PH2100 Formula Sheet Knight

Concepts of Motion

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time interval spent traveling}}$$

$$\Delta\vec{r} = \vec{r}_{\text{f}} - \vec{r}_{\text{i}}$$

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t}$$

Kinematics in One Dimension

For Motion in a Straight Line:

$$v_s = \frac{ds}{dt} = \text{slope of position-time graph}$$

$$a_s = \frac{dv_s}{dt} = \text{slope of velocity-time graph}$$

$$s_{\text{f}} = s_{\text{i}} + \int_{t_{\text{i}}}^{t_{\text{f}}} v_s\,dt = s_{\text{i}} + \begin{cases}\text{area under velocity curve} \\ \text{from } t_{\text{i}} \text{ to } t_{\text{f}}\end{cases}$$

$$v_{\text{fs}} = v_{\text{is}} + \int_{t_{\text{i}}}^{t_{\text{f}}} a_s\,dt = v_{\text{is}} + \begin{cases}\text{area under acceleration curve} \\ \text{from } t_{\text{i}} \text{ to } t_{\text{f}}\end{cases}$$

Uniform Motion:

$$s_{\text{f}} = s_{\text{i}} + v_s\Delta t$$

Uniformly Accelerated Motion:

$$v_{\text{fs}} = v_{\text{is}} + a_s\Delta t$$

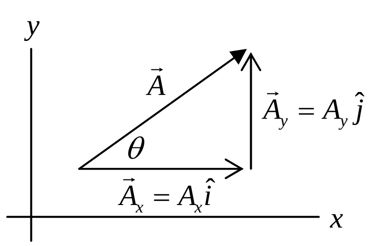
$$s_{\text{f}} = s_{\text{i}} + v_{\text{is}}\Delta t + \tfrac{1}{2}a_s\left(\Delta t\right)^2$$

$$v_{\text{fs}}{}^2 = v_{\text{is}}{}^2 + 2a_s\Delta s$$

Free fall: $a_y = -g = -9.80\text{ m/s}^2$

Motion on an Inclined Plane: $a_s = \pm g\sin\theta$

Vectors and Coordinate Systems



$$\vec{A} = \vec{A}_x + \vec{A}_y = A_x\hat{i} + A_y\hat{j}$$

In the figure above:

$$A_x = A\cos\theta \qquad A_y = A\sin\theta$$

$$A = \sqrt{A_x^2 + A_y^2} \qquad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Kinematics in Two Dimensions

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = a_x\hat{i} + a_y\hat{j}$$

Constant Acceleration:

$$x_{\text{f}} = x_{\text{i}} + v_{\text{ix}}\Delta t + \tfrac{1}{2}a_x\left(\Delta t\right)^2 \qquad y_{\text{f}} = y_{\text{i}} + v_{\text{iy}}\Delta t + \tfrac{1}{2}a_y\left(\Delta t\right)^2$$

$$v_{\text{f}x} = v_{\text{ix}} + a_x\Delta t \qquad v_{\text{fy}} = v_{\text{iy}} + a_y\Delta t$$

Projectile Motion:

$$x_{\text{f}} = x_{\text{i}} + v_{\text{ix}}\Delta t \qquad y_{\text{f}} = y_{\text{i}} + v_{\text{iy}}\Delta t - \tfrac{1}{2}g\left(\Delta t\right)^2$$

$$v_{\text{f}x} = v_{\text{ix}} = \text{constant} \qquad v_{\text{fy}} = v_{\text{iy}} - g\Delta t$$

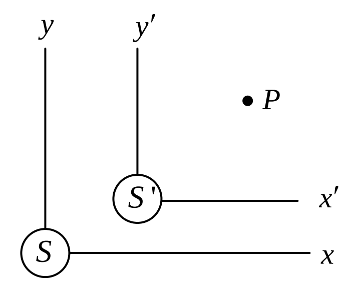
$$v_{\text{fy}}^2 = v_{\text{iy}}^2 - 2g\left(y_{\text{f}} - y_{\text{i}}\right)$$

Relative Motion:

$$\vec{r}_{PS} = \vec{r}_{PS'} + \vec{v}_{S'S}t$$

$$\vec{v}_{PS} = \vec{v}_{PS'} + \vec{v}_{S'S}$$

$$\vec{a}_{PS} = \vec{a}_{PS'}$$



Circular Motion:

$$\theta = \frac{s}{r}$$

$$\text{average angular velocity} = \frac{\Delta\theta}{\Delta t}$$

$$\omega = \frac{d\theta}{dt}$$

$$v_t = \omega r$$

$$\alpha = \frac{d\omega}{dt}$$

Uniform Circular Motion (constant ω):

$$v_t = \frac{2\pi r}{T} \qquad \omega = \frac{2\pi\text{ rad}}{T}$$

$$\theta_{\text{f}} = \theta_{\text{i}} + \omega\,\Delta t$$

$$a_r = \frac{v_t{}^2}{r} = \omega^2 r$$

Nonuniform Circular Motion (constant a_t):

$$a_t = \frac{dv_t}{dt} = \alpha r$$

$$\theta_{\text{f}} = \theta_{\text{i}} + \omega_{\text{i}}\Delta t + \tfrac{1}{2}\alpha\left(\Delta t\right)^2$$

$$\omega_{\text{f}} = \omega_{\text{i}} + \alpha\Delta t$$

$$\omega_{\text{f}}^2 = \omega_{\text{i}}^2 + 2\alpha\Delta\theta$$

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Force and Motion

$$\vec{a} = \frac{1}{m}\vec{F}_{\text{net}}$$

where $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots$

Dynamics I: Motion Along a Line

$$\vec{F}_{\text{net}} = \sum_i \vec{F}_i = m\vec{a}$$

$$\vec{F}_{\text{G}} = \left(mg,\ \text{downward}\right)$$

$$w = mg\left(1 + \frac{a_y}{g}\right)$$

A Model for Friction:

Static: $\vec{f}_{\text{s}} = \begin{pmatrix} 0 \text{ to } \mu_{\text{s}} n \\ \text{direction as necessary to prevent motion} \end{pmatrix}$

Kinetic: $\vec{f}_{\text{k}} = \left(\mu_{\text{k}} n,\ \text{direction opposite the motion}\right)$

Rolling: $\vec{f}_{\text{r}} = \left(\mu_{\text{r}} n,\ \text{direction opposite the motion}\right)$

Drag: $\vec{D} \approx \left(\tfrac{1}{4} A v^2,\ \text{direction opposite the motion}\right)$

Newton's Third Law

$$\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$$

Dynamics II: Motion in a Plane

$$\left(F_{\text{net}}\right)_x = \sum F_x = ma_x$$

$$\left(F_{\text{net}}\right)_y = \sum F_y = ma_y$$

$$\left(F_{\text{net}}\right)_r = \sum F_r = ma_r = \frac{mv^2}{r} = m\omega^2 r$$

$$\left(F_{\text{net}}\right)_t = \sum F_t = \begin{cases} 0 & \text{uniform motion} \\ ma_t & \text{nonuniform motion} \end{cases}$$

$$\left(F_{\text{net}}\right)_z = \sum F_z = 0$$

Impulse and Momentum

$$\vec{p} = m\vec{v}$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$J_x = \begin{cases} \int_{t_{\text{i}}}^{t_{\text{f}}} F_x(t)\,dt = \text{area under force curve} \\ F_{\text{avg}}\Delta t \end{cases}$$

$\Delta p_x = J_x$ (impulse-momentum theorem)

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots$$

$\vec{P}_{\text{f}} = \vec{P}_{\text{i}}$ (conservation of momentum)

Energy

$$K = \tfrac{1}{2} m v^2$$

$$U_{\text{g}} = mgy$$

$$E_{\text{mech}} = K + U$$

$K_{\text{f}} + U_{\text{f}} = K_{\text{i}} + U_{\text{i}}$ (conservation of mechanical energy)

$$\left(F_{\text{sp}}\right)_s = -k\Delta s \qquad U_s = \tfrac{1}{2} k \left(\Delta s\right)^2$$

Perfectly Elastic 1-D Collisions (m_2 initially at rest):

$$\left(v_{\text{f}x}\right)_1 = \frac{m_1 - m_2}{m_1 + m_2}\left(v_{\text{i}x}\right)_1 \qquad \left(v_{\text{f}x}\right)_2 = \frac{2m_1}{m_1 + m_2}\left(v_{\text{i}x}\right)_1$$

Work

$$W = \begin{cases} \int_{s_{\text{i}}}^{s_{\text{f}}} F_s\,ds = \text{area under the force-position curve} \\ \vec{F} \cdot \Delta\vec{r} = F\Delta r\cos\theta \quad \text{if } \vec{F} \text{ is a constant force} \end{cases}$$

$\Delta K = W_{\text{net}} = W_{\text{c}} + W_{\text{diss}} + W_{\text{ext}}$ (work-kinetic energy theorem)

$$\Delta U = U_{\text{f}} - U_{\text{i}} = -W_{\text{c}}\left(\text{i} \to \text{f}\right)$$

$$F_s = -\frac{dU}{ds}$$

$$E_{\text{th}} = K_{\text{micro}} + U_{\text{micro}}$$

$$\Delta E_{\text{th}} = -W_{\text{diss}} = f_{\text{k}}\Delta s$$

$$E_{\text{sys}} = K + U + E_{\text{th}}$$

$$K_{\text{f}} + U_{\text{f}} + \Delta E_{\text{th}} = K_{\text{i}} + U_{\text{i}} + W_{\text{ext}}$$

$$P = \frac{dE_{\text{sys}}}{dt} \qquad P = \frac{dW}{dt} = \vec{F} \cdot \vec{v} = Fv\cos\theta$$

Newton's Theory of Gravity

$$F_{M \text{ on } m} = F_{m \text{ on } M} = \frac{GMm}{r^2}$$

$$g_{\text{surface}} = \frac{GM}{R^2}$$

Circular Orbit: $v = \sqrt{\frac{GM}{r}} \qquad T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$

Physical Constants

$$g = 9.80 \text{ m/s}^2$$

$$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$$

$$M_{\text{earth}} = 5.98 \times 10^{24} \text{ kg}$$

$$R_{\text{earth}} = 6.37 \times 10^6 \text{ m}$$

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