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Test #2
MA3160, Spring '09

NAME: _____

Please **show work** or give reasoning for **every** answer. (No credit will be given for correct answers without an indication of how you arrived at your conclusion.)

If you obtain an answer or part of an answer with your **calculator**, please indicate what you punched into your calculator and what the output was.

If you use a **formula**, please write down the formula that you are using.

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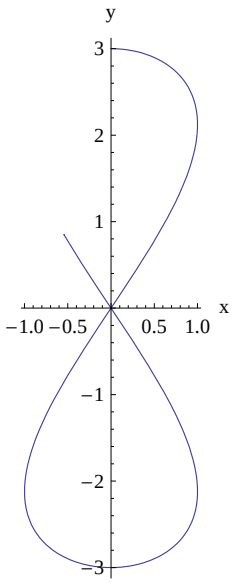
1. A skater's position at time t is given by $\begin{cases} x(t) &= \sin(2t) \\ y(t) &= 3\cos(t) \end{cases}$ $0 \le t \le 5,$

   where t is in seconds and x and y are in meters,
   and the inputs to sin and cos are in RADIANS.

   A graph of her path is shown at right.



   (a) Where is the skater at time $t = .5$?

   (b) Find the skater's <u>velocity</u> as a function of time, $v(t)$.

   (c) What is the skater's <u>speed</u> at time $t = .5$?

   (d) Find a vector which points tangent to the path at the time $t = 0.5$.

   (e) Set up (but do NOT evaluate) an integral which would give the <u>total distance traveled</u> by the skater from $t = 0$ until $t = 5$.

2. Give an equation for a vector field $\vec{F}(x, y)$ having BOTH of the following properties:

- The vectors are parallel to the x-axis ("horizontal").
- The vectors increase in magnitude as you move away from the y axis.

$\vec{F}(x, y) =$

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3. Give an equation for a vector field $\vec{V}(x, y)$ having BOTH of the following properties:

- All the vectors point toward the origin.
- Each vector has unit length.

$\vec{V}(x, y) =$

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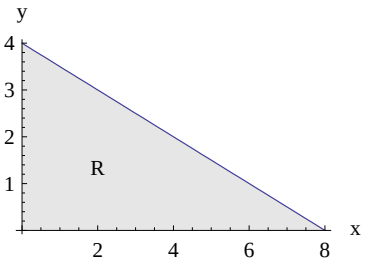
4. (a) In the figure shown at right, label all three boundaries of the triangle with equations.
   (b) Using Cartesian coordinates, set up the iterated integral $\int_R f(x, y)dA$ of the function $f(x, y) = e^{xy}$ over the triangular region shown.



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5. Consider the integral in polar coordinates: $\int_R f \, dV = \int_0^{\pi} \int_1^2 r \sin(r^2) \, dr \, d\theta.$

   (a) Write the function f in rectangular coordinates.

   $f(x, y) =$_____

   (b) Sketch R, the region over which f is integrated. (If your picture isn't clear, describe it briefly in words.)

6. Set up a Riemann sum which will give an OVERestimate for the integral $\int_R f(x,y)\ dA$ when R is the rectangular region with $0 \le x \le 2$ and $0 \le y \le 4$ and values of f are in the table below.

$y\backslash x$	0	1	2
0	9	5	4
2	8	4	3
4	6	3	1

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7. Kooky Curt was doing his section 16.3 homework and had written down the following integral:

$$\int_0^{\sqrt{2}} \int_1^{z^2} \int_{5+x^2}^{5-x^2} \sin(y) dx\ dy\ dz.$$

How do you know that Curt set up the integral WRONG, without even knowing what the question was? (Be as explicit as you can.)

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8. (a) Using Cartesian coordinates,
give an equation for the x-y plane.

(b) Using CYLINDRICAL coordinates,
give an equation for the x-y plane.

(c) Using SPHERICAL coordinates,
give an equation for the x-y plane.

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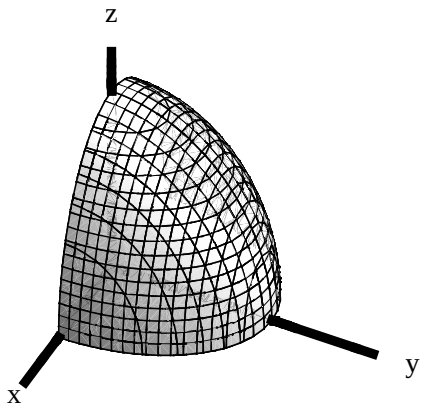
9. (a) Using Cartesian coordinates, give an equation for
a sphere of radius 5 centered at the origin.

(b) Using CYLINDRICAL coordinates, give an equation for
a sphere of radius 5 centered at the origin.

(c) Using SPHERICAL coordinates, give an equation for
a sphere of radius 5 centered at the origin.

10. A solid region W is one quarter of a sphere of radius 5 centered at the origin, with $y \geq 0$ and $z \geq 0$, as shown to the right.



You will set up the integral of the function

$$f(x, y, z) = \frac{z}{1 + x^2 + y^2}$$

over this region.

(a) Set up the integral $\displaystyle\int_W f(x, y, z)\, dV$ in Cartesian coordinates.

(b) Set up the integral $\displaystyle\int_W f(x, y, z)\, dV$ in cylindrical OR spherical coordinates. (Circle your choice.)