Test #1 MA3160, Spring '09

Please **show work** or give reasoning for **every** answer. (No credit will be given for correct answers without an indication of how you arrived at your conclusion.)

If you obtain an answer or part of an answer with your **calculator**, please indicate what you punched into your calculator and what the output was.

If you use a **formula**, please write down the formula that you are using.

1. L(x, y) is a LINEAR function of two variables. Some values of L(x, y) are listed in the table.

(a) What is $\nabla L(1,2)$ the gradient of L at (1,2)?



NAME:

- (b) Finish filling in the table. \longrightarrow
- (c) Find a formula for L(x, y).

(d) If we graphed z = L(x, y), what would the graph look like? (Describe the general shape.)

(e) Consider the graph of z = L(x, y) near the point (1, 2, 7). What is the slope in the direction of $\vec{v} = 4\vec{i} + 2\vec{j}$?

(f) Identify a vector which is perpendicular to the graph of z = L(x, y).

- 2. For the function $f(x, y) = ye^{xy}$,
 - (a) Give a formula for $f_y(x, y)$ (the y partial derivative of f)

(b) Find the exact value of $f_y(2, 1)$.

(c) Check your answer to the previous questions by using a difference quotient with $\Delta y = .01$ to approximate the value of $\frac{\partial f}{\partial y}$ at (2, 1).

(d) If f is measured in dollars, x is measured in meters, and y is measured in seconds, what are the units of $\frac{\partial f}{\partial y}$?

3. Consider a function g(x, y) which has the following derivatives at the point (2, -1):

$$g_x(2,-1) = 0, \qquad g_y(2,-1) = 0$$

$$g_{xx}(2,-1) = -2,$$
 $g_{yy}(2,-1) = -3$ $g_{xy}(2,-1) = 2$

Can you use the second derivative test (the discriminant) at the point (2, -1)?

– If not, explain why not.

- If so, show how to use it: What do you know about the shape of the graph of z = g(x, y) near the point (2, -1)?

4. Give an equation whose graph (in 3-D) is a cylinder of radius 5 centered on the x axis.

5. The graph of the equation

$$(x+3)^2 + (y+2)^2 + (z+2)^2 = 7?$$

is a sphere.

- (a) What are the center and radius of the sphere?
- (b) The sphere intersects the x-z plane in a circle. Find the equation for this circle.

- 6. Consider the graph of the function f(x, y) shown to the right. The x axis points out of the page (toward you).
 Decide on the sign (positive, negative, or approximately zero) for each of the following derivatives at the point shown. Identify the feature of the graph which determines your answer.
 - (a) $\frac{\partial f}{\partial x}$
 - (b) $\frac{\partial f}{\partial y}$
 - (c) $\frac{\partial^2 f}{\partial x^2}$
 - (d) $\frac{\partial^2 f}{\partial y^2}$



 $y \rightarrow$

The remaining questions refer to $f(x, y) = -8xy - \frac{1}{4}(x-y)^4$ and $g(x, y) = x^2 + y^2$. Level curves for f are shown at the bottom of the page, along with the graph of g(x, y) = 3.5.

7. Sketch a rough graph of the cross-section of f with x = 1.



- 8. In the first graph at the bottom of the page, put a big dot at each **critical point** of f(x, y).
- 9. Label each of your dots with "Max" or "Min" or "Saddle" (in the first graph below), to indicate if it is a local maximum, a local minimum, or a saddle point of f.
- 10. List the equations you would need to solve in order to find the critical points of f. (DO NOT SOLVE the equations, just write them out explicitly in terms of the variables. How many equations? How many unknowns?)

11. Suppose you solved the three equations: $\begin{cases} f_x(x,y) = \lambda g_x(x,y) \\ f_y(x,y) = \lambda g_y(x,y) \\ g(x,y) = 3.5 \end{cases}$. In the third graph below, put a

big dot at each solution. (Based on the graphs, how many solutions would you expect to get?)

