NAME: ______________________

TEST #2
MA3160, Fall '05

Please **show work** or give reasoning for **every** answer.

If you obtain an answer or part of an answer with your **calculator**, please indicate what you punched into your calculator and what the output was.

If you use a memorized or programmed **formula**, please write down the formula that you are using.

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1. Which of the following three points are critical points of the function $f(x, y) = xe^x(y^2 - 4)$? (The *Mathematica* output at right may be helpful.)

   (a) Is $(0, 0)$ a critical point of f?

   (b) Is $(-1, 0)$ a critical point of f?

   (c) Is $(0, 2)$ a critical point of f?

```
f[x_, y_] = x*E^x*(y^2-4)
D[f[x,y],x]  \\Factor
D[f[x,y],y]  \\Factor
```

Out[1]= $e^x x(-4 + y^2)$
Out[2]= $e^x(1 + x)(-2 + y)(2 + y)$
Out[3]= $2e^x xy$

```
D[D[f[x,y],x],x]  \\Factor
D[D[f[x,y],x],y]  \\Factor
D[D[f[x,y],y],x]  \\Factor
D[D[f[x,y],y],y]  \\Factor
```

Out[4]= $e^x(2 + x)(-2 + y)(2 + y)$
Out[5]= $2e^x(1 + x)y$
Out[6]= $2e^x(1 + x)y$
Out[7]= $2e^x x$

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2. If you know that the partial derivatives of g satisfy

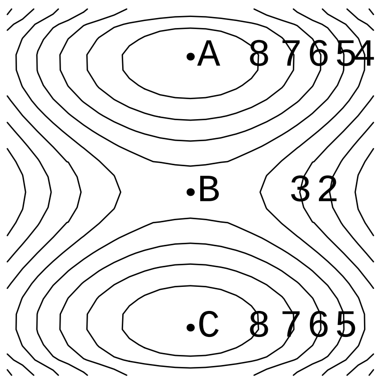
$$g_x(1, 3) = 0, \quad g_y(1, 3) = 0$$

and

$$g_{xx}(1, 3) > 0, \quad g_{yy}(1, 3) > 0, \quad g_{xy}(1, 3) = 0,$$

what can you conclude about the behavior of the function g near the point $(1, 3)$?

3. The graph below shows contours of a function $f(x, y)$ and three critical points of f. For each of these points, identify the type of critical point: local minimum, local maximum, saddle point, or none of these. What features of the contour plot are relevant here?



A is __________ because

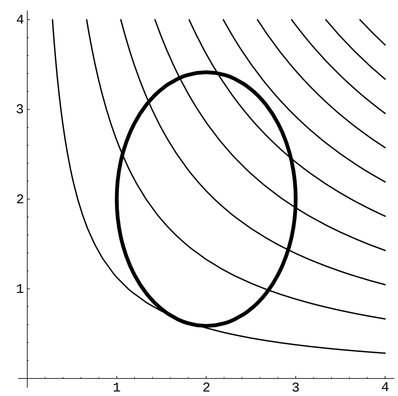
B is __________ because

C is __________ because

4. Suppose we want to maximize the function $f(x, y) = x + y$ subject to the constraint that $2x^2 + y^2 = 2$.

(a)

The bold curve shown at right is the graph of the constraint ($2x^2 + y^2 = 2$) and the other lines are level curves for the function $f(x, y) = x + y$. Clearly **label the point(s)** on the constraint curve at which f takes its maximum value. What is the relationship between the level curves and the constraint at this point?



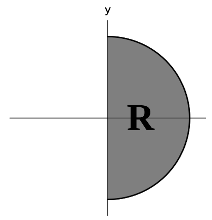
(b) Show how to use Lagrange multipliers to find exact values for the coordinates of the point you identified above.

5. Values of a function $h(x, y)$ are tabulated below. Using a Riemann sum with four subdivisions, compute an upper bound for the volume under the graph of $h(x, y)$ and above the rectangle R with $0 \le x \le 2$, $0 \le y \le 4$.

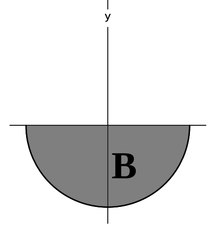
			x	
		0	1	2
	0	2	2	2
y	2	2	4	6
	4	2	6	10

6. Decide whether the following integrals are positive, negative, or zero. The regions R and B are the right and bottom halves of a circular disk centered at the origin in the x-y plane (as shown). Give a brief reason for your answer.

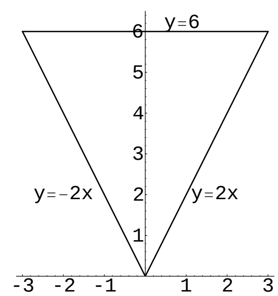
(a) $\int_R (y^3 + y^5)\, dA$



(b) $\int_B (y^3 + y^5)\, dA$



7. Set up an iterated integral for $\int_T f(x, y)\, dA$, where $f(x, y) = x^2 + y^2$ and T is the triangular region shown.



8. Show how to evaluate the following integral by hand:

$$\int_0^4 \int_0^2 (1 + xy)\, dx\, dy.$$

(Include enough details to show which integration should be performed first and which variable is treated as a constant in each integration.)

DO ANY **TWO** OF THE REMAINING THREE PROBLEMS

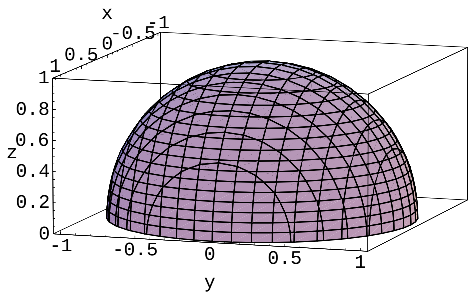
They are triple integrals of the same function over the same 3-D domain, using different coordinates.

9. Use RECTANGULAR (Cartesian) coordinates to set up the iterated triple integral for

$$\int_W f dV,$$

where $f(x, y, z) = \sin(x^2 + y^2 + z^2)$

W is the top half of a sphere of radius 1 described by $x^2 + y^2 + z^2 \leq 1, \quad 0 \leq z$.



(a) Label the top and bottom surfaces with equations.
Sketch and label a slice
(Which variable is held constant?)
(What are the equations for the boundaries of your slice?).

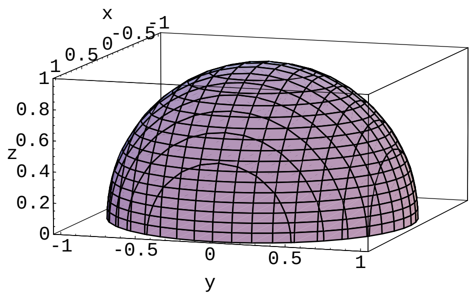
(b) Write the iterated integral for $\int_W f\, dV$ in Cartesian coordinates.

10. Use CYLINDRICAL coordinates to set up the iterated integral for

$$\int_W f\, dV,$$

where $f(x, y, z) = \sin(x^2 + y^2 + z^2)$

W is the top half of a sphere of radius 1 described by $x^2 + y^2 + z^2 \le 1, \quad 0 \le z$.



(a) Label the top and bottom surfaces with equations involving only the variables r, θ, and z. Write the function to be integrated in terms of r, θ, and z. What is dV?

$f =$ ________________ $dV =$ ________________

(b) Graph a slice here or make a sketch which shows which integration will be performed first.

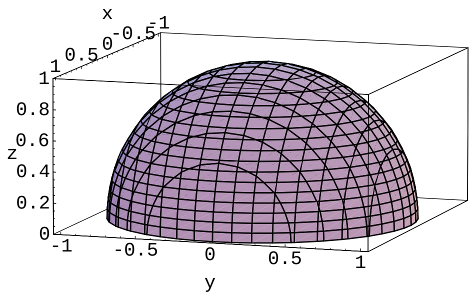
(c) Write the iterated integral for $\int_W f\, dV$ in cylindrical coordinates.

11. Use SPHERICAL coordinates to set up the iterated integral for

$$\int_W f\, dV,$$

where $f(x, y, z) = \sin(x^2 + y^2 + z^2)$

W is the top half of a sphere of radius 1 described by $x^2 + y^2 + z^2 \leq 1, \quad 0 \leq z$.



(a) Label the top and bottom surfaces with equations involving only the variables ρ, θ, and ϕ. Write the function to be integrated in terms of ρ, θ, and ϕ. What is dV?

$f =$________________ $dV =$________________

(b) Graph a slice here or make a sketch which shows which integration will be performed first.

(c) Write the iterated integral for $\int_W f\, dV$ in spherical coordinates.