NAME: ____________________

TEST #3
MA3160, Fall '05

Please **show work** or give reasoning for **every** answer. (No credit will be given for correct answers without an indication of how you arrived at your conclusion.)

If you obtain an answer or part of an answer with your **calculator**, please indicate what you punched into your calculator and what the output was.

If you use a memorized or programmed **formula**, please write down the formula that you are using.

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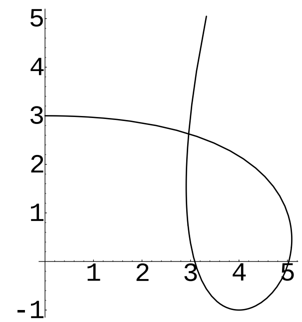
1. A bug is moving in the x-y plane according to the parametric equations



$$\begin{aligned} x &= t^3 - 6t^2 + 10t \\ y &= t^3 - 3t^2 + 3 \qquad 0 \le t \le 4. \end{aligned}$$

(x and y are measured in inches (from the origin) and time t is in seconds.)

(a) What are the coordinates of the bug at time $t = 1$?

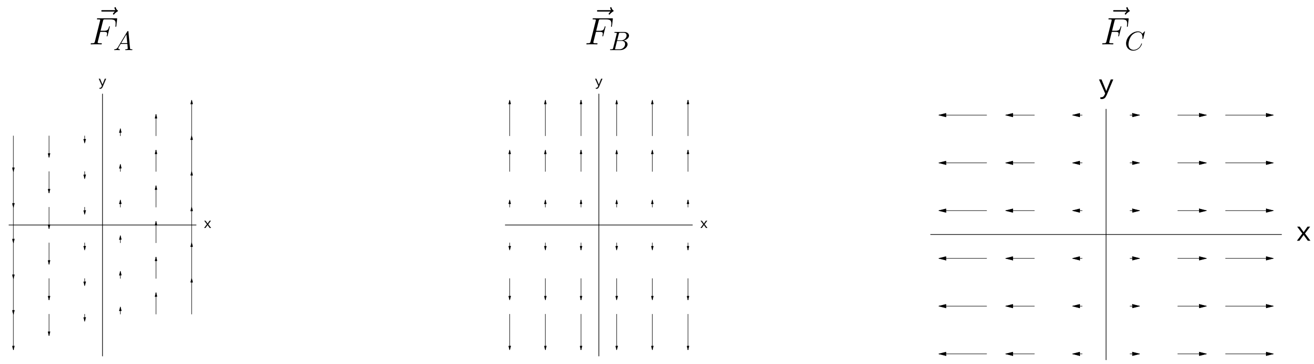
(b) Find the velocity of the bug (as a function of t).

(c) Find the speed of the bug at time $t = 1$.

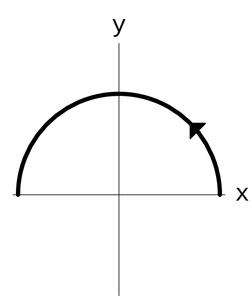
(d) Find a vector which is tangent to the curve at time $t = 1$.

(e) SET UP (but do not evaluate) an integral which would give you the total distance the bug travels (from $t = 0$ until $t = 4$).

2. Three vector fields are graphed here:



(a) Of these three, which could be the graph of $\vec{F}(x, y) = x\vec{\jmath}$?
(What feature of the graph corresponds to which part of the formula for $\vec{F}(x, y)$?)



(b) Suppose you computed the line integral of each of these vector fields along the semicircular path $\mathcal{C}$, shown at right (starting at $(1, 0)$ and ending at $(-1, 0)$). Would the line integral be positive or negative or zero?

A:

B:

C:

(c) If the curve $\mathcal{C}$ is parameterized by the equations

$$\begin{aligned} x(t) &= \cos(t) \\ y(t) &= \sin(t), \qquad 0 \leq t \leq \pi, \end{aligned}$$

SET UP the line integral of $\vec{F}(x, y) = x\vec{\jmath}$ along the path C.

3. The vector field $\vec{F}(x, y, z) = 2x\vec{\imath} + 6z^2\vec{k}$ is a gradient field.

   (a) Find a potential function $g(x, y, z)$ such that $\vec{F} = \nabla g$.

   (b) Find the value of the line integral $\int_C \vec{F} \cdot d\vec{r}$ where C is the line from $(1, 1, 1)$ to $(0, 2, 2)$. Use the fundamental theorem for line integrals.

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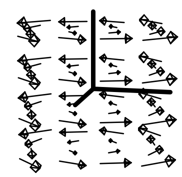
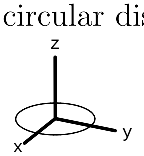
4. The vector field $\vec{V}(x, y, z) = -y\vec{\imath} + x\vec{\jmath} + 0\vec{k}$ swirls around the z axis as shown.

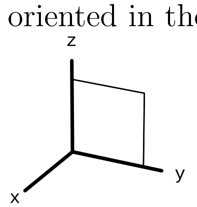


   Find the flux of $\vec{V}$ through each of the two surfaces below.
   (Give reasoning or show computation for each answer.)

   (a) a circular disc of radius three in the x-y plane, centered at the origin and oriented upwards.



   (b) a square of side length 1 in the y-z plane, with one corner at the origin and the other at $(0, 1, 1)$, oriented in the direction of the positive x axis.

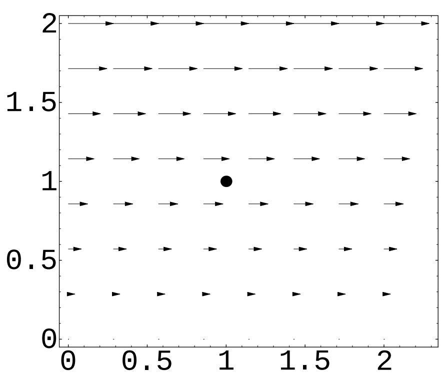


5. The vector field $\vec{V}(x,y) = x\sin(y)\vec{\imath} + y\sin(x)\vec{\jmath}$ is NOT a gradient field. Explain how you can tell.

6. Compute $\operatorname{div}\vec{F}$ using partial derivatives for $\vec{F}(x,y,z) = (x^2+y^2)\vec{\imath} + 2\vec{\jmath} + \sin y\vec{k}$.

7. Consider the vector field $\vec{v}$ shown below. Is $\operatorname{div}\vec{v}$ positive, negative, or zero at the point $(1,1)$? What property of the graph tells you this?

(Note: vectors get larger as you move upwards.)

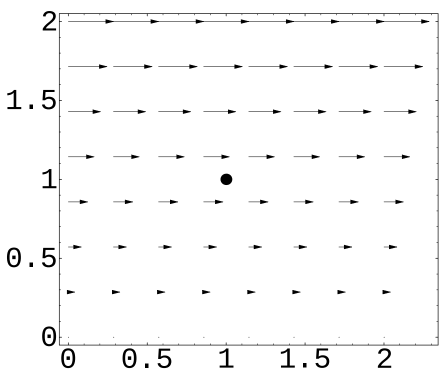


8. Compute $\operatorname{curl}\vec{F}$ using partial derivatives for $\vec{F}(x,y,z) = (x^2+y^2)\vec{\imath} + 2\vec{\jmath} + \sin y\vec{k}$.

9. Consider the vector field $\vec{v}$ shown below. (Assume that $\vec{v}$ has no z-component and is independent of z.)

At the point $(1,1,0)$, does $\operatorname{curl}\vec{v}$ point **into the page** (toward your desk, in the direction of $-\vec{k}$), **out of the page** (towards your face, in the direction of $\vec{k}$), or does it have **magnitude zero**?
What property of the graph tells you this?

(Note: vectors get larger as you move upwards.)



10. Use the divergence theorem to compute the flux of $\vec{F} = yz\vec{\imath} + 3y\vec{\jmath} + \sin(x^3y)\vec{k}$ out of the unit sphere (the sphere of radius 1, centered at the origin).