Test #1
MA2160, Spring '08, T.Olson

NAME: _____

Please **show work** or give reasoning for **every** answer. (No credit will be given for correct answers without an indication of how you arrived at your conclusion.)

If you obtain an answer or part of an answer with your **calculator**, please indicate what you punched into your calculator and what the output was.

If you use a **formula**, please write down the formula that you are using.

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1. These questions all involve the vector
$$\vec{v} = 2\vec{\imath} + 3\vec{\jmath}.$$

   (a) What is the length of $\vec{v}$?

   (b) Find a vector which points in the same direction as $\vec{v}$ but is twice as long.

   (c) Find a vector which is perpendicular to $\vec{v}$.

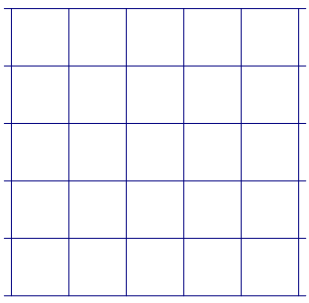
   (d) What is the dot product of $\vec{v} = 2\vec{\imath} + 3\vec{\jmath}$ with $\vec{u} = 2\vec{\imath} - 3\vec{\jmath}$?

   (e) What is the angle between $\vec{u}$ and $\vec{v}$?

   (f) What is the projection of $\vec{v}$ onto the line in the direction of $\vec{u}$?

   (g) On the grid, illustrate the fact that $\vec{v} + \vec{u} = 4\vec{\imath}$.
   (The positive x-direction is toward the right and the positive y-direction is toward the top of the page.)

   Label $\vec{v}$ and $\vec{u}$ and $\vec{v} + \vec{u}$ on your diagram.



2. Find a vector which is perpendicular to BOTH of $\vec{v} = 2\vec{\imath} + 2\vec{\jmath} + \vec{k}$ and $\vec{w} = 5\vec{\imath} - 9\vec{\jmath} + 4\vec{k}$.

3. Give an equation for the plane which goes through the point $(4, 5, 6)$ and is perpendicular to $\vec{v} = 2\vec{\imath} + 2\vec{\jmath} + \vec{k}$. (Your answer should be an equation involving x, y, and/or z.)

4. Suppose we know that the dot product of two vectors is a negative number: $\vec{v} \cdot \vec{w} < 0$.
What does this tell us about the geometric relationship between $\vec{v}$ and $\vec{w}$?

5. Fred and Ethel and Ginger were trying to find the vector perpendicular to a given plane and came up with three different answers.

$$\text{Fred: } \vec{n} = 4\vec{\imath} - 2\vec{\jmath} - 8\vec{k}$$
$$\text{Ethel: } \vec{n} = 4\vec{\imath} - 2\vec{\jmath} + 8\vec{k}$$
$$\text{Ginger: } \vec{n} = -2\vec{\imath} + \vec{\jmath} + 4\vec{k}$$

Which answer(s) is(are) most likely correct? (How did you come to your conclusion?)

6. Suppose we try to approximate the value of a definite integral

$$\int_1^9 f(x)\,dx.$$

We obtain the following approximations using the left Riemann sum, the right Riemann sum, and the midpoint rule, each with $n = 300$ subdivisions.

$$\begin{aligned} \text{Left Sum} &= 3.33851 \\ \text{Right Sum} &= 3.28518 \\ \text{Midpoint Sum} &= 3.31182 \end{aligned}$$

(a) What value would you get for the approximation of this integral using the TRAPEZOID rule with $n = 300$ subdivisions? Show how you computed your answer.

(b) Is the integrand $f(x)$ increasing or decreasing? (Assume f doesn't change from one to the other.) How can you tell?

(c) Is the integrand $f(x)$ concave up or concave down? (Assuming the concavity doesn't change.) How can you tell?

(d) Notice that LEFT(300) is accurate to one place beyond the decimal point. Approximately how many subdivisions would you need to have in order for LEFT(n) to be accurate to four places beyond the decimal point?

(e) Guess the exact value of the integral. (Make an educated guess to get full credit.) (If you guess the first 10 digits correctly, you'll win 5 extra credit points!).

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7. Show how to use a trig. substitution ($t = \sin w$ or $t = \tan w$) to rewrite $\int \sqrt{1 - t^2}\,dt$ in terms of w.

For the remaining problems, SHOW HOW to change the form of the given integral using a **substitution** or **integration by parts**.

8. $\int xe^{x^2}\,dx$

9. $\int xe^{x}\,dx$

10. $\int x\sqrt{1-x}\,dx$
(HINT: $w = 1 - x$)

11. $\int x\sqrt{1-x^2}\,dx$

12. $\int \frac{\ln x}{x}\,dx$

13. $\int \ln x\,dx$
(HINT: integration by parts.)

14. $\int (1-\sin^2 x)\cos x\,dx$