## Test #2 NAME: MA2160, Spring '08, T.Olson

Please **show work** or give reasoning for **every** answer. (No credit will be given for correct answers without an indication of how you arrived at your conclusion.)

If you obtain an answer or part of an answer with your **calculator**, please indicate what you punched into your calculator and what the output was.

If you use a **formula**, please write down the formula that you are using.

1. For each of the following, indicate if it is a proper integral or an improper integral (circle one).

 $\int_{1}^{3} \frac{2}{\sqrt{3-x}} dx \text{ proper or improper}$  $\int_{0}^{3} \sin(x) dx \text{ proper or improper}$  $\int_{2}^{50} \frac{x}{4+x^{2}} dx \text{ proper or improper}$  $\int_{2}^{\infty} \frac{x}{4+x^{2}} dx \text{ proper or improper}$  $\int_{-1}^{\infty} \frac{x}{4+x^{2}} dx \text{ proper or improper}$  $\int_{-1}^{1} \frac{1}{x^{2}} dx \text{ proper or improper}$  $\int_{0}^{2} \frac{2-x}{\sqrt{3-x}} dx \text{ proper or improper}$ 

2. Show how to evaluate the following improper integral by **rewriting it as a limit of a proper integral** (or integrals) and using the Fundamental Theorem of Calculus.

$$\int_0^2 \frac{1}{(x-2)^2} \, dx$$

Does this integral converge?

3. If f(x) is measured in people per square mile and x is measured in miles, what are the units of  $\int_0^{10} f(x) dx$ ?

4. <u>Set up</u> an integral which will give the total arc length of the graph of  $f(x) = 1 + x^2$  between (0, 1) and (2, 5).

5. For your homework, you computed the total force of water on a dam using an integral that came from slicing. Which direction do your slices go (horizontal or vertical), and **why do you slice it this way**?

6. Write a definite integral representing the area of the region, using the strip shown (i.e., slicing parallel to the x axis).



- 7. A rod of length 3 meters with density  $\delta(x) = 100 + x^2$  grams/meter is positioned along the positive x-axis, with its left end at the origin and x measured in meters.
  - (a) Set up an integral which gives the total mass of the rod.

(b) Set up an integral which gives the center of mass of the rod.

- 8. Suppose we generate a 3-D solid by rotating the shaded triangular region about the y axis.
  - (a) Sketch the 3-D region.(Describe briefly in words if your sketch isn't clear.)



(b) If we wanted to compute the volume by slicing, which way do the slices go? Is the thickness of the slice "dx" or "dy"?(Draw a picture if your answers are not clear.)

## Do <u>either</u> problem 9 $\underline{OR}$ problem 10

9. A three-dimensional solid sits above the semicircle of radius  $\sqrt{10}$  shown in the *x-y* plane below. Crosssections of the solid parallel to the *y*-axis (at fixed *x*-value) are equilateral triangles. Set up an integral that gives the volume of the solid.

(Note: the equation for the circular boundary is  $x^2 + y^2 = 10$ .) (Note: the area of an equilateral triangle of side-length b is  $\frac{\sqrt{3}}{4}b^2$ .)



10. The conical tank shown is full of water, which has a weight density of  $62.4 \text{ lb.}/(\text{ft})^3$ . The cone is 10 feet high and has a radius of 10 feet at the top.

How much work is done to pump all the water up over the lip of the tank?

