NAME: _____

Test #3
MA2160, Spring '08, T.Olson

Please **show work** or give reasoning for **every** answer. (No credit will be given for correct answers without an indication of how you arrived at your conclusion.)

If you obtain an answer or part of an answer with your **calculator**, please indicate what you punched into your calculator and what the output was.

If you use a **formula**, please write down the formula that you are using.

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1. For each of the following, decide if it is a geometric series.

	geometric?	
$1 + 1/2 + 1/3 + 1/4 + 1/5 \ldots$	yes	no
$1 - x + x^2 - x^3 + x^4 \ldots$	yes	no
$3 + 3z^2 + 3z^4 + 3z^6 + 3z^8 \ldots$	yes	no
$2 + 5 + 8 + 11 + 14 + \ldots + 32$	yes	no
$b + b^2 + b^3 + b^4 + b^5 \ldots b^{10}$	yes	no
$5 + 5/x + 5/x^2 + 5/x^3 + 5/x^4 \ldots$	yes	no

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2. Compute the sum of each of the following:

   (a) $3 + 3/4 + 3/16 + 3/64 + \ldots$

   (b) $2/3 + 4/9 + 8/27 + \ldots + (2/3)^{20}$

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3. Give an example of a geometric series which does NOT converge.

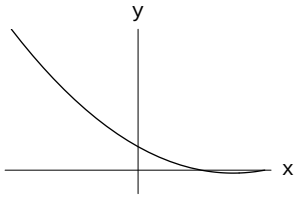
4. The following is the Taylor polynomial of degree 3 for a function $f(x)$:

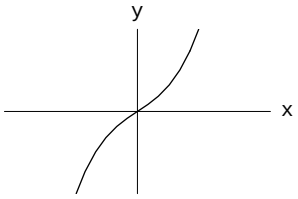
$$P_3(x) = 1 - 3x + 2x^2 + x^3.$$

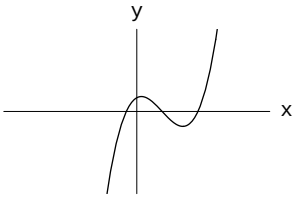
(a) Find the equation of the line tangent to $f(x)$ at $x = 0$.

(b) What is $f'''(0)$?

(c) Which of the following could be the graph of the function $f(x)$ near $x = 0$?







(d) What additional information would we need in order to get a bound on the error when approximating $f(x)$ by $P_3(x)$ on the interval $0 \leq x \leq 5$?

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5. What degree Taylor polynomial would we need in order to estimate $\cos(x)$ on the interval $[-10, 10]$ with an error of less than 0.1? (Show how to use the error bound to compute your answer.)

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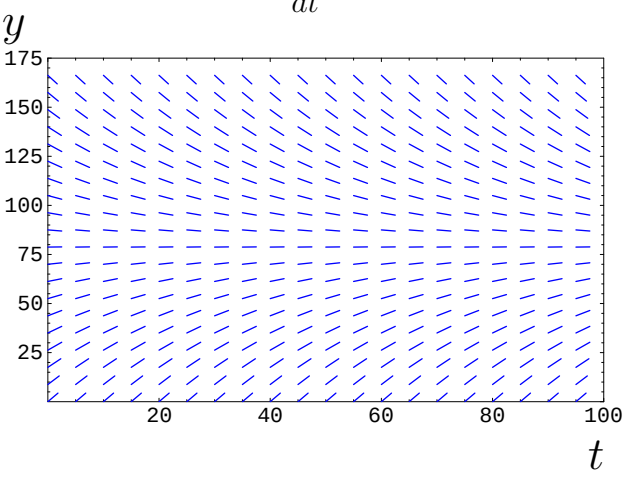
6. What is the interval of convergence for the series $\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \ldots$?

7. A cup of coffee is left outside on a summer morning. Its temperature after t minutes is given by some function $y(t)$ (y is measured in degrees Fahrenheit). Because the rate of change of temperature of the coffee is proportional to difference between air temperature and coffee temperature, $y(t)$ satisfies the differential equation $\frac{dy}{dt} = .03(80 - y)$. The slope field for this equation is shown here:



(a) Silly Sadie thinks this is the slope field for the equation $\frac{dy}{dt} = .03(y - 80)$. Explain how you can tell it is NOT.

(b) In the graph of the slope field above, sketch the solution curve that goes through the point $(t, y) = (0, 150)$.

(c) What is the temperature outside?

(d) If the coffee is $150^o F$ at time $t = 0$, use two steps of Euler's method to compute the approximate values for $y(2)$ and $y(4)$. (Do two steps of Euler's method with step size $\Delta t = 2$.)
Record the details in the table.

t	y	slope	Δy
0	150		
2			
4		XXX	XXX

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8. Determine which of the following differential equations can be solved by separation of variables. Do not solve.

(a) $y' = x$

(b) $y' = y$

(c) $y' = xy$

(d) $y' = x + y$

(e) $y' = \ln(xy)$

9. Consider the differential equation

$$x\frac{dy}{dx} = y + x^2.$$

(a) Show how to check (by hand) that $y = x^2 + Cx$ is a solution for any constant C. (Do not re-solve the equation using separation of variables or your calculator).

(b) Find a solution to the initial-value problem

$$x\frac{dy}{dx} = y + x^2, \quad y(1) = 5.$$

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10. Show how to solve the following equation using separation of variables.

$$\frac{dz}{dx} = z^2 + xz^2$$