NAME: ____________________

TEST #3
MA3160, Spring '06

Please **show work** or give reasoning for **every** answer. (No credit will be given for correct answers without an indication of how you arrived at your conclusion.)

If you obtain an answer or part of an answer with your **calculator**, please indicate what you punched into your calculator and what the output was.

If you use a memorized or programmed **formula**, please write down the formula that you are using.

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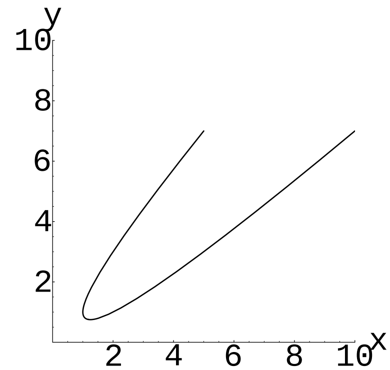
1. Suppose the position of a car is given by

$$\begin{aligned} x(t) &= t^2 - 4t + 5 \\ y(t) &= t^2 - 5t + 7 \qquad 0 \le t \le 5 \end{aligned}$$



where x and y are measured in meters and t is measured in seconds. (The car's path is shown at right.)

(a) Where is the car at time $t = 1$?

(b) What is the velocity of the car at time t?

(c) What is the speed of the car at time $t = 1$?

(d) Find a vector pointing in the direction tangent to the curve at the point you found in part (1a).

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2. Give parametric equations for the line segment from $(9, 8, 7)$ to $(4, 5, 6)$. (Include limits for t.)

3. Give a formula for a two-dimensional vector field with both of the following properties:

- All vectors are parallel to the y-axis, and
- the length of the vectors increases as you move away from the y-axis.

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4. Is $\vec{G}(x,y) = 2xy\vec{\imath} + (x^2 + y^3)\vec{\jmath}$ a gradient field? How can you tell?

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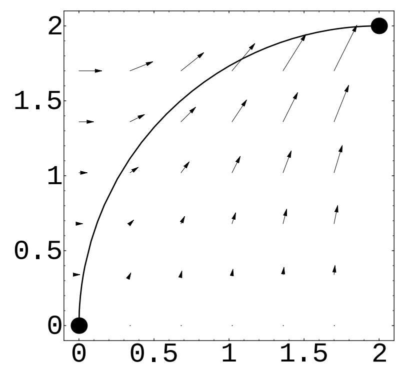
5. Suppose we want to compute the line integral, $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x,y) = (y^2)\vec{\imath} + (2xy)\vec{\jmath}$ and C is the circular arc from $(0,0)$ to $(2,2)$ shown below.

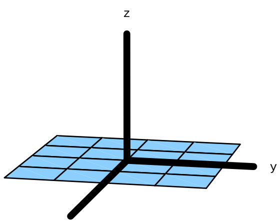
(a) Based on the graph alone, do you expect the line integral, $\int_C \vec{F} \cdot d\vec{r}$ to be positive or negative? Why?

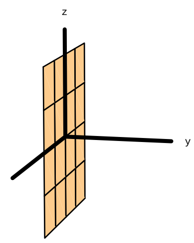


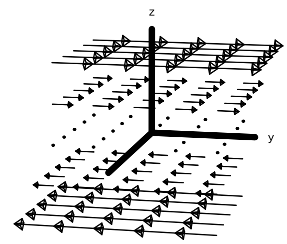
(b) $\vec{F} = (y^2)\vec{\imath} + (2xy)\vec{\jmath}$ is a gradient field.
Identify its potential function (i.e., find a scalar-valued $g(x,y)$ such that $\vec{F} = \nabla g$).

(c) Use your answer to part (5b) and the fundamental theorem for line integrals to evaluate $\int_C \vec{F} \cdot d\vec{r}$.

6. Freddy Ornot wanted to compute the flux of the vector field $\vec{F}(x, y, z) = z\vec{j}$ through each of the two surfaces shown.
Each surface is a square centered at the origin (in the x-y plane, oriented in the positive z direction, or in the x-z plane, oriented in the positive y direction). The vector field is plotted to the right.







After looking at these graphs, he declared that "Both fluxes will be zero, but <u>for different reasons</u>." Explain why, based on the graphs.

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7. Find the flux of the vector field $\vec{F}(x, y, z) = z\vec{j}$ through the surface of the square in the x-z plane with $0 \leq x \leq 2$ and $0 \leq z \leq 2$, oriented in the positive y direction.

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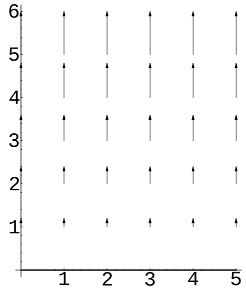
8. Consider the vector field $\vec{v}$ graphed below. Is the divergence of $\vec{v}$ positive, negative, or zero at the point shown? Justify your answer without referring to formulas (make it clear that you understand the geometric definition of the divergence).



(Note: vectors are vertical and increase in length as you move upward.)

9. For the vector field $\vec{v}(x, y, z) = (x^2 - y^2)\vec{\imath} + 2xy\vec{\jmath} + \sin(z)\vec{k}$,

(a) Find $\text{div}\vec{v}$.

(b) Find $\text{curl}\vec{v}$.

(c) Suppose $\vec{v}(x, y, z)$ represents the velocity of a fluid at the point (x, y, z). If you placed a small object in the fluid at the point $(0, -1, 0)$, would the fluid cause it to spin? (How do you know?)

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10. Suppose that $\vec{F} = F_1(x, y)\vec{\imath} + F_2(x, y)\vec{\jmath}$ and its derivatives are continuous everywhere. Write two statements which are equivalent to

"*$\vec{F}$ is a path-independent vector field.*"