Sections 20.2 and 20.4:
The Divergence Theorem and Stokes' Theorem

The Divergence Theorem
If W is a solid region enclosed by S, then

$$\int\int\int_W \operatorname{div}\vec{F}\, dV = \int\int_S \vec{F}\cdot\vec{n}\, dA,$$

provided that $\vec{F}$ and its partial derivatives are continuous in and on S, and S is piecewise smooth and oriented outwards from W.

Suppose we are interested in computing the total flux of the vector field

$$\vec{F} = (2xy - 2x^2y)\vec{\imath} + (y^3 - y)\vec{\jmath} + xz\vec{k}$$

outwards through the surfaces of the unit cube bounded by the three coordinate planes and the three planes $x = 1$, $y = 1$, and $z = 1$.

1. The Divergence Theorem tells us that this flux is equal to the triple integral of what function over what 3-D region? (Give an explicit formula for the function and the limits of integration.)

2. Compute the triple integral.

3. Compute the total flux through all six surfaces of the cube.
   (Help: The only face with a non-zero value for $\vec{F}\cdot\vec{n}$ is the top.(Isn't your teacher nice?!))

Stokes' Theorem
If S is a surface bounded by C, then

$$\int\int_S \mathrm{curl}\vec{F} \cdot \vec{n}\, dA = \int_C \vec{F} \cdot d\vec{r},$$

provided that $\vec{F}$ and its partial derivatives are continuous on S and C, and C is piecewise smooth and oriented by the right-hand rule relative to the orientation of S.

Suppose we want to compute the circulation of

$$\vec{F} = (2x - z)\vec{\imath} + y^2\vec{\jmath} + (x + 4z)\vec{k}$$

around the circle of radius 10 in the x-z plane, centered at the origin, starting on the x axis and moving upwards.

1. Stokes' theorem says that this circulation is equal to the integral of what function over what 2-D surface?

2. Compute the 2-D surface integral.

3. Compute the circulation by parameterizing the curve and computing the line integral.