TEST #1
MA3160, Fall '05

Please **show work** or give reasoning for **every** answer. I need some evidence that you understand the topics. (No credit will be given for correct answers without an indication of how you arrived at your conclusion.)

If you obtain an answer or part of an answer with your **calculator**, please indicate what you punched into your calculator and what the output was.

If you use a memorized or programmed **formula**, please write down the formula that you are using.

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1. * Give the equation of the sphere of radius 5 centered at $(1, 2, 3)$.

2. Sketch the graph of the equation $x^2 + z^2 = 4$ in three dimensions (x-y-z space). Put tick marks on the axes to show the size. Describe it briefly in words if your picture isn't clear.

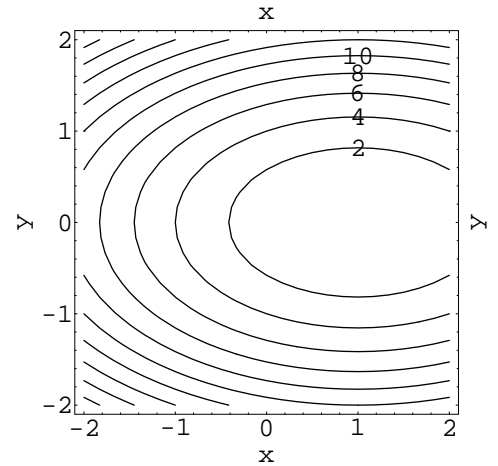3. Find an equation for the plane tangent to the graph of $f(x, y) = x^2 y$ at the point $(3, 1)$.

4. At right is a set of level curves for a function $f(x, y)$.

(a) Is $f(0, 1)$ positive or negative?
What feature of the contours tells you this?

(b) Is $f_y(0, 1)$ positive or negative?
What feature of the contours tells you this?

(c) Is $f_{yy}(0, 1)$ positive or negative?
What feature of the contours tells you this?

(d) Sketch a graph of the cross section of f with $x = 0$.
(Show the general shape and concavity of the cross section.)

(e) If we compute the quantity $f_y(0, 1)$, what feature of **your** graph (the cross-section) have we calculated? (Be as precise as you can.)
(You do not need to compute $f_y(0, 1)$, just tell me the meaning of the number.)

5. The table contains function values for a linear function of two variables.

(a) What is the gradient of the function at the point $(10, 6)$?

|		y		
		4	6	8
	5	3	6	9
x	10	7	10	13
	15	11	14	17

(b) Write an equation for the function.

6. Suppose a function $g(x, y)$ satisfies $g_x(x, y) = 0$ for all x and y. What does this say about the formula for g? Be as specific as you can; give an example if it helps to illustrate your answer.

7. The plot below shows the level curves of a function $g(x, y)$. Determine the sign (positive, negative, or zero) of each of the following partial derivatives at the point P. Assume the x- and y- axes are in the usual orientation.

(a) g_y (b) g_{xy}
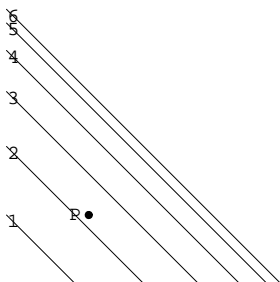
8. The monthly mortgage payment in dollars, P, on a 30-year mortgage is a function of two variables:

$$P = f(A, r),$$

where A is the amount borrowed (in dollars) and the interest rate is r percent.

(a) If $\frac{\partial P}{\partial r}\big|_{(92000,14)} = 72.82$, what is the meaning of the number 72.82? (Include units.)

(b) Would you expect $\frac{\partial P}{\partial A}$ to be positive or negative? Why?

9. Find the following partial derivatives:

(a) f_y for $f(x, y) = x^2 \sin(x) + x^2 \sin(y)$

(b) f_x for $f(x, y, z) = \frac{x}{y^2 + z^2}$

(c) $\frac{\partial}{\partial y}\left(e^{x^3 y}\right)$

10. Suppose you are hiking, and your altitude h (in feet) depends on the distance east and north of your starting point.

$$h(x, y) = 300xy + 100y^2$$

$$\text{altitude} = h(x, y), \quad x = \text{miles east}, \quad y = \text{miles north}.$$

You are currently 2 miles east and 3 miles north of your starting point.

(a) What is the altitude at your current location?

(b) Give an equation for the level curve (of h) through the point $(2, 3)$.

(c) What is the slope as you head east?
(Units of "feet per mile" are fine.)

(d) In which direction should you go if you want to go up as fast as possible?

(e) If you head north-by-northeast (in the direction of the vector $\vec{i} + 2\vec{j}$), what would the slope be?

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h[x_, y_] = 300*x*y + 100*y^2;
ContourPlot[h[x, y], {x, 0, 4}, {y, 0, 4}
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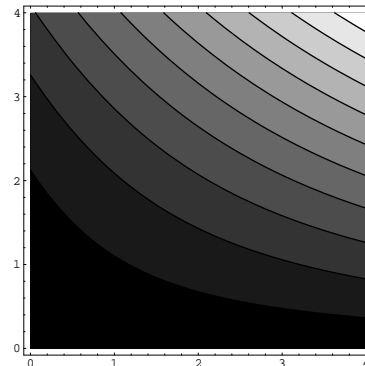

You might want to use the *Mathematica* output (at right) to CHECK your answer.

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11. For the function $g(x, y, z) = x^2 y^2 + y - z$

   (a) If you are at the point $(x, y, z) = (2, 1, 6)$, which level surface are you on?
      (Write an equation for the level surface.)

   (b) Find a vector which is normal (perpendicular) to this surface at the point $(2, 1, 6)$.

12. Let $F(u, v)$ be a differentiable function of two variables. If $h(x) = F(5x, x^2)$, find $h'(x)$.
    (Your answer will depend on the partial derivatives F_u and F_v as well as on x.)

13. Write a symbolic equation or inequality which is equivalent to saying that "A cross-section of $f(x, y)$ at constant x-value is concave up." Your answer should include some kind(s) of partial derivatives of f.