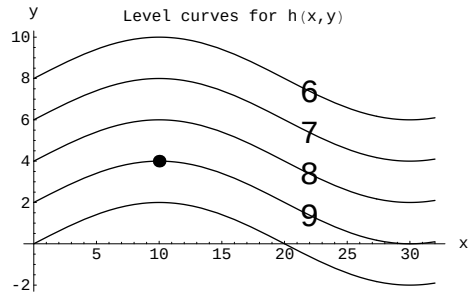
3. Suppose that $g(t)=f(t,t^2)$, where $f(x,y)$ satisfies $f_x(2,4)=3$ and $f_y(2,4)=5$. What is $g'(2)$?

4. Suppose you are walking on a surface over the x-y plane with height $h(x, y)$, where h has the level curves shown. (All quantities are measured in meters, the x-axis points east, and the y-axis points north.)



Justify your answers to the following questions by referring to a specific feature of the contours graph.

(a) As you walk north (in the y direction) from the point $(10, 4)$, are you walking uphill or downhill? What information does this tell you about a partial derivative of h?

(b) Is $h_{yy}(10, 4)$ positive, negative, or approximately zero? (Why?)

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5. Suppose a function $g(x, y)$ satisfies $g(0, 1) = 7$ and its partial derivatives are $g_x(0, 1) = 2$ and $g_y(0, 1) = 3$.

(a) What is $\nabla g(0, 1)$?

(b) Find an equation for the plane tangent to the graph of g at $(0, 1, 7)$.

(c) Find $g_{\vec{v}}(0, 1)$, the directional derivative of g in the direction of the vector $\vec{v} = 3\vec{\imath} + 4\vec{\jmath}$.

6. (a) Is $(0, 0)$ a critical point of $f(x, y) = x^2y - 4y$?
(Show how you can tell.)

(b) $(2, 0)$ is a critical point of $f(x, y) = x^2y - 4y$.
Does f have a local maximum at $(2, 0)$? ... a local minimum at $(2, 0)$?
(Show how you can tell.)

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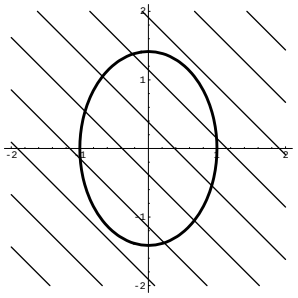
7. Suppose we want to find the maximum value of $f(x, y) = 3x + 3y$ subject to the constraint that $g(x, y) = 0$.
The lines graphed at right are the level curves of f, and the dark oval is the graph of $g(x, y) = 0$.

(a) On the graph, put a big dot at the point where the constrained maximum occurs.

(b) At the point you found, in what direction does the gradient of g point?



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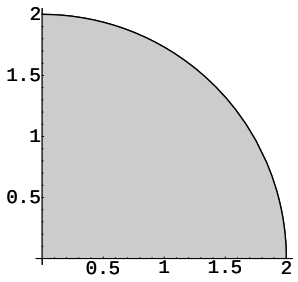
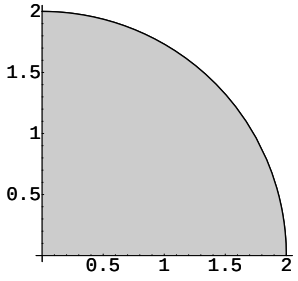
8. Set up the iterated integral of $f(x, y) = x + y$ over the first-quadrant part of a circle of radius 2 (as shown).
Use Cartesian (rectangular) coordinates.



9. Set up the iterated integral of $f(x, y) = x + y$ over the first-quadrant part of a circle of radius 2 (as shown).
Use polar coordinates.



10. Using spherical coordinates, set up the integral of the function $f(x, y, z) = \dfrac{1}{x^2 + y^2 + z^2}$ over the hemispherical region beneath the graph of $x^2 + y^2 + z^2 = 1$ and above the x-y plane.

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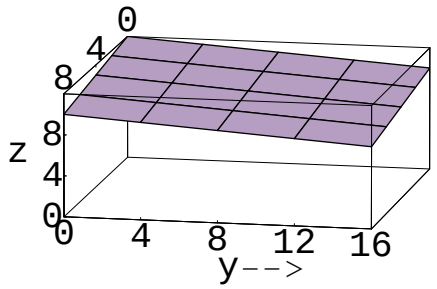
11. A barn is 8 meters wide and 16 meters long, with a slanted roof, as sketched below. The interior of the barn is described by $0 < x < 8$, $0 < y < 16$, and the roof is where $z = 12 - x/4 - y/8$, where x and y are distances from the west and south walls and z is the height above the floor (all measured in meters).

    (a) Set up a double integral which will give the volume of the barn.



    (b) If the density of dust in the barn is given by $\delta(x, y, z) = 1000ye^{-z}$ particles per cubic meter , set up an integral which gives the total amount of dust in the barn.

The questions on this page all involve the parameterized path given by

$$C : \begin{cases} x(t) = 3\cos(t) \\ y(t) = 3\sin(t) \end{cases} \qquad 0 \le t \le 2\pi.$$

12. Sketch and/or describe the path C, including orientation/direction.

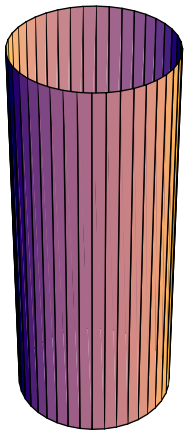
13. Set up the line integral, $\int_C \vec{F} \cdot d\vec{r}$, of the vector field $\vec{F}(x, y) = y^2\vec{\imath}$ around C.

14. Green's theorem says that this line integral of $\vec{F}(x, y) = y^2\vec{\imath}$ around C is equal to the double integral of what function over what region? Set up the double integral.

15. A fluid is flowing through a cylindrical pipe of radius 1, centered on the z-axis. The velocity of the fluid at a radial distance r from the center of the pipe is $\vec{v} = 10 * (1 - r^2)\vec{k}$, i.e.,

$$\vec{v}(x, y) = 10(1 - x^2 - y^2)\vec{k}, \qquad 0 \leq x^2 + y^2 < 1.$$

(a) Find the flux of $\vec{v}$ through a circular cross-section of the pipe, oriented upwards.



(b) Find the divergence and curl of $\vec{v}$ at an arbitrary point (x, y) in the pipe.

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16. The vector field, $\vec{F}$, is graphed at right.

(a) If possible, draw an oriented curve C from P to Q which results in a negative value for the line integral $\int_C \vec{F} \cdot d\vec{r}$.

(b) Is $\vec{F}$ a path-independent (conservative) vector field?
Justify your answer.