Tips on

Writing Proofs

T. Olson

Restate the hypotheses and introduce the notation from the statement of the theorem. Make it clear which symbols represent an arbitrary element from a set and which represent specific elements with specific properties. For example, if the theorem starts with "For any real number $a > 0 \ldots$", then your proof will start with something like "Let $a \in R$ with $a > 0$." If the theorem is supposed to hold "for all integers m and n", then start your proof with "Let m and n be integers."

Use the hypotheses. Make it clear exactly where each hypothesis is needed.

Define new variables in terms of variables already defined. If the new variable is obtained algebraically, give the formula. If the new symbol comes from a theorem (like the "Denseness of Q"), you must show that the conditions of this theorem are satisfied and tell the reader which theorem you are invoking. When defining a new variable, use the words "for some" or "for any" to make it clear whether you are talking about a specific number or any number from a certain set.

Be precise. Writing merely "by definition" or "Contradiction" isn't helpful unless you specify which definition is applied to which variable or which conclusion contradicts which other. Statements like "it is obvious that $\ldots$" are both unhelpful and insulting.

Prepare your reader for the type of proof that follows by indicating the type of proof (particularly if it is by induction or by contradiction). Also, if you need to prove an intermediate result, mention what you will be proving first. (For example, "First we will show that $r + \sqrt{2}$ is irrational for any rational number r.")

Justify every step in your proof. If there is anything more than simple algebra involved, cite a theorem or inequality or identity which explains the step. (If the theorem doesn't have a name, or you can't remember it, at least rephrase it so the reader knows what information has led you to that conclusion. For example, "Since $< x_n >$ is an increasing sequence that is bounded above, it has a limit.")

Tell the reader what you have shown. After every major piece of a proof, summarize your results so far with a statement like "Now we have shown that $\ldots$ (stuff) $\ldots$ whenever $\ldots$."