

**Test #1**  
**MA3160, Spring '06**

NAME: \_\_\_\_\_

Please **show work** or give reasoning for **every** answer. (No credit will be given for correct answers without an indication of how you arrived at your conclusion.)

If you obtain an answer or part of an answer with your **calculator**, please indicate what you punched into your calculator and what the output was.

If you use a **formula**, please write down the formula that you are using.

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1. Suppose the size of Barney's snow statue  $S$  depends on the amount of rain ( $x$ ) and the amount of direct sunlight ( $y$ ) hitting the statue during January, so the height (in meters) of his statue is given by a function

$$S = g(x, y)$$

where  $x$  is measured in inches and  $y$  is measured in hours.

- (a) Explain in words the meaning of the statement “ $g(2, 0) = 82$ . ”  
(Include units for all three numbers.)

- (b) If  $g_y(2, 0) = -6$ , what are the units of the “ $-6$ ”?

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2. The table contains function values for a linear function of two variables,  $l(x, y)$ .

- (a) Finish filling in the table.

- (b) Write a formula for  $l(x, y)$ .

			$x$	
		4	6	8
	0	1	2	
$y$	1	5		
	2			

3. Consider the point  $\mathbf{P} = (2, 3, 5)$ .

(a) How far is the point  $\mathbf{P}$  from the  $y$ - $z$  plane?

(b) How far is the point  $\mathbf{P}$  from the  $z$  axis?

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4. Suppose you know the following facts about a function  $h(x, y)$ :

$$h(10, 20) = 2, \quad h_x(10, 20) = 3, \quad h_y(10, 20) = 4.$$

(a) What is  $\nabla h(10, 20)$ ?

(the gradient of  $h$  at  $(10, 20)$ )

(b) Estimate  $h(9.5, 18)$ .

(Show how you get to your answer.)

(c) Compute  $h_{\vec{v}}(10, 20)$ , the directional derivative of  $h$  in the direction of the vector  $\vec{v} = 5\vec{i} + \vec{j}$ .

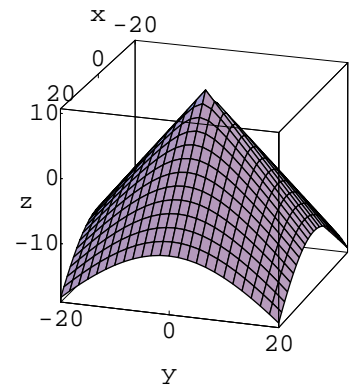
(Show how you get to your answer.)

(d) If  $h$  is measured in dollars and  $x$  and  $y$  are both measured in meters, what are the units for  $h_{\vec{v}}$  (the directional derivative of  $h$ )?

5. The following questions refer to the function

$$f(x, y) = 10 - \sqrt{x^2 + y^2}.$$

A graph of  $z = f(x, y)$  is shown at right.



(a) Sketch a cross-section of  $f$  with  $y$  fixed. Label the axes (“ $x$ ” and/or “ $y$ ” and/or “ $z$ ”).

(b) Find an equation for the cross-section of  $f$  with  $y = 8$ .

(c) Sketch a set of level curves for  $f$ . Show the correct shape and spacing. (You do not need to label the levels.)

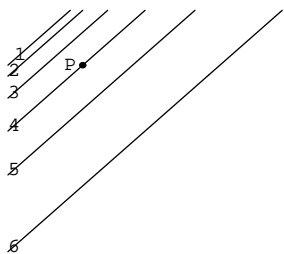
(d) Find a vector which is perpendicular to the level curve of  $f$  at the point  $(3, 4)$ . (Show how to get to your answer.)

(e) Find a vector which is perpendicular to the graph of  $f$  at the point  $(3, 4, 5)$ . (Show how to get to your answer.)

6. Find an equation for the intersection of the surface  $z = 10 - \sqrt{x^2 + y^2}$  with the  $x$ - $y$  plane. Sketch and/or describe the intersection graphically.

7. The following shows a set of level curves for a function  $g(x, y)$ . Determine the sign (positive, negative, or zero) of each of the following partial derivatives at the point  $P$ , and justify your answer. (Assume the  $x$ - and  $y$ - axes are in the usual orientation.)

(a)  $g_x$



(b)  $g_y$

(c)  $g_{xx}$

contours for  $g(x, y)$

(d)  $g_{xy}$

8. Given  $z = f(x, y)$ ,  $x = u + v$ ,  $y = uv$ , calculate  $z_u(3, 2)$  using the data shown:

$$f(3, 2) = 2$$

$$f(5, 6) = 4$$

$$f_x(3, 2) = 1$$

$$f_x(2, 4) = 7$$

$$f_x(5, 6) = 3$$

$$f_y(3, 2) = 5$$

$$f_y(2, 4) = 9$$

$$f_y(5, 6) = 6$$