Test #2
MA3160, Spring '06

NAME: ____________________

Please **show work** or give reasoning for **every** answer. (No credit will be given for correct answers without an indication of how you arrived at your conclusion.)

If you obtain an answer or part of an answer with your **calculator**, please indicate what you punched into your calculator and what the output was.

If you use a **formula**, please write down the formula that you are using.

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1. Find all critical points of the function $f(x,y) = x(y^2 - 4y)e^{-x}$.
   (Note: You do NOT need to classify the critical points as maxima or minima.)
   You may use the Mathematica code/output shown.

```
f[x_, y_] = x*(y^2-4*y)*E^(-x);
D[f[x,y],x]
D[f[x,y],y]
```

Out[2]= $-e^{-x}(-1+x)(-4+y)y$
Out[3]= $2e^{-x}x(-2+y)$

```
D[D[f[x,y],x],x]
D[D[f[x,y],x],y]
D[D[f[x,y],y],y]
```

Out[4]= $e^{-x}(-2+x)(-4+y)y$
Out[5]= $-2e^{-x}(-1+x)(-2+y)$
Out[6]= $2e^{-x}x$

2. Consider a function $g(x,y)$ which has the following derivatives at the point $(1,-1)$:

$$g_x(1,-1) = 0, \qquad g_y(1,-1) = 0$$

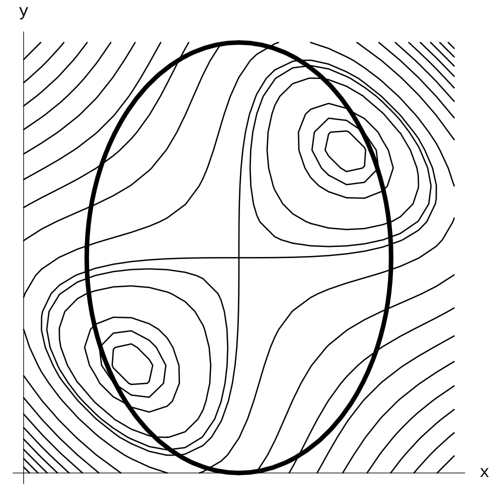
$$g_{xx}(1,-1) = 2, \qquad g_{yy}(1,-1) = 3 \qquad g_{xy}(1,-1) = 4$$

   What can you say about the shape of the graph of $z = g(x,y)$ near the point $(1,-1)$?

3. The figures below show the graph of $g(x, y) = 5$ (in **bold**), along with the level curves of a function $f(x, y)$.

(a) Suppose we solve the system of equations:

$$\begin{aligned} g(x,y) &= 5 \\ f_x(x,y) &= \lambda g_x(x,y) \\ f_y(x,y) &= \lambda g_y(x,y) \end{aligned}$$

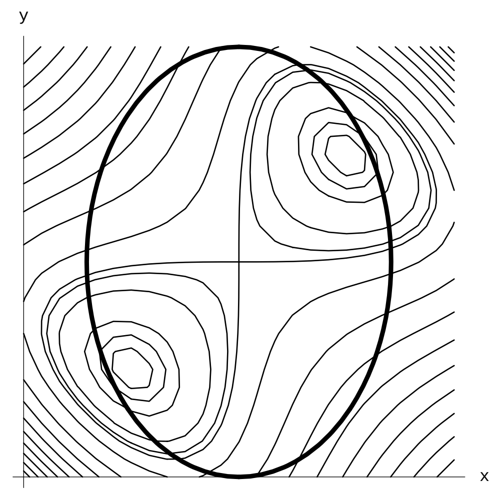


i. Put a big dot (or dots) on the graph showing the approximate location of our solution(s) $\longrightarrow$

ii. Is the second derivative test (the discriminant "D") valid at the point(s) we found?
If so, what additional information could we get from it?
If not, why not?

(b) Suppose we solve the system of equations:

$$\begin{aligned} f_x(x,y) &= 0 \\ f_y(x,y) &= 0 \end{aligned}$$



i. Put a big dot (or dots) on the graph showing the approximate location of our solution(s) $\longrightarrow$

ii. Is the second derivative test (the discriminant "D") valid at the point(s) we found?
If so, what additional information could we get from it?
If not, why not?

4. Yoopie was doing his section 16.3 homework and had written down the following integral:

$$\int_0^{\sqrt{2}} \int_x^{x^2} \int_{-\sqrt{y^2+z^2}}^{\sqrt{y^2+z^2}} x^2 e^y \, dx \; dy \; dz.$$

How do you know that Yoopie set up the integral WRONG, without even knowing what the question was? (Be specific about what is wrong.)

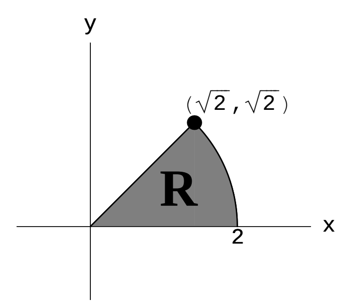
5. For the integral $\int_{-1}^{1} \int_{-x}^{1} \cos(y)\, dy\, dx$, sketch and/or describe the region over which the integration is being performed.

6. Consider the following integral (written in polar coordinates):

$$\int_{\pi/2}^{\pi} \int_{0}^{1} r^3 \sin(\theta)\, dr\, d\theta.$$

Sketch and/or describe the region in the x-y plane over which the integration is being performed.

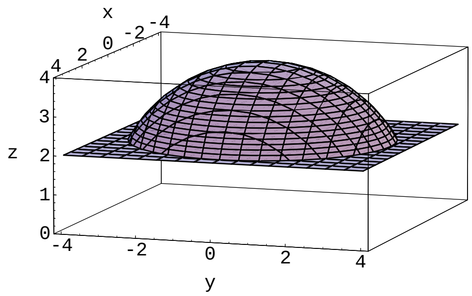
7. Suppose we want to evaluate the integral of $f(x, y)$ over the region R, where $f(x, y) = xy$ and R is the region shown at right (shaped like a wedge of pie).



(a) Set up the iterated integral in polar coordinates.

(b) Set up the iterated integral in rectangular coordinates.

8. A three-dimensional region W is the "cap" of a spherical ball of radius 4, as shown. The top surface bounding W is a sphere of radius 4 centered at the origin and the bottom surface is a horizontal plane 2 units about the x-y plane.



(a) Write one equation for the top surface and another for the bottom surface in terms of CYLINDRICAL coordinates.

TOP: ______________________

BOTTOM: ______________________

(b) Write one equation for the top surface and another for the bottom surface in terms of SPHERICAL coordinates.

TOP: ______________________

BOTTOM: ______________________

(c) SET UP the iterated integral for $\int_W \frac{xz}{y}\, dV$.